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PRACTICAL ESSAYS

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BY ROBERTSON BUCHANAN, ENGINEER.

WITH

NOTES AND ADDITIONAL ARTICLES,

CONTAINING NEW RESEARCHES ON VARIOUS MECHANICAL SUBJECTS,

BY THOMAS TREDGOLD, C.E.,

MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS.

AND NOW

REVISED INTO A THIRD EDITION WITH ADDITIONS,

BY GEORGE RENNIE, ESQ. C.E. F.R.S.

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GENERAL PREFACE

TO THE

FIRST EDITION.

While my *Essay on the Teeth of Wheels* was in the Press, the idea occurred to me of publishing a series of Essays, all of a practical nature, on Mill Work, and other Machinery, each forming in itself a whole, at the same time that there would be a natural connexion, so that each Essay might again form a part of a general system. The Essay on the *Teeth of Wheels* I considered as the first of the series. The next in order that occurred to me was on the *Shafts of Mills.*

This is a subject of much practical importance; and it cost me much time and labour to collect and arrange facts respecting gudgeons and journals, with a view to deduce rules to guide the Millwright in a part of his business, which there is reason to suppose has hitherto in most cases been conducted at random. An Essay on this subject I accordingly sent to the press in 1808, but when a considerable part of it was printed off, an unfortunate occurrence to the printer and publisher prevented it until now from being finished.

Meanwhile I had at intervals gone on writing other Essays, including that *on the Shafts of Mills.* I also corrected the Essay on the Teeth of Wheels, and supplied some additional tables and a second appendix.

With a view to practical utility, I have endeavoured to adapt the style of these Essays to the comprehension of such operative mechanics as have not had the advantage of mathematical instruction; but at the same time I have given reference to authors for the demonstrations of such propositions as I found it necessary to introduce, in order to give such workmen some notion of the principles on which their work should be conducted.

For any repetitions, want of unity, and other imperfections, which will doubtless too readily appear in these Essays, I may offer the same apology which I did on a former occasion, that they were written at many different and distant intervals, occasioned by interruptions from professional and other engagements.

NOTE.—The Second Edition was superintended by the late Mr. Tredgold; and the principal facts noticed in it are incorporated in the following Preface.

PREFACE

TO THE

THIRD EDITION.

The Essays of Robertson Buchanan on Practical Mechanics have been long known, and duly appreciated by the public.

They consist of a series of treatises, seven in number, on several of the elementary parts of machinery; such as the Teeth of Wheels, published in 1808; on the Shafts of Mills, in 1809; and on Millwork and other Machinery, in 1814. The copious Index of the contents of the present edition, drawn up by Dr. Jamieson, sufficiently explains the nature of the work.

In perusing the Essay on the Configuration of the Teeth of Wheels, we are at once struck with its resemblance to the admirable treatise of Camus, published in 1782[a], and which the author duly acknowledges. The subject is divided into two parts; firstly, the principles as laid down by Camus; secondly, the application of these principles to different kinds of spur and bevel gear.

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[a] Cours de Mathématique.

The first application of the epicycloidal curve to the teeth of wheels is generally ascribed to Roemer, a Danish mathematician, in 1674[a], although De la Hire[b] claimed the merit, and demonstrated that if a tooth of either a wheel or pinion be formed by a portion of an exterior epicycloid, described by a generating circle of any dimensions whatever, the tooth of its follower will be properly formed by a portion of an interior epicycloid, described by the same generating circle. The object he had in view, was to secure a perfect uniformity of pressure and velocity to the machine, so that, in all positions, the wheels which transmit the power should act equally and similarly, and that the surfaces of the teeth, by touching in a point, should roll over each other when in motion, and thus avoid all friction, a desideratum hitherto impracticable to accomplish.

The general properties of the cycloid and epicycloid, and the modes of generating these curves, both geometrically and mechanically, have been given by various authors, and Buchanan applied the principles of Camus to the forms of the surfaces of the teeth of wheels and pinions acting against each other under different circumstances; such as the wheel and trundle, the wheel and pinion, the rack and pinion, conductor or conducted, external or internal, spur or bevel gear, so as to render them comprehensible by the general reader. The author next investigates the action of conical, or bevel wheels, under the different circumstances of the inclination of their axles, applying the same exterior forms of the teeth of spur wheels to the teeth of bevel wheels, with the exception that the curves should be a spherical epicycloid. The principles of bevel wheels had been already pointed out by De la Hire in the year 1666[c], although long known and applied previously[a], in the case of a conical trundle.

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[a] Wolfii Opera Mathematicis.

[b] Traité des Epicycloides, 1694.

[c] Memoires de l'Academie, 1666. 1669.

De la Hire[b] was not only among the first to apply the epicycloid to the configuration of the teeth of wheels, but he considered the involute of a circle, as the best of the exterior epicycloids, and which it may be proved to be, if we consider the generating straight line as a curve of infinite radius, and which would strictly apply to a pinion acting on a rack, and *vice versâ* to the teeth of a rack. Euler, in 1760, treated the case of the involute very generally[c].

Kaestner[d], in 1771, shewed a method of describing and applying the involute to the teeth of wheels. Professor Robison applied the involute to the wheels of a mill near Edinburgh, but the result was any thing but satisfactory. The same principle has since been advocated by Ferguson[e], Professor Airey[f], and Professor Willis, whose valuable paper appears in the Appendix.

We are also indebted to several continental writers, but particularly to M. Hachette[g], for his elaborate investigation of the curves most applicable to the teeth of wheels. Professor Airey states, "That in order that the mechanical effect which one wheel will produce upon another, may in all positions be the same, it is necessary that the line perpendicular to the surfaces of the teeth at the point of contact, intersect the line joining the centres at a fixed point, which divides that line into two parts, the ratio of which is the mechanical power; when this holds, the proposition of the angular velocity will be constant." Mr. Airey then demonstrates the case mathematically, and advises that the teeth be made to work a little before and after the line of centres, and thinks that a tooth formed by the union of an epicycloid and hypocycloid is preferable to any form whatever, for the line of action is always very nearly perpendicular to the radius, by which means, not only is the friction made much less, but also the strain upon the axles is considerably diminished. The same applies to bevel wheels and rack-work, with reference to uniformity of motion and action, which he conceives to be of far greater consequence than any diminution of friction, which can never be reduced to nothing, except the part of contact be always in the line of centres, a condition which may be satisfied only by means of an infinite number of curves, and amongst others by two logarithmic spirals, but the mechanical action, and the motion would be dreadfully irregular.

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[a] Bessoni, 1582. Theatrum Machinarum.

[b] Traité des Epicycloides, and Nove Comment. Petropol. 1754, 1755.

[c] Comment. Petropol. 1754, 1755.

[d] De Dentibus Rotorum Reg. Soc. Gottingensis.

[e] Sir David Brewster, edition of 1807.

[f] Cambridge Philosophical Transactions, Vol. II.

[g] Traité Élémentaire des Machines, 1811.

This question is now little more than one of mere curiosity, arising from the smallness of the teeth of wheels now made, and the greater perfection of workmanship in the materials, in consequence of the use of iron wheels, and the accuracy with which the teeth are formed and adjusted by the most simple method of templates and compasses; and the approximation of the form thus generated to the form presented by theory is very close. The determination of the strength of the teeth to the power to be transmitted, is given in the fourth chapter on the principles of proportioning the strength of the teeth of wheels.

The rule adopted by millwrights for finding the depth of the teeth from the bottom to the pitch line, and from the pitch line to the top of the tooth, is simply to multiply the pitch by 5, and divide by 9, and *vice versâ.*

The curves of the exterior and interior surfaces of the teeth are sometimes traced by means of a tracer fixed in the radius line of one of two pieces of board, the edges of which are cut out to suit the primitive circles of the wheels required; then, by fixing a template to one edge, and dividing the teeth accurately, the tracer will, by the rolling of the two circles, describe the curves required.

The usual mode of describing the teeth of wheels by arcs of circles, is admitted to approximate to the true curve, if the centre and radius of the wheel be determined correctly. In the best establishments, this is invariably done, and the result of many years' practice has proved the goodness of the system.

With respect to Buchanan's tables on the pitches of wheels, and the strength of the teeth with the corresponding numbers and horses' power moving at the pitch line at different velocities, it will be observed that the prevailing proportion is, that the pitch is about double the thickness of the tooth, and the length rather longer than the thickness, but three to four times the thickness is more usually adopted.

The investigation of the proper curves to be given to the teeth of wheels, by Professor Willis, has been added by way of an appendix to the concluding chapter on the Teeth of Wheels. The first section gives a succinct account of the curves adapted to practice, and shews, by way of corollary, that if for a set of wheels of the same pitch a constant describing circle be taken, and employed to trace those portions of the teeth which project beyond each pitch line by rolling on an exterior circumference, and those which lie within it, by rolling on its interior circumference; then any two wheels of this set will work correctly together. Professor Willis then shews how this can be accomplished, and then gives a form of increased strength to the backs of the teeth, but which are only suited to move in one direction. The second section of this paper shews how the practical approximation to the true form can be accomplished by arcs of circles, a form which approaches to modern practice. An instrument termed an *odontagraph*, together with tables for facilitating its use, and for forming cutters for shaping the teeth, is proposed, and a theory given of the nature of the motion which is produced by the pressure of one circular arc upon another, when disposed so as to work in the manner of teeth.

The Essay on the Shafts of Mills is divided into five chapters, containing a general description of shafts most employed in mill-work, and the strains to which they are subject from lateral stress and torsion; the strength and stiffening of shafts, journals, and gudgeons, with reference to the strength of materials, according to the experiments of different authors. The subject of torsion is briefly examined, in conjunction with lateral stress. It is shewn that, in general, the strength of a cylinder or solid axle to resist the force of torsion, is as the cube of its diameter, and that the length of a cast iron shaft has no influence on its resistance to torsion, whatever may be the exception with wooden shafts. The power of a cast iron shaft to resist torsion is calculated from Mr. Tredgold's formula, which considers the resistance the same as from the lateral stress. The Table of Shafts, at the end of the fifth Chapter, takes into consideration the two kinds of resistance.

The use of iron in machinery previously to its adoption in England, is evidenced by referring to the works of Ramelli, Bockler, and Bessoni, where there are representations of iron wheels, and portable mills and cranks of the same metal, but it was only used in this country about the year 1550.

Iron pipes came into use in France about the year 1672, and from the year 1782 to 1784, cast iron was used in machinery at Colebrook Dale, Rotherham, and at most of the great iron works in England. Cast iron wheels were also in use at Manchester, Liverpool, Netherby, and several other places.

The theory of torsion has been investigated by several writers, but with very little effect. Coulomb was the first to direct the attention of mathematicians to this kind of resistance. If a cylindrical body, such as a line or series of lines or fibres be suspended vertically, but having its upper end fixed, be turned round through any angle by the existence of some lateral force, and if its elasticity be not impaired, it will, after the deranging influence has ceased, return to its former position, and may perform this retrocession in a certain time. Professor Leslie[a], in considering this question, conceived the cylinder to consist of a series of thin discs, which, when twisted, each successive disc will make an angular advance till this accumulation at a certain distance amounts to a complete circuit, and such revolutions will be repeated at every like interval. The torsion is thus proportionate to the angle of deviation and the fourth power of the diameter of the cylinder, and inversely as its height.

If the exterior particles of a line or series of lines be overstrained, then their cohesion will be destroyed, and the line return no longer to its pristine state. If the strain be within the limits of the elasticity of the exterior particles, the oscillations will be perfectly isochronous.

The application of this principle by Coulomb[b] led to the construction of an exquisite balance for detecting and measuring the smallest forces. Thus, by establishing a connexion between the elastic and passive resistance of metals, and the resistance from torsion, we are led to an approximation to the true theory. The experiments of

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[a] Elements of Natural Philosophy.

[b] Memoires de l'Academie des Sciences, 1784.

Minard[a] and Désormes, Lagerhjelm[b], Bornet[c], Seguin[d], and Ardant[c], however, effected much towards the solution of the question.

The experiments of Savart[e] on the torsion of different bars show the following results:

1st. That the angles of torsion are in every case proportionate to the forces of torsion within the limits of elasticity.

2dly. That in bars of the same section, the forces of torsion are directly proportionate to the length of the bars, the length having little other influence than increasing the angle of torsion.

The experiments by Duleau, Banks, Dunlop, and by the Editor of this work, approximate pretty nearly the mean values.

On the dynamical effect of men and horses, Buchanan quotes the estimates of Desaguliers, Emerson, Smeaton, and Watt, as also a table in units of force from Dr. Young. Buchanan's own experiments of the effective power of men working in different positions, had already been published in the fifteenth volume of the Repertory of Arts, for 1801. Of the authors who have written upon this subject, the names De la Hire[f], Amontons[g], Lambert[h], Bernoulli[i], Coulomb[k], and Schulze, are most distinguished. De la Hire considered the question almost entirely with reference to the muscular powers of animals. Amontons with reference to their velocity. Deparcieux their weight and muscular action conjointly, and Lambert reduced the problem to an equation, which gives the relation between the weight, velocity, burthen, and path of inclination pursued by the animal. Bernoulli maintained that the quantity of fatigue was always proportional to the quantity of action, whatever be the nature of the work performed.

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[a] Annales de Chimie.

[b] Du Fer dans les Ponts suspendus.

[c] Poncelet, Mécanique Industrielle, 1839.

[d] Des Ponts en Fils de Fer, 1826.

[e] Annales de Chimie.

[f] Mémoires de l'Academie des Sciences, 1699.

[g] Mémoires de l'Academie des Sciences, 1709.

[h] Mémoires de l'Academie de Berlin, 1776.

[i] Mémoires de l'Academie des Sciences, 1760.

[k] Prix de l'Academie, tom. vii. 1753.

Coulomb proved the absurdity of this position, by shewing that the daily quantity of action is variable according to the circumstances under which it is developed, and by a series of valuable experiments, comprehending almost every case, he assigned values to each kind of action.

Schulze's investigations on the absolute and mean effect of human labour to lift, draw, and push weights, have furnished us with standards of value which may be safely taken as the average of continental labour.

The more modern investigations of Gerstner[a], Morin, Fourier, Devilliers, and others on the continent, and of Tredgold, Palmer, Sylvester, Bevan, M'Neill, and Field, in this country, besides many experiments which have been made by ourselves[b] on the extreme and mean powers of men and horses, have established sufficient data whereon to found

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[a] Treatise on Mechanics, translated from the German, 1834.

[b] The following is a brief statement of the dynamical effect of human and horse power severally applied to walking wheel-cranes, crabs, cranes, pile-driving engines, horse runs, &c., for raising different materials.

MANUAL LABOUR.

BUILDING MATERIALS.

	lbs.
1stly. One man in 67 journeys raised a weight of 16,342 lbs. (including the weight of himself) to the height of 30 feet in 10 hours, equal to a weight of . . . . raised 1 foot high per minute.	817

formulæ suitable to every case of the application of animal power to perform mechanical operations.

2dly. One man in 47 journeys raised a weight of 11,374 lbs. (including his own weight) to a height of 50 feet in 10 hours, equal to a weight of . . . . lbs. 974.8
raised 1 foot high per minute.

Ordinary Cranes.—Experiments made at the West India Docks.

3dly. The power of six men applied to a crane is capable of raising 224,000 lbs. 15 feet high in eight hours, equal to a weight of 1166.6
raised 1 foot high per minute by 1 man.

4thly. The power of 6 men applied to a crane is capable of raising a weight of 262,080 lbs. 12 feet high in 8 hours, equal to a weight of 1092
raised 1 foot high per minute by 1 man.

N.B. By experiment, the friction of these cranes varied from $\frac{1}{10}$th to $\frac{1}{14}$th of the absolute weight.

Ordinary Cranes.—Experiments made at the London Docks.

5thly. By the walking wheel-crane, worked by 6 men, a weight of 787,920 lbs. was raised 7 feet in 8 hours, equal to 1915
raised 1 foot high per minute by 1 man.

6thly. Again, by the walking-wheel crane, worked also by 6 men, a weight of 911,680 lbs. was raised 8 feet in 11 hours, equal to 1841
raised 1 foot high per minute by 1 man.

7thly. By 2 crabs worked by 6 men each, a weight of 728,000 lbs. was raised to a height of 16 feet in 8 hours, equal to . . 2012
raised 1 foot high per minute by 1 man.

HORSE POWER.

8thly. The dynamical effect of a horse power applied to a pile-driving engine worked by 2 horses, was found to be equal to a weight of 42,536 lbs. raised 1 foot high in 35 seconds, or a weight of 36,459
raised 1 foot high per minute per horse.

The Appendix to the Second Essay contains tables, by Tredgold, on the properties of materials, and the influence of alloys in increasing the tenacity of metals. These data have been further extended by the experiments of Messrs. Fairbairn and Hodgkinson on the relative strength of hot and cold blast iron[a], and on the compression of cast iron columns[b]. But our knowledge of the elastic properties of materials, the laws of the elongation and compression, and the effect of temperature upon their cohesion, is as yet but imperfectly known. The experiments of Rondelet, Dupin, Tredgold, Barlow, Bramah, Gerstner, and Adam Burg on the flexure and resilience of wood and iron have furnished some valuable facts on this subject, but it is to Messrs. Minard, Désormes, and Ardant that we are principally indebted, for determining the law of elongation by the direct tension of the fibres of wood and iron. Vicat showed, in the case of a cubical prism of lead, that the law of compression is constant from a constant augmentation of pressure. Peclet proved that for cast iron the molecular displacement of the crystals did not, in the first instant of compression, exactly follow the compression in proportion to the resistance, and our own experiments in the compression of several of the softer metals have shewn the

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9thly. Again, the power of a horse applied to working runs for raising earthwork up a run or inclined plane, the base of which was 60 feet, and the vertical height 40 feet, was equal to a resistance of 410lbs. travelling through a space of 72 feet in 1 minute by two horses, which is equal to . . . lbs. 14,760 raised 1 foot high per horse power per minute; a result very inferior to the last, arising from the inconstant nature of the work.

[a] Seventh Report of the British Association for the Advancement of Science.

[b] Experimental Researches on the Strength of Pillars of Cast Iron, and of other Materials.—Philosophical Transactions, 1840.

densities to have increased in a greater ratio than the compression.

The influence of temperature, so far as the temperature of the atmosphere is concerned, appears to exercise very little influence, but when carried beyond the limit of atmospherical temperature, the experiments of Messrs. Tremery and Poirier have shewn that, at a dull red heat, (450° Fahrenheit,) the tenacity of a bar of iron had lost one sixth of its original strength.

M. Savart[a] proved, by means of a series of ingenious experiments on the sonorous vibrations of different materials, the influence of time in the aggregation of the particles in cooling of substances, apparently homogeneoús; and Messrs. Vicat, Minard, and Désormes, and ourselves[b], have, by means of iron bars loaded to within the limits of their absolute strength, shewn that permanent set or loss of elasticity, and even rupture, takes place when influenced by time.

On the subject of Shafts and Couplings, a new era had arisen. The introduction of the textile fabrics in the country, by Lombe and others, and the inventions of Wyatt, Arkwright, and Watt, led to a new system of machinery. The necessity of producing high velocities occasioned a corresponding diminution in the dimensions of shafts, and those ponderous masses of wood, cast iron, and their enormous bearings and couplings, gave place to slender rods of wrought iron and light frames or hooks for suspending them. In like manner, wheels and pulleys of large diameters were replaced by pulleys and straps of moderate diameters and dimensions, and by uniting the pulleys in series of different diameters, and alternating their positions opposite to each other, a greater variety of velocities were obtained, and a great deal of friction and noise done away with, without taking into consideration the economy resulting from the lighter kind of machinery and the less quantity of power than formerly required to put the whole in motion. These improvements are in a great measure due to Messrs. Fairbairn and Lillie[a]. To use the words of Dr. Ure[b], "The method of increased velocities in the driving arms of factories is undoubtedly one of the most remarkable improvements in practical dynamics. It diminishes greatly the inertia of the mass to be moved, by giving to much lighter shafts and wheels the same momentum, and it permits the pulleys or drums which immediately impel the machines by straps to be reduced to a size much nearer to that of the steam pulleys fixed on the main axes of these machines."[c]

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[a] Annales de Chimie et de Physique, sur les Vibrations longitudinales des Corps, tome 65.

[b] On the Effects of Temperature on the Arches of Bridges. Transactions of the Institution of Civil Engineers, Vol. III.

The same improvements have taken place with regard to the couplings, which are now reduced to simple rings of wrought metal keyed to the circular ends of the abutting ends of the shafts.

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[a] In a letter to the Editor of this publication, Mr. Fairbairn dates the introduction of the new system of gearing from the year 1815: at that time, says he, "the shafts of our cotton mills were moving at 40 and 50 revolutions per minute, whereas at the present day we have none under 60, and as many as 300 and 350. The same number of revolutions are applicable and now in use for flax and silk. The extensive use of wrought iron for shafts, and the slide lathe, have given wonderful facilities to the production of shafts, and increased velocities and reduced friction by the transmission of great power through a comparatively small section. In some of the more recent mills of my construction, we have shafts only 2½ inches diameter overcoming the power of a 40-horse engine. Another improvement was our system of coupling, and the mode of suspending shafts from the main beams and ceilings of rooms, &c. In the first instance the couplings never get loose, and in the second, the shafts are strung like wires along the ceiling, and with small iron pulleys transmit the motion to the machinery without crowding the room or obstructing the light."

[b] Philosophy of Manufactures.

[c] Ibid.

As respects iron, cast iron pipes and cranks and pumps were used in the old London Bridge Waterworks, by Sorocold, in the time of Charles II., and mention is made of a cast iron wheel, 4 feet in diameter, which worked into a pinion 6 inches in diameter; and he adds, "If the teeth of the wheel be of brass, and the teeth of the leaves of the pinions of iron, the machine will work more equally." It seems generally believed, however, that Smeaton was the first to introduce cast iron wheel work in machinery at the Carron Iron Works, for the purpose of boring cannon, about the year 1769, although he had previously applied a cast iron axis for a windmill in 1754; but the founder's art was so imperfect, that Smeaton was obliged to proceed cautiously: and it was not until the years 1784 and 1785, when the Albion Mills were built, that cast iron was applied to all parts of machinery, and the late Mr. Rennie was the first to introduce accuracy in the forms of the teeth of wheels, by turning and adjusting the teeth, and causing the iron to work into wooden cogs. The subsequent progress which has been made in the later period of his life, introduced a new era in mill machinery, which, in point of accuracy and smoothness of workmanship, has not been exceeded, even under the automatic system of self-acting tools. Arkwright used iron bevel wheels and band pulleys, at the cotton spinning mills at Cromford and Belper, in 1775.

The Fourth Essay of Buchanan, on the Method of Disengaging and Re-engaging Machinery while in Motion, may be fairly included in the Third Essay on Couplings, with the exception of the fast and loose pulleys and friction clutches, which are found to be the simplest and best for engaging and disengaging machinery without shocks. The friction plate inclosed between two other plates, introduced some years back by ourselves, has been found to answer all the conditions in point of simplicity and effect required by a friction pulley, and does away with all the inconveniences of the cones.

The Fifth Essay on the Mechanism for equalizing the Motion of Mills, relates to the changes of velocity to which every first mover is subject, either from an increase or diminution in the supply of power, or where the power is uniform, from the increase or diminution of the resistances required to be overcome. This is accomplished by means of double or conical pendulums and balls, either for regulating the supply of wind, water, or steam, according to the quantity of action required.

The Appendix to the Fifth Essay is extracted from a paper communicated by Buchanan, in the year 1799, to the Philosophical Society of Edinburgh, and afterwards to the Editor of the Philosophical Magazine, on the Velocity of Water Wheels. The author negatives the conclusions of Banks, viz. that the velocity of an overshot wheel is as the cube root of the quantity of water it receives, by contrasting his own experiments on water wheels moving with their common velocity and half that velocity; and the result was, that the last half required just half the quantity that the first did; and this he confirms by two letters from Mr. Roberton, in which the author contrasts the maximum velocities of Smeaton and Banks's water wheel; and says that while Smeaton, by his maximum velocity of three feet, lost only one-fourth of the original effect, Banks, at his maximum velocity of one foot per second, reduced it to one half of that velocity, thus making the same quantity of water produce four times the quantity of work, or twenty times the quantity of work which Smeaton could perform with the same quantity of water. The continuation of Buchanan's Appendix shews that the mechanical effect depends on the wheel's diameter, the height of the fall, and on the velocity of the circumference of the wheel; and it is shewn that a water wheel will produce the greatest effect when the diameter of the wheel is proportioned to the height of the fall, so that the water flows upon the wheel at a point about 52¾ degrees distant from the summit of the wheel.

The subject of water wheels has been fully treated, both theoretically and practically, by many authors both on the continent and in this country; suffice it to mention the names of Pitot, Deparcieux, Lambert, Borda, Bossut, Eytelwein, Morosi, &c., &c., among the former, and of Smeaton, Robison, Fenwick, and Banks among our own countrymen; and in more modern times by Navier, Poncelet, Morin, Fourneyron, &c., and by several eminent mechanicians in this country.

Of the several classes of overshot, breast, and undershot wheels, a great diversity of opinion prevailed. By Pitot it was maintained that the float boards of undershot wheels should be continued in the line of the radius. By Deparcieux, that the floats should be inclined to an angle of 15 or more degrees. Bossut was of a contrary opinion. Borda, Bossut, and Robison considered that the maximum velocity of the wheel's circumference should be one third of the velocity of the current. Smeaton made the maximum velocity of the wheel between one third and one half of the current. Banks differs from all the authorities. Navier, Poncelet, and Morin[a], make it one half, whether the floats are on the line of the radius of the wheel, or curved. Again: as regards the diameter of the wheel, it was maintained by some that the diameter of the wheel should never exceed the height of the fall, and by others that the diameter should in all cases exceed the height of the fall, in which latter opinion Smeaton coincides; for, says he, "the higher the wheel is in proportion to the whole descent, the greater will be the effect." The same diversity of opinion existed relative to the proper number of floats; Pitot maintaining that the number of floats should be equal to 360° divided by the arc of the circle plunged in the channel, and Fabre and others, that the number should be as great as possible. Bossut found the best number of floats to be forty-eight. Smeaton from 24 to 40, for a wheel of 20 feet diameter, and Poncelet 30 to 36 floats for a wheel of 16 to 18 feet diameter[*].

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[a] Expériences sur les Roues Hydrauliques à Aubes, Planes, et sur les Roues Hydrauliques à Aujets, 1836. Subsequent experiments have raised the useful effect to 75 per cent. of the absolute expenditure.

In considering the action of the water on the vertical or radial floats of the common undershot wheel, M. Poncelet was of opinion that one of the causes of the small effect produced by undershot wheels, arose from the imperfect action of the fluid by shocks in entering the wheel, and by gravity in quitting it; whereas, if the floats were so constructed as to admit of the water entering and quitting the wheel quietly, the effect would be a maximum.

The inclination of the floats has long been a favourite project with mechanicians, but in so far as our own experience goes, little or no benefit has been derived from that arrangement; and in this opinion we are confirmed by the experiments of Bossut, who found the effect, at different angles, to be rather disadvantageous than otherwise. But M. Poncelet's curved floats produced the following results.

[*] The following experiment was made in the year 1820, by the editor of this work, upon two water wheels, each 19 feet 4 inches diameter, and 6 feet in width; one wheel had 40 floats, and the other 48 floats, and the fall 12 feet 2 inches in height. The machinery consisted of two wheels, and two pinions of cast iron, and two pairs of 4 feet diameter French burr stones.

The result was, that the wheel with 40 floats, ground, in 31 hours, 359 bushels of wheat; and the wheel with 48 floats ground, in 32 hours, 392 bushels of wheat. Hence, the wheel with 40 floats ground the same quantity of wheat (by experiment) with 1·43 per cent. less water than the wheel with 48 floats. It was also proved that 58·33 lbs. of water ground 12·74 lbs. of wheat per minute, and 650·62 cubic feet of water, falling one foot, ground one bushel of 60 lbs. weight of wheat, for the wheel with 40 floats.

1stly. That the maximum velocity of a wheel with curved floats, was 0·55 of the velocity of the stream.

2dly. That the dynamic effect for small falls and large openings, and 0·65 for large falls and small openings, and, generally speaking, the effect of the wheel with the curved float, compared with the effect of the wheel with vertical floats, was as 0·60 to 0·50 of the power expended.

M. Poncelet's researches on this subject have placed it upon its true basis. But it is to the experiments of the committee of the Franklin Institution[*] that we are indebted for the most detailed information we possess on the subject of water wheels.

The great defect of all former experiments, is the small scale upon which they have been made.

The experiments of the Franklin Institution were made with wheels of 20, 16, 10, and 6 feet diameter, respectively, and all the conditions of friction and other retarding forces were strictly attended to. The results prove that the maximum and mean effects of large wheels are greater with moderate velocities (double what Smeaton assigned as the maximum velocity for the mean circumference of a water wheel). The maximum and mean effects are diminished with an increased velocity, as the wheels are diminished in diameter.

	Maximum effect.	Correspond. velocity.	Mean effect.	Correspond. velocity.
[*] 20 feet diameter wheel: 128 Experiments................	·800	5·48	·784	6·01
15 feet diameter wheel: 88 Experiments..................	·692	5·87	·609	5·73
10 feet diameter wheel: 180 Experiments................	·643	5·88	·562	7·90
6 feet diameter wheel: 178 Experiments................	·567	7·59	·484	8·18

In the case of vertical water wheels, the water acts either by its impulse or gravity. But with horizontal wheels with inclined or curved floats, the motion is produced by the impulse and gravity of the water conjointly. The experiments of Messrs. Piobert and Tardy[a] on several wheels of this description, in the south of France, have given very feeble results, seldom exceeding one fourth of the power expended, and averaging much less. The reaction of a column of water upon the curved floats of a horizontal wheel has been found to be more effective, and the recent experiments of M. Morin[b] upon the Turbine of Fourneyron have shewn this new and curious machine, when properly constructed and moving at its maximum velocity, to be equally effective (if not more so) with the best vertical wheels. The effect of the reaction of a column of water had previously attracted the attention of Euler and Bernoulli in 1750[c], and a machine was proposed by Euler in 1754, upon the principle of the steam wheel of Hero of Syracuse. This machine was further improved by Mannoury D'Hectot[d], who constructed several in the neighbourhood of Paris, with bent tubes, originally suggested by Euler. The theory of the reaction of a column of water against the sides or circumference of an upright tube when allowed to flow through a hole or pipe fixed in its base, has often been investigated by philosophers. Daniel Bernoulli, in his Hydrodynamica in 1788, and John Bernoulli, in his Hydraulica, and in the St. Petersburg Transactions, proposed a very ingenious and elegant method of determining the impulse of a column of fluid falling perpendicularly upon a plane surface infinitely extended. The former considered the curve described by every filament of fluid as a channel in which a body moves, and which experiences at each point the action of a centrifugal and tangential force, which varies according to a given law. He then calculated all these forces, and found that the impulsion of a fluid against a horizontal plane is equal to the weight of a column of fluid, whose base is equal to the section of the fluid vein, and whose altitude is equal to twice the height of the fall due to the velocity of the fluid. The theory was afterwards very fully verified by a series of experiments. The question of water flowing from a cylindrical or any other shaped vessel was also treated by Maclaurin in his Fluxions, published in 1742. But the application of the principle of reaction to produce motion in machines, is due to Segner[a], professor of mathematics at Gottingen, who first constructed the machine, commonly known as Barker's mill[b]. The celebrated Euler made this machine of Segner the object of his investigation, in a paper published in the Memoirs of the Academy of Berlin, in the years 1750 and 1751, and shewed that, in order to produce the greatest effect, as well from the pressure as from the centrifugal force of the effluent water, it was necessary to curve the horizontal arms or tubes of the machine, so that the apertures should be in a line with the radius of the wheel. In 1754, he again turned his attention to the subject, and constructed a machine with two systems of wheels, the upper wheel or cylinder which received the water being fixed, and the lower one moveable and attached to a vertical axis; the water then flowed from the upper cylindrical to the lower conical wheel, and from thence through twenty small conical pipes fixed into its circumference, into the air,

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[a] Expériences sur les Roues Hydrauliques à Axe vertical, Paris, 1840.

[b] Ibid., Turbines Metz, 1838.

[c] Memoires de l'Academie de Berlin.

[d] Journal des Mines, 1813.

[a] Exercitationes hydraulicæ, Gott. 1747.

[b] It was called Segnersche Wasseraad, in Germany.

and caused the machine to revolve[a]. Mather de la Cour, and Waring[b], proposed to introduce the column of water from below at once into the horizontal arms; and a patent for a similar application of this principle has recently been taken out in Scotland. As regards the effect of these machines, opinions are various; Banks does not estimate it at above one third. Waring concludes that the greatest effect will be produced when the velocity of the orifice is half that of the issuing water, and that this effect will be nearly the same as that of a well constructed undershot water wheel.

Mr. Ewart[c] estimates the maximum effect to be considerably greater than the same quantity of water applied to an undershot wheel, but less than that which it would produce if properly applied to an overshot wheel. In 1824 M. Burdin invented a modification of Segner's machine, which he termed *turbine a réaction*[d]. It received the water in the upper part of a cylindrical drum, and allowed it to issue at its base through a series of helical channels wound round the outer surface of the drum, and from these through three pyramidal pipes issued horizontally into the atmosphere. This machine was found to produce an effect of from 65 to 75 per cent. of the power expended.

It was reserved, however, to M. Fourneyron to bring the turbine to its present perfection, and this he has accomplished, after the most unremitting perseverance of many years devoted to the subject. As before stated, the turbine consists of a horizontal wheel with curved floats, which are set in motion by the pressure of the water issuing from the centre to the circumference, or *vice versâ*, and which, having performed its office, quits the floats horizontally. But as the problem requires that the water should enter the wheel without shocks, and leave it without velocity, a peculiar kind of construction both of the wheel and floats is necessary, and it is the practical determination of the curves, derived from experience alone, which has led M. Fourneyron to the solution of the question. Most of the turbines established by M. Fourneyron in France and Germany have been submitted to the investigations of M. Morin, and the results have so far exceeded the expectations of men of science, as must eventually lead to a very considerable modification in hydraulic engines as first movers; and the report of the Commissioners, Messrs. De Prony, Arago, Gambey, and Savary, appointed by the Royal Academy of Sciences at Paris, on the experiments of M. Morin, entirely adopts his conclusions. M. Morin's experiments were made upon the turbines erected at Moussay, Mülbach, Lépine, Inval, and at St. Blaise *.

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[a] Journal de Rozier.

[b] Transactions of the American Philosophical Society of Philadelphia.

[c] On the measure of Moving Force; Vol. II. Memoirs of the Literary and Philosophical Society of Manchester, 1808.

[d] Annales des Mines, Tom. III., 1828. A more improved machine of this description erected by M. Burdin at Pontgibaud in France, called a Turbine à Évacuation alternative, when submitted to the test of the friction lever of Prony, produced an useful effect equal to 67 per cent. of the power expended, and performed the same quantity of work with one third of the water formerly required by a horizontal wheel worked by the percussion of the water.

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* The first series of experiments was made on the turbine of Moussay, in 1837. The diameter of the wheel was ·085 metres, or 33½ inches; the height of the fall was 7½ metres, or 24 feet 8 inches; and the number of turns made by the wheel varied from 76 to 240 per minute, according to the opening of the sluice; the relation between the effective and theoretical expenditure of water was 0·910. The maximum effect corresponded to a velocity of 180 to 190 turns per minute, and the useful effect was 0·690, or from 31 to 52½ horses' power; but at velocities of 140 and 230 turns per minute, this relation varied only from 0·650 to 0·690 of the absolute power expended, or a variation of only $\frac{1}{17}$, thus showing that the effect of the wheel was not altered materially by variations in its velocity. The wheel also was not affected when working submerged in tail water.

The wheel at Mülbach of only 2 metres, or 6½ feet diameter, and a fall of 3½ metres, or about 11½ feet, with a volume expended of 2½ cubic

The Sixth Essay relates to changing the velocity in machinery by means of lathes, alternating pulleys, alter-

metres, yielded a useful effect of 91 horses' power, or 78 per cent. of the expenditure. In this case the number of revolutions of the wheel varied from 50 to 100 per minute.

The turbine of Lépine, with a fall of 2 metres, or 6½ feet, and a velocity of from 60 to 100 revolutions per minute, yielded a power of 12 horses.

Finally, the turbine at St. Blaize, with a fall of 108 metres, or 354 feet, and a wheel under 22 inches diameter, made 2300 turns in a minute, and transmitted a force equal to 40 horses.

M. Morin concludes from his experiments:—

1stly. That turbines are equally adapted to great as to small falls of water.

2dly. That they are capable of transmitting an useful effect equal to from 0·70 to 0·78 of the absolute power.

3dly. That their velocities may vary very considerably from the maximum effect, without differing very sensibly from it.

4thly. That they will work nearly as effectually when drowned to the depth of one or two metres, as when free.

5thly. That consequently, they will make use of the whole of the fall when placed below the level of extreme low water.

6thly. That they may receive variable quantities of water without altering the ratio of the power to the effect.

And if to these properties be added simplicity, economy, and compactness, together with the facility of communicating high velocities to machinery without the intervention of wheels or pulleys, the turbine, he considers, ought to rank among the best hydraulic machinery in use.

At St. Maur, near Paris, four turbines have been erected for the purpose of grinding corn. Each turbine is 5 feet 2 inches in diameter, and 8 inches in thickness, and makes 50 revolutions per minute, driving 10 pair of stones 3 feet 4 inches in diameter at the rate of 200 revolutions per minute, each turbine doing the work of 40 horses' power.

At Corbeil, about 16 miles from Paris, M. D'Arblay has recently replaced two out of four vertical iron wheels upon the best principles, and replaced them with two turbines of similar diameters as those at St. Maur, and they are now working each 10 pairs of stones with the greatest regularity and satisfaction.

For further information on this subject, see Experiences sur les Roues à Axe vertical, par M. Arthur Morin, Metz, 1838. Also Versuche mit Horizontalen Wasserräden von Herrn Wedding und Herrn Carliczect, Berlin, 1837.

nating cones, friction wheels, mules, and double speeds, as applied to cotton spinning. The theory of mechanical motions has been very little examined until recently. Some of the early writers, such as Ramelli, Bessoni, Zonca, &c., describe the various continuous or alternate motions used in machines; but these motions were scarcely classified until 1794, when Monge produced his Elements of Machines for the use of the Polytechnic School. It was afterwards treated by Hachette[a], Lanz and Betancourt[b], Ampère[c] and Borgnis[d], Whewell[e] and Willis[f].

In the Traité de Mécanique of Borgnis, mechanical organs are divided into six classes.—1st. Receptors, under which are classed every description of machine moved by the power of animals.—2dly. Hydraulic receptors, such as vertical and horizontal wheels, machinery moved by the reaction or pressure of water, or by heat, vapour, or wind.

Under the second and third classes, or communicators and modifiers, are machines for transmitting and modifying motion, such as toothed wheels, eccentrics, inclined or curvilinear surfaces, chains, levers, pulleys, wheel eccentrics, screws, cams, &c., together with the machines for producing continuous, or variable, or alternate motions. The fourth class comprehends simple supports for maintaining vertical or horizontal axles, and rotative supports for maintaining motions of translation in one or more directions; and under the third class in this division are comprehended *tools*. The fifth order relates to regulators, such as fly wheels, governors, counter weights, horological scapements, eccentric wheels, curvilinear motions, friction levers, and conical pulleys and wheels, (alluded to by Buchanan.) The sixth, or last class, termed operators, comprehends every kind of machine for blowing air, for agitating liquids, or semifluids, or solids; for compressing substances by means of rollers and presses, or for stretching or extending metals: again, for operating by friction, such as grinding, polishing and filing. Fourth sub-division, again, by shocks, such as hammers, stampers, pile engines, wedges, &c. And under the last or fifth sub-division, come the operators by separation, such as rakes, scribbling and carding machines, knives, chisels, scrapers and boring tools.

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[a] Traité elementaire des Machines, 1812.

[b] Composition des Machines, 1808.

[c] Essai sur la Philosophie des Science, 1835.

[d] Traité de Mécanique.

[e] Philosophy of the Inductive Sciences, 1840.

[f] Principles of Mechanism, 1841.

A new work, however, by Professor Willis, has just appeared, the object of which, to use his own words, "has been to form a system that would embrace all the elementary combinations of mechanism, and at the same time admit of a mathematical investigation of the laws by which their modifications of motion are governed. I have therefore, says he, confined myself to the elements of pure mechanism, that is, to those contrivances by which motion is communicated purely by connexion of parts, without requiring the essential intermixture of dynamical effects. Instead of considering a machine to be an instrument by means of which we may change the direction and velocity of a *given motion*, I have treated it as an instrument by means of which we may produce any *relations of motion* between two pieces."[*]

The system adopted by Professor Willis is condensed, in a synoptical table of the elementary combinations of pure mechanism, into five divisions and three sub-divisions:

The first class comprehends motion by rolling contact, such as toothed wheels, annular wheels, racks, sectors, face gearing, hook gearing, and wheels in general for producing constant or variable velocities, or a combination of both.

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[*] Willis's Principles of Mechanism, 1841.

The second division includes motion produced by sliding contact, such as cones, screws, and worms, pin and slit levers, spiral, and other curved surfaces, under the different circumstances of constant or variable motion.

The third division shews how the same motion can be produced by wrapping connectors, such as guide pulleys, gearing chains, curvilinear pulleys, and fusees.

The fourth division includes the motion produceable by link work, such as cranks, joints, ratchet wheels, and intermittent link work.

And the fifth or last division includes reduplication by means of tackle of ropes, either parallel or unparallel.

The aggregate combinations and velocities, and adjustments of machinery, are treated with that ingenuity, precision, and order, which might be expected from the author. As regards the practical application of the various motions used in machinery, we need only adduce the early inventions introduced into the textile fabrics by Arkwright and Crompton, Wyatt, Hargreaves, of Watt, of Boulton, of Huddart, and others who have illustrated the history of mechanical inventions, not to mention invidiously inventors and men of science who in modern times have carried the art to the highest perfection.

The Seventh Essay treats of the framing of mill work and small machinery, according to the principles of Robison and Tredgold.

The object of framings in mill work, is to support and maintain the different parts of machines in their proper and relative distance, so that all the wheels shall work as smoothly as possible, and without shocks or vibrations; for this purpose it is necessary that the framing be made in conformity to the strictest rules of science; that is, with reference to the composition and resolution of forces, that the resultants of these forces should be represented by ties or struts; in short, that all pressures should be so distributed and resisted as to maintain a perfect state of equilibrium throughout. In obtaining a knowledge of these principles, it is necessary that we understand the properties of the materials with which we have to deal; their strength and stress in all positions, their durability, and their powers to resist decay. These properties will be found in our table of the strength of materials, and it is on the judicious distribution of these materials that much, if not the whole of the art of the mechanician depends. In all cases of tension, to use wrought iron, and in those of compression, cast iron; to observe the proper forms best suited to the pressure or tension they are to undergo, and to avoid as much as possible the use of framing in all heavy machinery, availing ourselves of masses of materials, such as stone, brick, concrete, or sand, in all cases where vibrations or shocks are to be resisted. For although cast iron, as a material, combines the advantages of stiffness, strength, and durability, and the facility of its being moulded into every possible form suited to the framing of small machinery, yet it is occasionally subject to break by unequal contraction in the cooling, and by blows or changes of temperature. Framing of wrought iron is, therefore, much used in marine steam engines.

The Eighth Essay, although not in the original edition of Buchanan's Essays, treats of the geometrical and practical methods for finding the centres of gravity of mill wheels, illustrated by examples of two, three, or four wheels keyed upon the same shaft. This subject has been so amply illustrated by Dr. Jamieson, but particularly in his *Mechanics for Practical Men*, that further comment is unnecessary.

A series of tables of square and cube numbers and roots, taken from Hutton's Course of Mathematics, closes the whole of Buchanan's work.

In the preceding observations we have confined our attention to a brief outline of the past and present state of our knowledge of the subjects treated by the Essays, and an imperfect review of the labours of those to whom we are so deeply indebted for the knowledge we possess of mechanical science. The labours of Buchanan are confined to the development of a few elementary principles connected with practical mechanics, excellent in themselves, but defective both in the extent and arrangement necessary to a complete system of mechanics.

The science of mechanics, which treats of the equilibrium and motion of solid or fluid bodies, and which, under its various divisions of statics, hydrostatics, dynamics, and hydrodynamics, comprehends the theory of action and reaction. Practical or technical mechanics, on the contrary, treats of forces as realities, and machines as material objects, capable of transmitting, regulating, or modifying motion. It also depends on a multitude of facts combined together, and establishes, by way of experiment, values to every element subservient to industry. Furthermore, it determines the value of animate and inanimate force, such as the force of men and animals; the force of gravity, such as weight, water, or other fluids; of elastic fluids, such as wind, steam, gas, &c., all of which forces are made sensible through the agency of machinery. By machinery we understand an assemblage of materials, particles or parts susceptible of receiving, communicating, or modifying motion. A machine may consist of a simple or compound lever, or assemblage of levers, revolving on a centre, such as band wheels or rollers, or any of the mechanical powers; or it may be divided into three parts,—the parts which receive, the parts which transmit, and the parts which communicate or perform the work: all these motions are effected by certain resistances which we term passive, such as inertia and friction, but which deduct or abstract from the absolute force in proportion to the perfection of and mode of applying the machine. Machines may be employed for displacing solid or fluid masses, for changing the forms of ductile and compressible materials by pressure, for separating masses of solid materials by friction, for producing changes of volume in solids by percussion, for separating solids into fragments by the same force, for mixing solids together by penetration, for separating filamentous substances from other extraneous substances with which they are interlaced, and for rearranging and interlacing them.

Whatever be the nature of the machine, it ought to be so combined that its useful effect be as great as possible; that it should be as simple in its construction as possible; that its parts should combine strength, stiffness, lightness, uniformity of action, and be as free as the nature of the resistance will permit from passive resistance; that it should act without shocks or sudden changes of motion; and that the communication between the power and resistance should be as simultaneous as possible. These important principles exact an intimate knowledge of the properties of materials, the modes of transmitting motion in all its varieties, of contact by means of the teeth, cams, and other curved surfaces, by bands and pulleys, or by direct or oblique pressure. Machines are the implements of manufacture, a word which applies to every product of art which is made by machinery, and with little or no aid from human labour. It forms a separate section, or rather a science, of automatic labour. It is the automatic science which has raised our country to its present elevated position in the world, as displayed in its cotton, silk, woollen, and flax manufactures; in its multitudinous and beautiful machines for shortening, multiplying, and even dispensing with the labour of man, evinced in the construction of automatic machines for creating the instruments of power whereby the elements are chained to perform their unremitting toil,—whereby the powers of wind and water, and steam and gas are rendered subservient to our uses, and ere long, let us hope, that mysterious power of electric magnetism, which seems to govern all. What have we not witnessed during the present century? If we turn to the triumphs of steam, we find that, whereas the duty of the pumping engines in Cornwall in the year 1808 was barely equal to 20 millions of pounds of water raised one foot high by a bushel of coals; in 1835, the duty performed by Mr. Austen's engines at the Fowey Consols and Lanescot mines, with an 80 inch cylinder, was upwards of 125 millions of pounds of water lifted by one bushel of coals weighing 94 lbs., and this has been confirmed more recently by the valuable experiments of Wicksteed[a]. Thus carrying out the ideas and principles of the great Watt, so fully detailed in his patent for 1782, and in the works of Robison[b], Tredgold[c], and Farey[d]. If we look to the marvels which have been effected in locomotion[e], both on sea and land, no longer subject to the uncertainty of the elements, the untiring machine impels the mighty fabric against the wind and waves, annihilating almost time and distance between the New and Old Worlds, while by its stupendous energies, and the art of the engineer, distances ae no longer measured by space.

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[a] Experimental Enquiry concerning the Relative Power and Useful Effect produced by the Cornish and Boulton and Watt Pumping Engine and Cylindrical Waggon-head Boilers. 1841.

[b] Robison, Article Steam Engine.

[c] Tredgold on the Steam Engine, 2 vols. Weale, 1838-40.

[d] Farey's Treatise on the Steam Engine.

[e] Comte de Pambour's Practical Treatise on Locomotive Engines, 1840.

Let us reckon upon the future, says M. Arago, in his historical éloge of James Watt.

"A time will come when the science of destruction shall bend before the arts of peace; when the genius which multiplies our powers, which creates new products, which diffuses comfort and happiness among the great mass of people, shall occupy, in the general estimation of mankind, that rank which reason and common sense now assign to it.

"Then Watt will appear before the grand jury of the inhabitants of the two worlds. Every one will behold him, with the help of his steam engine, penetrating in a few weeks into the bowels of the earth, to depths which, before his time, could not have been reached without an age of the most toilsome labour, excavating vast mines, clearing them in a few minutes of the immense volume of water which daily inundates them, and extracting from a virgin soil the inexhaustible mineral treasures which nature has deposited there. Combining delicacy with power, Watt will twist, with equal success, the huge ropes of the gigantic cable by which the man-of-war rides at anchor in the midst of the raging ocean, and the microscopic filaments of the aërial gauze and lace. A few strokes of the same engine will bring vast swamps into cultivation, and fertile countries will also thus be spared the periodical returns of deadly pestilential fevers, caused in those places by the heat of the summer sun.

"The great mechanical powers which had formerly to be sought for in mountainous districts, at the foot of rapid cascades, will, thanks to Watt's invention, readily and easily arise in the midst of towns, on any story of a house. The extent of these powers will vary at the will of the mechanician; it will no longer depend, as heretofore, on the most inconstant of natural causes, on atmospherical influences. . . . .

"Installed in ships, the steam engine will exercise a power a hundredfold greater than the triple and quadruple ranks of rowers . . . . . . and by the help of a few bushels of coal, man will vanquish the elements; he will play with calms and contrary winds and storms.

"Lastly: The steam engine drawing in its train thousands of travellers, will run on railroads with far greater speed than the swiftest race-horse."[a]

"And, in conclusion, let us quote the opinion of Sir John Herschel. On the importance of practical mechanics (he says, in his admirable treatise on the Study of Natural Philosophy,) "Practical mechanics is in the most preeminent sense, a *scientific art*, and it may be truly asserted, that almost all the great combinations of modern mechanism, and many of its refinements and nicer improvements, are creations of pure intellect, grounding its exertion upon a moderate number of elementary propositions in theoretical mechanics and geometry. On this head we might dwell long, and find ample matter both for reflection and wonder. But it would require not volumes merely, but libraries, to enumerate and describe the prodigies of ingenuity which have been lavished on every thing connected with machinery and engineering. By these we are enabled to diffuse over the whole earth the productions of any part of it, to fill every corner of it with miracles of art and labour in exchange for its peculiar commodities; and to concentrate around us, in our dwellings, apparel, and utensils, the skill and workmanship not of a few expert individuals, but of all who, in the present and past generations, have contributed their improvements to the processes of our manufactures."[b]

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[a] The annals of racing record several wonderful feats performed by race-horses.—Eclipse once ran 2 miles in 2 minutes, and on another occasion 4 miles in 6 minutes and 2 seconds. Flying Childers ran over the Newmarket course, 7420 yards, in 7½ minutes. Greyhounds have been known to run nearly as fast as race-horses.

[b] Preliminary Discourse on the Study of Natural Philosophy, pages 63 and 64.

ON TOOLS.

The subject of tools has been so amply illustrated by Mr. James Nasmyth, in the Appendix, that little remains to be added. By tools, we understand instruments employed in the manual arts for facilitating mechanical operations by means of hammers, punches, chisels, axes, adzes, planes, saws, drills, files, &c., by means of percussion, penetration, separation and abrasion of the substances operated upon; for all of which operations various motions are required to be given either to the tool or to the work. In handicraft work the tool receives motion, but in self-acting or automatic tools, motion may be given to either. In the case of the turning lathe, the tool remains fixed, and the object moves. In that of the planing machine, the tool may remain fixed, or be made to move according to the duty required to be performed. In almost all the other machines, such as the slotting, the key-grooving, the punching, the drilling, the nut-cutting, the teeth of wheels cutting, the boring, the screw-cutting machines, the tools receive motion. In the screw, bolt, and nut machines the tool is either moveable or fixed. The use of handicraft tools is coeval with the earliest periods of antiquity, and the recent researches of modern travellers have proved the ancients to have been acquainted with almost all the tools now in use[a]. The potter's wheel, the axe, the chisel, the saw, &c. attest the perfection to which the mechanical arts were carried by the Greeks and Romans, and subsequently in the arts of turning exhibited by the Dondi family, in the construction of their clocks and machines for spinning silk[b], in the middle of the

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[a] Manners and Customs of the Ancient Egyptians, by Sir Gardner Wilkinson, F.R.S., 1837.

[b] Histoire des Sciences Mathematiques, par Guillaume Libri, Vol. I., 1838.

13th century in Italy, and afterwards by Bessoni[a], De la Hire[b], De la Condamine[c], Grand Jean[d], Plumier, and Morin[e]. The three plates of Bessoni shew the different modes of turning and cutting screws of all sorts of fancy work. De la Hire shews how all sorts of polygons may be made by the lathe, and Condamine shews how a lathe may be made to turn all sorts of irregular figures by means of tracers moved over the surface of models and sculptures, medals, &c., and this is perhaps the first idea of the machine called the *Tour à Portrait.*

The work of Plumier enters most extensively into the art of turning, for he shews the construction of the lathe and its different parts, the art of making, hardening, tempering, and sharpening tools, the different kinds of motions which may be given to the lathe by means of wheels, excentrics, and models, and the different inventions relative to works of art which have been performed by the lathe, among which may be mentioned the moveable or slide rest. In the common rest which supports the tool, the idea of fixing the tool and pushing it in the direction of the parallel bed of the lathe, so as to cause the tool to traverse the work parallel to it, must have been obvious, and as this could have been easily effected by means of the screw and handle, it required little ingenuity to carry out the idea to its fullest extent, by constructing a rest to allow of the slide traversing the horizontal or vertical plane in any direction. The machine described by Plumier is neither more nor less than the slide-rest and planing machine combined: it consists of two parallel bars of wood or iron connected together at both extremities by bolts or keys of sufficient width to admit of the article required to be planed; a moveable frame being placed between the two bars, and motion being given to it by a long cylindrical thread, is capable of giving motion to any tool which may be put into the sliding frame, and consequently either causing the screw, by means of a handle at each end of it, to push or draw the point or cutting edge of the tool either way. If also motion be given to the tool by means of guides upwards or downwards, it is evident that any kind of reticulated form can be given to the work, as in the machine described by Plumier, which was intended for ornamenting the handles of knives, and which is called by Plumier, *Machine à manché de Couteau d'Angleterre*[a], from its having been an English invention. The *Machine à Conneler* described by Bergeron[b], a mode of grooving columns, is probably derived from the same source, from its resemblance to the English machine. We have given a plate and description of Nicolas Forq's machine in Plate 45 of the present work, and we have a drawing of a similar machine which was used in Germany many years back. The origin of the planing machine, in more recent times, is said to have arisen from the grooving or fluting of the drawing rollers used in cotton machines shortly after the introduction of Arkwright's inventions. The patent of Sir Samuel Bentham[c] in 1793, for various new methods for working wood, metal, and other materials, certainly contemplates the working of tools similarly to the tools employed in the planing machine. The patent comprehends giving all sorts of motion to tools, and the patent of Joseph Bramah[d], taken out in 1802, was for machinery for producing straight, parallel, and smooth surfaces and other materials requiring truth, in a manner more expeditious and perfect than can be performed by the use of axes, screws, planes, and other cutting instruments used by hand in the usual way.

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[a] Theatrum Machinarum, 1582.

[b] Machines Approuvées par l'Académie, 1719.

[c] Ibid. 1733.

[d] Machines Approuvées par l'Académie, 1733.

[e] L'Art de Tourner en perfection, 1749.

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[a] See pages 155, 156, and Plates 54, 55, 56, Plumier l'Art de Tourner. Paris, 1754.

[b] Manuel du Tourneur, Paris, 1816.

[c] Repertory of Arts, 1793, Vol. X.

[d] Repertory of Arts, 1802.

Billingsby[a], of Birkenshaw, took out a patent in 1802, for boring cylinders in a vertical position, although horizontal machines had their advantages. The boring of large cylinders by horizontal machines had long been practised by Smeaton, Wilkinson, Walker, Darby, and Boulton and Watt, and at Butterley and other great iron works; but it was only within the last few years that the vertical boring machines came into use.

As respects the introduction of the first planing machines which have been used during the present century, opinions are at variance. Messrs. Fox, of Derby, the eminent tool makers, state that the first machine employed for this purpose was constructed by Mr. Fox, senior, in the year 1821, for the purpose of planing the wrought and cast iron bars used in the lace machines. The machine was capable of planing an article 10 feet 6 inches in length, 22 inches in width, and 12 inches in depth; others give the credit of the invention to Manchester, and we ourselves put in our claim for constructing a planing machine with a moveable bed, urged by an endless screw and rack, and furnished with a revolving tool, so early as 1820, having several years previously employed the principle for grooving and planing parallel bars.

Mr. Bramah, in 1811, employed the revolving cutter to plane iron. Mr. Clement[b] states that he made a planing machine, for planing the sides of weaving looms and the triangular bars of lathes, previously to 1820. He afterwards constructed a beautiful machine for planing large and small work with the greatest accuracy. The bed moved on rollers, and the tools cut both ways. The beautiful work executed by this tool, for Mr. Babbage's calculating machine, evinces the perfection of its performance. It is thus by the aid of automatic tools that we are enabled to produce the greatest precision and identity of parts in machinery, which could never before be attained by machines made by hand labour; and it is hoped that, ere long, the chisel, the file, and the grindstone will be banished from the factory, and that nicety of parts and uniformity and silence of action, blended with the science of construction, will eventually supersede the expensive and imperfect construction of the handicraft system.

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[a] Repertory of Arts, Vol. II., 1803.

[b] 46th and 49th Volumes of the Transactions of the Society of Arts.

We might enlarge upon this subject, by detailing the nature and properties of the materials required for tools; the forging, hardening, and tempering of them; the velocities at which they should be made to move through the different materials, such as wood, iron, brass, copper, and tin. We might give the principles of the action of the different machines employed to produce different effects[a]; but we have exceeded our limits, and it only remains to express our great obligations to the several gentlemen who have so liberally assisted us on the present occasion. To Professor Willis, for his article on the Teeth of Wheels. To Mr. James Nasmyth, for his Paper on Tools, and his numerous and beautiful drawings of the tools which bear the name of Nasmyth and Gaskell. To the late lamented Mr. Francis Bramah, we owe the original drawing of the first slide rest of his father, in 1794, the work of the late Mr. Maudslay, and it is yet in use; and also for the drawing of the lathe for turning spheres. To Mr. Fairbairn, for his plate riveting and punching machine, and for his advice and assistance on several occasions. To Mr. Whitworth, for the information we have derived from his various pamphlets on plane metallic surfaces, and on the proper mode of preparing them; likewise, on an uniform system of screw threads[a]. To Messrs. Fox, for their screw-cutting machine, and other information. To Messrs. Benjamin Hick and Son, of Bolton, for the liberal assistance of the drawings for the plates which bear their name; and we take this opportunity of noticing their ingenious machine for cutting the teeth of the largest sized wheel used in mill-work, and their mandril for holding rings; their steel belts, as a substitute for leather bands, are used very successfully. To Mr. Francis Lewis, of Manchester, we are equally indebted for the drawings of the different machines, placed by that gentleman at our disposal. To Messrs. Maudslay and Field, for the liberal present of the drawing of their self-acting punching machine, by which accuracy is insured in the heretofore neglected art of boiler-making; it is one among the proofs of the high state of excellence to which those gentlemen have brought the mechanical arts in this country. A table of references, and an ample description of the different tools, by our assistant, Mr. George Pinchbeck, will, we trust, explain the different details. With respect to the plates, it is sufficient to state, that they are engraved by Lowry, a name too well known to need further comment. The liberality with which the whole has been got up by its spirited publisher, will, it is trusted, be acceptable to the public.

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[a] Experiments of M. Morin, on the Measures of the Dynamic Effects of Human and Animal Power, and on Machines in general.

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[a] On an Uniform System of Screw Threads. 8vo, 1841.

PUBLISHER'S ADDRESS.

The production of Works specially devoted to Engineering is, in this country, frequently attended with difficulty, not arising from the scarcity of subject-matter or the disinclination of practical men to facilitate its arrangement, but from their inability to find time to render their willing aid. Delay, as in the instance of the present work, is in consequence unavoidable. A publisher's risk is increased in no trifling degree, when he ventures upon publications of a scientific character unaided by an author or editor of experience in the matters of which they treat; but it is his duty to select such useful and novel subjects as shall not only be of practical help to the engineer, but afford a clear view of elementary principles to the student; and as there are now amateurs in Engineering as well as in other departments of art, such works are peculiarly acceptable.

Keeping this twofold object steadily before me, it has been my constant endeavour for many years to render works of practical reference as complete as possible, especially by an adequate number of engraved illustrations of examples; and it is with grateful feelings that I acknowledge the liberality of many gentlemen, whose names appear as contributors of drawings in my numerous published works, some few of which are here mentioned, together with an account of their sales: viz.—Tredgold on the Steam Engine, 2300 copies since October, 1838; Public Works of Great Britain, 975 copies within the same period; Papers of the Royal Engineers, 1000 copies of each of the volumes 3 and 4; 1750 copies of the two editions of Comte de Pambour's Practical Treatise on Locomotive Engines; 870 copies of the work on Bridges; 500 copies, in a few months, of the work of Mr. Clegg, Jun., on Coal Gas; and within one month upwards of 500 of Mr. Wicksteed's Experiments on the Cornish Engine were sold, which, with many others, are testimonies of the esteem in which such works are held at the present time.

In the present instance it affords me much pleasure gratefully to acknowledge and publicly state the liberality of Mr. George Rennie, the editor of this work, who, although having multitudinous professional engagements, has (ardent in the love of his art) found the necessary time for the arrangement, the addition to, and editing of this new edition. This has been done GRATUITOUSLY, and it is hoped that the Subscribers, in receiving the work consequently so much cheaper, will, in the acknowledgment of its utility, respond to the Publisher's thanks now expressed for the kindness conferred.

December 1, 1841. JOHN WEALE.

LIFE OF BUCHANAN.

We are indebted to the late Thomas Tredgold for whatever is known of the life of Buchanan. It was furnished by his friends, and though brief, the life of a man of genius is always interesting.

Robertson Buchanan was born on the 14th of July, 1769, at Glasgow, and was connected by birth with some of its principal citizens. His father was nephew to Neil Buchanan, who, in the year 1768, represented Glasgow in Parliament; and his mother was daughter of Arthur Robertson, who for many years was chamberlain of that city. Buchanan lost his mother at his birth, and his father when he was only fifteen. His father had not been fortunate in business, and the son was left unprovided for, but he had already shewn some talent for drawing and mechanics, which induced his maternal uncle to place him with a house-carpenter at Glasgow. The genius of Buchanan sought its native field in a short time, for we afterwards find him working with a millwright, and subsequently crossing the border for London. After a short time he quitted the metropolis, returned to Glasgow, and commenced business there as a millwright; in the year 1791 he gave it up to take the management of the new cotton-mill then building at Rothesay in the Isle of Bute. There he invented his pump which is not liable to choke, and for which he obtained a patent in the year 1796. In the same year he wrote some papers, which were published in the Repertory of Arts and Manufactures;—one on the improvement of cattle mills, another on preventing carding machines from injuring the health of those employed to attend on them.

He left Bute in the year 1801, much impaired in health by the anxiety of a responsible situation in a losing business, and returned to London, with a view of deriving some benefit from the pump he had invented; but in this he never succeeded. He was introduced, however, to Count Rumford and Professor Pictet, by whom his attention was directed to the heating of rooms, and in the year 1807, he published an "Essay on Warming Buildings by Steam." He had previously been engaged in preparing the "Essay on the Teeth of Wheels," but when a considerable part of it was printed off, an unfortunate occurrence to the printer and publisher delayed the publication until the year 1808. In the year 1810 he published his work on heating buildings in an improved form, with the title of "Practical and Descriptive Essays on the Economy of Fuel and Management of Heat." In the year 1814 appeared the Six Essays on Mill Work, which, with that on the Teeth of Wheels, constitute the present work.

In the year 1816 he published a practical essay on propelling vessels by steam, a work full of new views and principles in that most important art, and the commencement of a new era of civilization in the annals of society at large. He also contributed the articles "Cotton-spinning" and "Arkwright" to the Edinburgh Encyclopædia, besides several papers on less important subjects.

He died in the 47th year of his age, at Creech St. Michael, in Somersetshire, on the 22d of July, 1816.

He was a man of amiable character, with a strong sense of religious and moral duty, and was greatly respected by all that knew him. His knowledge in mechanics was very extensive. He was a close and accurate observer, extremely assiduous in collecting every fact or experiment which came under his notice, and he was unquestionably one of the few practical men who have shewn inclination, or sought leisure, to reason on facts in general with accuracy and judgment, and always with a view of rendering the information thus acquired, an available source for unforeseen emergencies. Buchanan was happy in the choice of popular subjects, and he fully compensates for want of system in handling them by the variety and utility of particulars no less interesting than abundant, whether learned from his predecessors, or derived from contemporaries.

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* Holtzapfel, of Charing Cross, makes this instrument.

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ON

THE TEETH OF WHEELS.

ESSAY I.

PREFACE.

Led from situation, as well as curiosity, to attend very minutely to some parts of practical mechanics, one of the objects which early attracted the notice of the author of the following short Essay, was the figure of the Teeth of Wheels. He observed, that, in forming these teeth, workmen followed rules for which they could assign no satisfactory reason. Nor did he then find in books the information he wanted: the subject seemed to him to require a detail and simplification, which no English writer, with whom he was acquainted, had given it. Afterwards, indeed, he found that some French mathematicians had treated it with much attention. But their works, though sufficiently clear to those who have studied mathematics, are too abstract to be of general utility. In the following Essay, therefore, such an elucidation of the subject has been attempted, as might render it plain to the operative mechanic—an object, which will appear the more important, the more we consider the great variety of useful purposes to which wheel-work is applied.

De La Hire and Camus are the two French writers, who have treated most extensively this branch of mechanics.—From the work of the latter, who has written more accurately, and more fully, the author has borrowed largely; nor has he scrupled to take from others, whatever he found to suit his purpose, and to make the fullest use of the communications of his friends.

Of the methods followed, it will be sufficient to remark, that the subject naturally suggested these two general divisions—First, the Principles of the Configuration of the Teeth of Wheels:—Secondly, the application of these to practice.

The first chapter contains the Principles—the second, their Application, with certain modifications—1st, to *Spur Gear*, under which are comprehended the *Wheel* and *Trundle;* the *Wheel* and *Pinion;* the *internal Pinion*, and the *Rack* and *Pinion*.—And, 2dly, to *Bevel Gear*.

A third chapter is added, which contains a manner of forming *Spur Wheels*, upon principles somewhat different from those considered in the preceding chapter.

In the following pages, no pretensions are made, either to invention or profound investigation. The writer has studied perspicuity alone, and will have completely attained his object, if he has only been fortunate enough to give such a view of the various kinds of teeth, as will enable the artist to form some judgment of their respective merits, and to execute any of them with accuracy and ease. For this purpose it has been his aim to divest every part of the subject of obscurity, and to accommodate it to those who possess not the advantages of a mathematical education. But he is far from saying, that they will not find some difficulties, particularly in the first chapter; nor will they, perhaps, fully understand the truths it contains, till they see their relation to practice pointed out in the second. He found, that without becoming exceedingly prolix, there was no avoiding the use of some mathematical terms, but of these he has given definitions, either as the terms themselves occur, or at the conclusion of the Essay *.

---

* This Preface was written several years before the translation of Camus was published.

GENERAL DEFINITIONS.

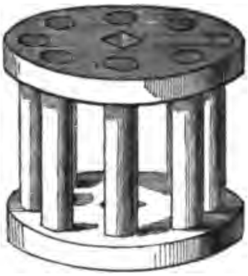
I.

1. **When** two toothed wheels act upon one another, the greater is called the *Wheel*, and the lesser the *Pinion*.

II.

2. Instead of the pinion, the trundle is sometimes used, such as is here represented. It is likewise known by the names of lantern and wallower.

Fig. 1.



III.

3. As pinions and trundles are employed for the same purposes, when the action of two wheels is spoken of, in general, the trundle is comprehended under the name pinion.

IV.

4. The teeth of wheels and of pinions, are comprehended under the general term, *Teeth*. Where the teeth are of the same piece with the body of the wheel, they are called, properly, *teeth*; when they are each of a particular piece, they are called *cogs*. The teeth of pinions are called *leaves*, and those of a trundle *staves*.

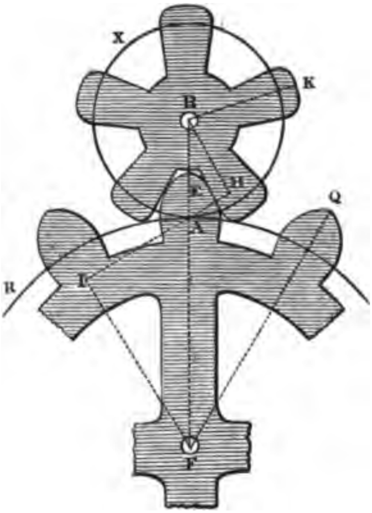
V.

5. When the action of wheels is spoken of in general, under the name *teeth*, are comprehended teeth, (properly so called,) *cogs*, *leaves*, and *staves*.

VI.

6. The straight line B F, which joins the centres B F, of a pinion and wheel, which act together, is called the *line of centres*.

Fig. 2.



VII.

7. When the line of centres B F is divided into two parts, A B, A F, proportional to the number of the teeth in the wheel, and in the pinion, these two parts, A B, A F, are named *proportional radii*.

It may be proper in this place to show, in what manner the line of centres is to be divided in the proportion of the number of teeth in the wheel and pinion; and for this purpose, we shall denote the length of the line of centres by l; the number of teeth in the pinion by p, and the number of teeth in the wheel by w; then by the definition, the line l is to be divided into two parts, having the ratio of p to w.

Let $x =$ the lesser segment, and $y =$ the greater. Then we have

$$p\,y = w\,x, \text{ and } x + y = l;$$

but by division and transposition, we obtain

$$y = \frac{w\,x}{p}, \text{ and } y = l - x;$$

and by comparing these values of y, we get

$$(p + w)x = p\,l, \text{ and therefore it is } x = \frac{p\,l}{p + w}.$$

and in like manner it is $$y = \frac{w\,l}{p + w}.$$

RULE

For the proportional radius of the pinion. Multiply the length of the line of centres by the number of teeth in the pinion, and divide by the number of teeth in both the wheel and pinion.

For the proportional radius of the wheel. Multiply the length of the line of centres by the number of teeth in the wheel, and divide by the number of teeth in both the wheel and pinion.

VIII.

8. If from the centres B F are described, with the proportional radii, the circles X A, R A; these circles represent two cylinders, which touch in the point A as if they had teeth infinitely small, or as if one of them were conducted by the other by contaction only. These circles I shall call *proportional circles;* or, as they are termed by millwrights, *pitch lines.*

IX.

9. The right lines, B K, F Q, drawn from the centres of the pinion and wheel, to the extremities of their respective teeth, are called *real radii.*

CHAPTER I.

OF THE PRINCIPLES OF THE CONFIGURATION OF THE TEETH OF WHEELS.

10. In the construction of machines, the proper formation of the teeth of wheels is an object of much importance. Though experience may often enable the merely practical mechanic to approach, in this respect, to some degree of perfection, yet, being ignorant of principle, his work is always conducted with uncertainty, and he generally produces machines expensive in working, and defective in regularity, effect, and duration.

For when the acting parts of a machine are not truly formed, it may be so loaded as just to be in equilibrio with its work in the most favourable situation of its parts, but when it changes into a less favourable situation, the machine will stop, or at least, stagger, hobble, or work unequally.

The best figure, therefore, which can be given to the teeth, is that which shall cause them always to act equally and similarly, in situations equally favourable, and which shall consequently give the machine the property of being moved uniformly by a power constant and equal; or, in other words, ensure an uniformity of pressure and velocity.

Were the teeth of wheels infinitely small, their action would be regarded as that of cylinders, simply touching, having the property required. The finite and sensible teeth given to wheels will, therefore, be of the most advantageous figure, when one wheel conducts another, as if they simply touched; or when their pitch lines have in every part of their revolution equal velocities.

That teeth have this property, when formed in a certain manner, will be evident from the following proposition and its connections*.

PROPOSITION.

11. When teeth are of such a form, that a perpendicular H E I† (Fig. 2. p. 2.) drawn to the tangent to the edge of the tooth in the point of contact E, cuts the line of centres at the termination A, of their proportional radii, *their pitch lines shall have in corresponding places, equal velocities,* whether the wheel drives the pinion, or the pinion the wheel; that is to say, that they will move each other as if they merely touched‡.

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* The manner of forming teeth of wheels here referred to by our Author, would ensure an equable communication of power or motion in the imaginary case when the rubbing parts have no sensible friction; but in no other; except it be possible to contrive a practicable form for teeth having the property of rendering the friction uniform during the action of each pair of teeth: this has not yet been accomplished. Hence it appears that practical men have not without reason been doubtful of the advantages of the kind of teeth proposed by mathematical writers; for that the friction of teeth is not uniform, Dr. Young has proved in a letter, which forms a valuable part of this Essay, (see Art. 66—71.) And we have a practical proof of the same thing in the unequal wear of teeth, (see Art. 40.) The best means of avoiding the inequality produced by friction seems to be, to make the teeth "as small and as numerous as is consistent with strength and durability," (Art. 65.) These limits, with respect to strength and durability, I will endeavour to establish in the additions to Art. 121, and those following it. And since, in adopting the principle of short teeth, the curved surface of each tooth will become so small that a circular arc may be employed instead of the proper curve, we shall, in the additions, point out the mode of describing circular arcs to answer this purpose.

† For the manner of drawing this perpendicular, see Art. 18.

‡ This being a fundamental proposition, it is of importance that it should be well understood; we shall therefore, in this note, attempt a popular illustration of it.

It is demonstrable that the line H B (Fig. 2.) has the same proportion to the line I F which A B has to A F. For since B H and F I are parallel, each of them being perpendicular to H I; it follows that the triangles A B H and A F I

We shall now proceed to show, that the epicycloid gives the property to the teeth of wheels required in the preceding proposition, and shall begin with some definitions respecting that curve.

Before we proceed with our Author, it will be an advantage to examine this proposition more particularly.

12. Let A H (Fig. 2.) be the direction of the force of the wheel to turn the pinion, and B H a line perpendicular to A H, drawn to the centre of motion B. The effect of the force to turn the pinion will be directly as the distance of its direction from the centre of motion, or as H B. Also, the angular velocity generated will be inversely as the distance of the direction from the same centre, or as $\frac{1}{\text{H B}}$. Consequently, the quantity of motion communicated to the pinion is as $\frac{\text{H B}}{\text{H B}}$; that is, in an invariable ratio; but by the same reasoning it may be proved that the force of the wheel at A is invariable; and therefore, the pinion will be moved in the same manner as if it were moved by contact at A, when H A is perpendicular to the common tangent of the surfaces in contact at E.

The same may be proved when the pinion drives the wheel. But this, as well as the more detailed investigations of Camus (on the Teeth of Wheels, Art. 521.) and his followers, neglects the effect of friction. Let the effect of the friction of the surfaces be represented by x, when the pressure and velocity of these surfaces are each equal

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are similar, or equiangular; but the sides about the equal angles of equiangular triangles are proportional: therefore, it is

H B : I F :: A B : A F.

Now let us suppose H B and I F to be levers, and H I a string, the one lever pressing from the other, would act upon it with just the same force that the pinion and wheel do at the point A, where the pitch lines touch; or, in other words, as if the circle X acted on the circle B, by means of a string, as pulleys do on each other by a band.

to unity, or 1; then the ratio will be $\frac{HB}{HB(1-x)}$, or as $\frac{1}{1-x}$; which is invariable only when the friction is invariable. But when the teeth are very short, and formed so that the motion would be uniform were the friction uniform, it is perhaps the best practical method of forming teeth.

DEFINITIONS.

I.

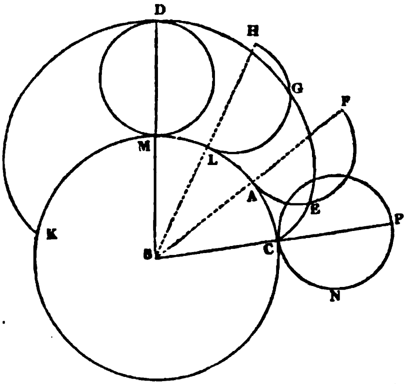
13. If upon the same immoveable plane are placed two circles, CNP, CALMK, (Fig. 3 and 4.) which touch each other in the point C, and the former, with a supposed style or tracer in its circumference at the point C, is made to revolve round the circumference of the latter, the style C, during the revolution, will describe upon the plane CALMK the curve CEGDK, which is called an *epicycloid.* The *epicycloid* therefore, is a curve generated by a point in one circle revolving about another, either on the concavity or convexity of its circumference, and thus it differs from the common cycloid, which is generated by the revolution of a circle along a straight line. The cycloid, however, has some times been assimilated with the epicycloid, by considering the straight line as the circumference of a circle of which the diameter is infinite.

II.

14. The circle CNP, which, in revolving describes the epicycloid, is called the *generating circle of the epicycloid,* and the arc CALMK of the immoveable circle, upon which the generating circle revolves, is called the *base of the epicycloid.*

EPICYCLOIDS are distinguished into two sorts, *exterior* and *interior.*

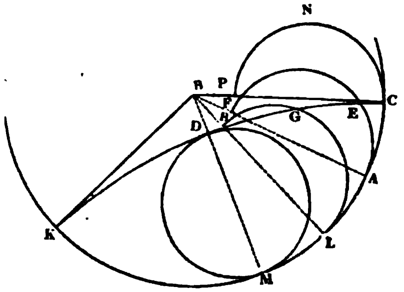
Fig. 3.



III.

15. When the generating circle revolves without the circle of its base, as in Fig. 3, the epicycloid is called an *exterior epicycloid.*

Fig. 4.



And when the generating circle rolls within the circle of its base, as in Fig. 4, the epicycloid is called an *interior epicycloid.*

COROLLARIES.

I.

16. As the generating circle in revolving from its first situation, CNP, to different portions, AEF, LGH, &c.; applies

successively all the parts of its circumference to those of its base, it is evident the base, CALMK, of the epicycloid is equal to the circumference of the generating circle CNPC, and each such portion, as CA, or CL, &c., of the base, is equal to each part EA, or GL, of the circumference of the generating circle.

Hence a method of drawing the epicycloid, by describing the circles AEF, LGH, &c., which have all the same radii as the generating circle CNP, and touch the base CALMK in any points A, L, &c.; and by making the length of the arcs AE, LG, &c., taken from the points of contact with the base, equal to the arcs AC, CL, &c*.

Having thus determined as many points, E, G, &c., as may be necessary, the curve CEGDK, which shall pass through them and the point C, where the supposed style of the generating circle was supposed to begin its tract, shall be an epicycloid †.

II.

17. When the generating circle CNP revolves within the circle of its base, (Fig. 5,) and has for its diameter the radius BC of its base, the point C, the place of the style during the revolution of the generating circle, will always continue in the diameter CBK. *Hence the epicycloid described by the style* C *is a straight line, and a diameter of*

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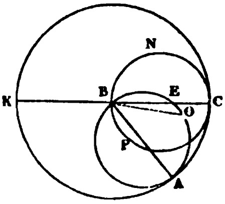
* In practice, this is most easily done, and with sufficient accuracy, by dividing each arc of the base, as at AC, into a number of small equal parts, and by setting off the same number upon each arc of the generating circle, as at AE.

† To draw the epicycloid mechanically, make the circle of the base and the generating circle of wood, and having fixed a tracer in the circumference of the generating circle, let the base remain at rest, and the tracer, during the rolling of the generating circle, will draw an epicycloid. In order to make circles move with more accuracy, a small piece of tape may have one of its ends nailed to the circumference of the one circle, and the other end to the other circle.

*the circle of its base**; and the circumference of the generating circle C N P being half that of the base, the commencement C and termination K of the epicycloid, must divide the circumference of the base into two equal parts, and the diameter A B of the generating circle being half that of K C of the base, when the generating circle is in the middle of its progress, the point C must be in the centre of the circle of the base C (or coincident with B); hence we have a point C at the origin B, in the middle, and K at the end of the epicycloid, which all lie in C K, the diameter of the base, and the whole epicycloid may be considered as coinciding with C K, the diameter of the base†.

FIG. 5.



III.

18. When the generating circle of the epicycloid, as in Fig. 6, is in any position, A, E, B, touching the circumference of its base in any point A, *a straight line, drawn from*

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* Upon this principle a parallel motion has been constructed. It is used by Messrs. Fenton, Murray, and Wood, in some of their smaller steam engines.

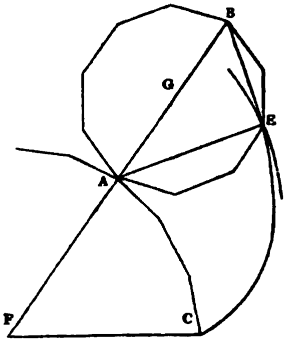
For a short account of it, see Gregory's Mechanics, vol. ii. p. 265.

† It would carry us too far into mathematics for many readers, were we strictly to demonstrate, that every point of the epicycloid must lie in the diameter of the base; what is said, however, will satisfy them of the truth. The mathematical reader will find a demonstration of this in "Cours de Mathematique, par Camus," iv. No. 538; or English Translation of that part which treats of the Teeth of Wheels, and from which Buchanan borrowed largely in this part of his work.

the point of contact A *to the point* E, *actually describing the epicycloid, will be perpendicular to it.*

This will be evident by supposing the circles to be polygons, having a great number of sides. For when turning on any of the summits, the tracer describes a small part of a circle from that summit as a centre, and will consequently be perpendicular to it.

Fig. 6.

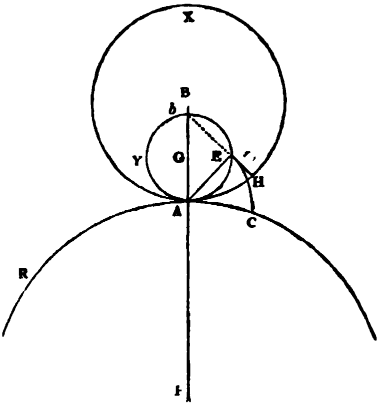


IV.

19. Let us imagine in the same plane three circles, R, X, Y, Fig. 7, which touch in the same point A, and which consequently have their centres, F, B, G, in a straight line, and are moveable round their centres only.

Suppose a style fixed in the circumference of the circle Y, and that the three circles are made to turn by the movement of one of them: if we make each of the arcs, A H, A C, equal to A E, *then the style placed in* E *shall have described on the plane of the circle* R, *a portion* C E *of an exterior epicycloid, and on the plane of the circle* X, *a portion* H E *of an interior epicycloid.*

Fig. 7.



The two epicycloids, C E, H E, *traced in the same time by the style* E, *touch in the point* E. For the straight line A E, drawn from the point A, where the generating circle Y touches its base R C, shall be perpendicular to the two epicycloids, and the straight line H E shall touch the epicycloid in the point E*.

V.

20. Let us next suppose, that the generating circle Y has for a diameter the radius A B of the circle X, within which it is placed, and that the three circles, R, X, Y, touch continually in the point A, as in Fig. 8.

The interior epicycloid H E, *which touches the exterior* C E, *shall be a straight line directed towards the centre* B *of the circle* X, Art. 16, and consequently a portion of the radius B H, which shall always touch the exterior epicycloid C E in the point E, where it shall be met by the perpendicular A E.

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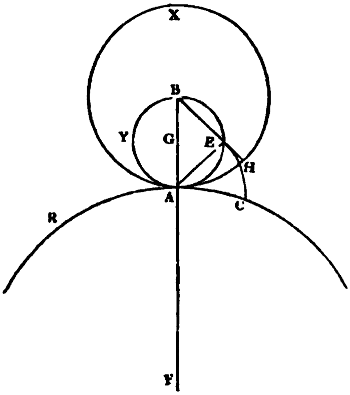
* Because any triangle which can be inscribed in a semicircle, is a right-angled triangle. For manner of drawing a perpendicular on the end of a line, see supplementary definitions, Art. 83.

FIG. 8.



Hence it follows, that when the two circles, R, X, touch continually, and the one causes the other to turn by contact at the point A, if we imagine a radius B H in the circle X ; and having made A C equal to A H, there will be described by the point C, an exterior epicycloid C E, which has for a generating circle Y, the diameter of which is equal to the radius B H, this radius B H, during the movement of the circles R, X, shall always touch the epicycloid in the point E, where this epicycloid shall be cut by the straight line A E perpendicular to its curve.

Thus instead of supposing, that one of the two circles R, X, turns forward the other by the point of contact A, let it be supposed, that the one is made to push forward the radius B H, of the circle X, by an epicycloid C E attached to the circle R, and described by the movement of the circle Y, the diameter of which is equal to the radius B H.

One may be able thus reciprocally to make the epicycloid C E, attached to the circle R, push forward by a radius B H a circle X ; and by means of the epicycloid C E, and of the radius, BH, the two circles, R, X, may be able to conduct themselves as if put forward by the point of contact A[*].

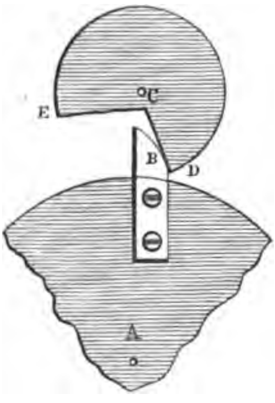
For suppose the radius, BH, and the epicycloid, CE, to be teeth of wheels, X and Y; and the perpendicular AE, from the touching surfaces in all situations, cuts the line of centres at the termination A of their proportional radii. But we saw, Art. 11, that when this was the case, the proportional circles must have equal velocities.

It is principally from this, that we shall deduce the best figure which can be given to the teeth of wheels and pinions, when one part of the wheel and pinion, or of both, ought to be a straight line tending to the centre of such wheel or pinion.

VI.

21. If in the same plane we have but two circles, R, Y, Fig. 10, which touch in the point A, and if the movement of the one communicate itself to the other, by this point of contact, any point E of the circumference of the circle Y, describes upon the plane of the moveable circle R, an epicycloid CE.

Fig. 9.



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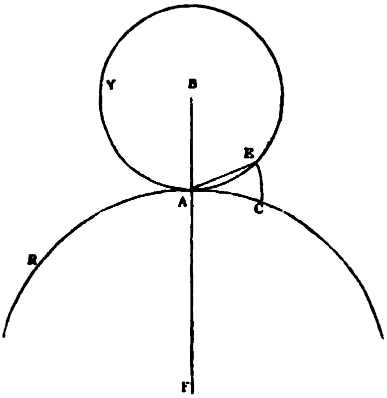
[*] To be satisfied of this experimentally, make any two circles of wood, as in Fig. 9; to the circumference of one of them A, fix a piece of wood B, formed into an epicycloid, generated by a circle half the diameter of C upon A as a base.

On the circle C, draw the line OD, and cut out the part bounded by that line and CE.

If you cause one of the circles to move the other by the parts B, CD, both circles will have the same velocity; as may be ascertained by putting a mark opposite any point in the circumference of each circle before they begin to move, and another after they stop, and the distance between which, measuring by the arcs, will be found equal.

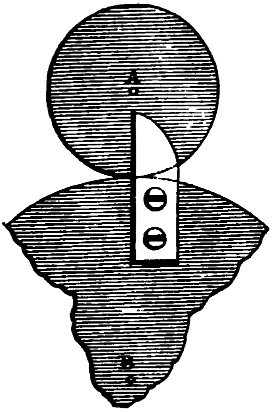
Fig. 10.



Suppose this epicycloid attached to the circle R, it (the epicycloid) shall conduct the circle Y, pushing it round by the point E of its circumference, in the same manner as the circle R might conduct the same circle Y in communicating motion to it by the point of contact A.

And in like manner, the point E of the circumference of the circle Y, turns the circle R, in pushing it by the epicycloid C E, supposed to be attached to R, in the same way that the circle Y would conduct the circle R in communicating its motion by the point of contact A*.

Fig. 11.



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* The experiment to prove this is similar to the former, but with this difference, that in the circumference of one of them, A, is fixed a fine needle, which is made to act against a piece of wood, formed into an epicycloid, fixed upon the other, B, which epicycloid is generated by A upon B as a base.

The same mode of proof applies here that did to the corollary immediately preceding.

This last corollary enables us to determine the best figure which can be given to the teeth of wheels, when the pinion shall be a trundle composed of staves.

We shall likewise determine from it the most advantageous figure which can be given to the teeth of a pinion, when the wheel shall have staves in place of teeth.

22. In addition to the properties of the epicycloid mentioned above, there are several others of a curious and scientific nature, which may perhaps be not improperly introduced in this place, although they may not be immediately applicable to the construction of the teeth of wheel work.

1. If the generating and quiescent circle have to each other any commensurable ratio, then is the epicycloid thus generated both rectifiable and quadrable; that is, both its length and area are exactly determinable.
2. If the generating and quiescent circles are incommensurable with each other, then the epicycloid is unquadrable, but it is still rectifiable; that is, the area in this case cannot be found in finite terms, although the length of the curve is exactly assignable.

23. To these we may also add the following rules for finding the lengths of epicycloidal curves, and the areas which they enclose.

RULE I.

As the semidiameter of the quiescent circle, is to the sum of the diameters of the two circles, so is double the versed sine of the arc of the generant, which has passed over any portion of the quiescent circle, to the length of the epicycloidal arc generated by the point which touched the quiescent circle or base at the beginning of the motion.

When the whole arc is required, the versed sine becomes the diameter of the generant. The length of any arc of an interior epicycloid is found in a similar manner, only using the difference of the diameters in the second term of the proportion instead of the sum.

RULE II.

To find the area of an epicycloid; it is, as the radius of the quiescent circle is to three times that radius, plus twice the radius of the generant, so is the circular segment AE, to the epicycloidal sector AEC. Or, so is the whole area of the generant, to the whole area of the epicycloid. This rule applies to both the exterior and interior epicycloid.

A general proposition for the area of all cycloids and epicycloids is given by Dr. Halley, and is as follows, viz.:

That the area of a cycloid or epicycloid, either primary, curtate or prolate, is to the area of its generating circle, as the sum of double the velocity of the centre, and velocity of the circular motion, to the velocity of the circular motion.

The same proportion holds good in reference to any parts generated in those curves, and the analogous segments of the generating circle.

CHAPTER II.

OF THE APPLICATION OF THE PRINCIPLES OF THE CONFIGURATION OF THE TEETH OF WHEELS.

24. HAVING endeavoured to show, that an epicycloid is a curve, whereby two circles may conduct themselves as if put forward by the simple contact of their circumferences, I shall now attempt a practical explanation of this curve, in giving the best form to the teeth of wheels.

SECTION I.

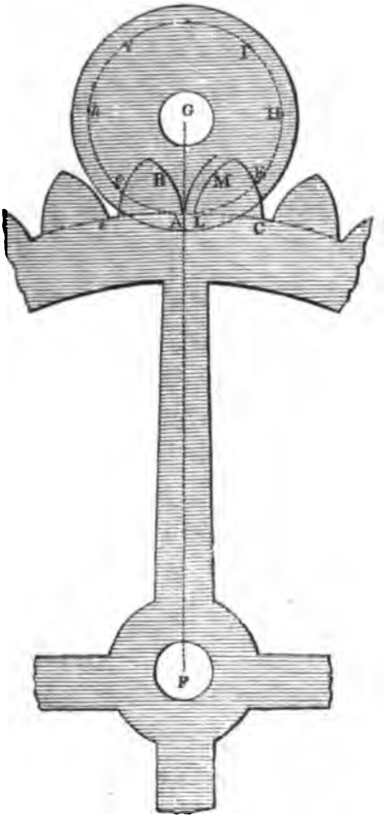
OF SPUR GEARS.

25. By *Spur Geers* is understood wheels acting together, and in the same plane, with their axes parallel; under this head the wheel and trundle come first to be considered.

OF THE WHEEL AND TRUNDLE.

26. To determine the figure of the teeth of the wheel, which depends always upon that of the staves of the trundle, we shall first suppose the staves to be indefinitely small, and represented (Fig. 12) on the end of the trundle by the points, A, E, H, &c.: when we have found the figure of the teeth proper to conduct the indefinitely small staves, (which are used for demonstration only,) we shall, by means of that figure, trace the true form which should be given to the teeth of wheels to conduct trundles with cylindric staves of some magnitude. Thus the solution of this case, naturally divides itself into two parts.

FIG. 12.



I.

TO FIND THE FIGURE OF THE TEETH WHEN THE STAVES ARE INDEFINITELY SMALL.

27. Draw the proportional circles, *c* A C and *e* A E, and divide each of them into the number of equal parts which it should have of teeth*.

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* This operation is called by millwrights *setting off the pitch.* By the pitch is understood the distance between the centres of two contiguous teeth.

We have seen*, if the circle *c* A C, which touches the circle *e* A E, have attached to its circumference an epicycloid, C E, described by the point E of the circumference of the circle *e* A E rolling upon the circle, *c* A C, the epicycloid conducts the circle *e* A E by the point E, as if conducted by contact at A, and consequently the circumferences of the two circles shall have the same velocity.

The epicycloid, C E, is then the best figure which can be given to the teeth of a wheel to conduct a trundle, the staves of which are indefinitely small, and therefore must move the stave E, in the direction from A towards E, until a second stave arrive, and be taken in the line of centres by a second epicycloid, A B, which shall in like manner conduct this stave, A, until the arrival of another stave, *e*, in the said line: and thus the other staves of the trundle shall be conducted by the other epicycloids of the wheel.

Here it may be observed, though perhaps already evident, that it is the convex side of the epicycloid which must be used: for though it be useful in some machines, to make the concave side of a single epicycloid conduct a point of a single piece moveable on a centre, yet were a number of teeth so formed, it would be impossible for them to act on a number of staves, for they would be so hooked and entangled as not to move forward in the smallest degree.

Were it wished that the wheel should move the trundle in both directions, it is obvious that each tooth of the wheel should have its opposite sides, C E, L M, formed into equal epicycloids.

As we have supposed the staves of the trundle indefinitely small, were the teeth of the wheel also perfect figures, and equally distanced, there would be no need of other than indefinitely small spaces between the adjacent teeth of the wheel; but as perfect precision is not to be expected, a space more or less, such as A L, must be left between them, to enable the wheel, notwithstanding the inequalities of the teeth and staves, to move the pinion.

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* See Chap. I. Article 21.

We have hitherto supposed the teeth of the wheel conducted by the staves of the trundle, but it is evident, had the teeth of the wheel the same figure, when conducted by the staves, the wheel and trundle would retain the property of moving with the same velocity. It may only be observed, that the staves of the trundle conduct the teeth of the wheel in approaching the line of centres, while the teeth of the wheel conduct the staves of the trundle in their progress from that line*.

II.

TO FIND THE FIGURES OF THE TEETH OF THE WHEEL, WHEN THE STAVES OF THE TRUNDLE ARE CYLINDERS OF A FINITE DIAMETER.

28. Consider the trundle at first as having infinitely small staves, represented by the centres of the staves, A, E, H, &c., and trace, as above mentioned, the teeth C L P, A Q N, &c. of the wheel, as if it had to conduct a trundle with infinitely small staves: observing to leave a small space, such as A L, between all the teeth, in order that they may act freely.

Describe, with the radius of the staves, upon the plane of each tooth, as many small arcs as may be convenient, having all their centres in the two epicycloids which form the teeth.

Trace, by means of these little arcs, two curves, such as R O, S O, parallel to the epicycloids, and then you will have inclosed the space, R O S, which is the figure all the teeth of the wheel ought to have beyond its proportional circle.

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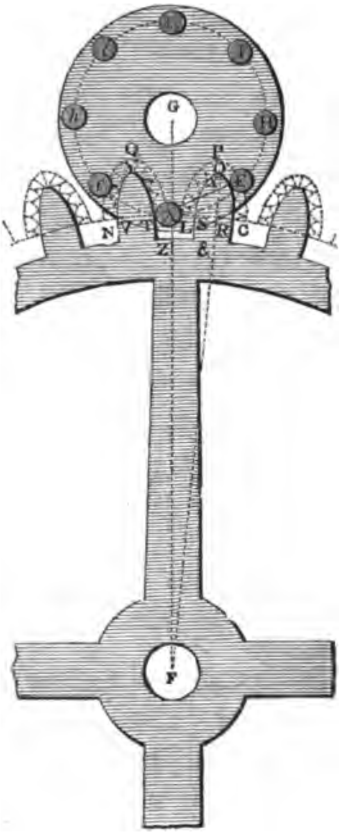
* See Article 33 of this chapter.

FIG. 13.

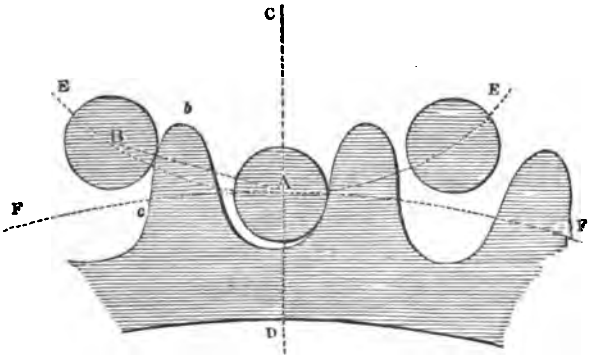


For if we suppose, that the centre E, of a stave, is conducted by the tooth CPL; the curve RO, which is parallel to the epicycloid CP, and which is placed at the distance of the radius of the stave E, shall always touch the circumference of that stave.

Thus the curve RO shall conduct the cylindric stave, as if the tooth CPL conducted the centre of that stave, and consequently the tooth ROS, shall be a proper figure to conduct the trundle, with cylindric staves.

The curved parts of the teeth of the wheel, being determined as above, the spaces TS, Z&, &c. should be cut out, in order to admit that part of the staves which extends beyond the proportional circle of the trundle.

Fig. *A*.



TO DESCRIBE THE TEETH OF A WHEEL, FOR A TRUNDLE, BY MEANS OF CIRCULAR ARCS.

29. Let CD be the line of centres; EE the pitch line of the trundle; and FF that of the wheel; and suppose the centre of the stave A to be in the line of centres CD; then place one foot of the compasses in the centre of the stave A, and describe the arc *bc*, which is the form of the tooth. The part of the teeth of the wheel, within the pitch line, may be described with circular arcs as in the figure.

Teeth formed in this manner will not sensibly differ from those described according to the principles laid down in the preceding articles, when the length of each tooth is not greater than is necessary. The reader will easily perceive, that the radius for describing the teeth, is equal to the pitch diminished by half the diameter of the stave; and also that the centres of those arcs will always be in the proportional circle, or pitch line of the wheel.

Some authors have imagined that the friction of the wheel and trundle might be reduced by making the staves revolve; but it could not be effected so far as to balance the extra labour of construction, and where the strain would be considerable, it would become quite impracticable. (See Emerson's Mechanics, prop. 119, rule 9; and Transactions of the Society of Arts, vol. xxxv. p. 128.)

Smeaton appears to have been very partial to the wheel and trundle, when the trundle was executed with cast iron staves; these he recommended to be of an oval figure, and made smooth by grinding them. (Smeaton's Reports, vol. i. p. 316; vol. ii. p. 391 and 423.)

It may be demonstrated that the least real radius of the wheel should be equal to the proportional radius added to half the pitch; when the necessary allowances are made for wear, (see Art. 43.) and when the staves are of the same diameter as the thickness of the teeth. But when the staves are larger than the teeth, as in the figure, a less real radius is required.

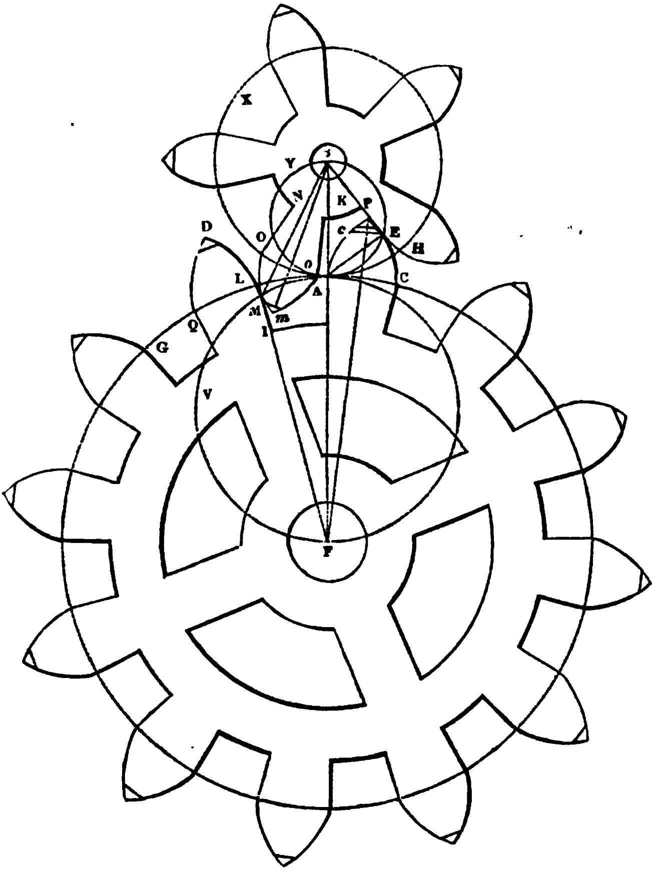
Having considered the case of a wheel and trundle, with cylindric staves acting together, we are now to explain that of a wheel and pinion, two sides of the figure of whose teeth are straight lines directed to its centre.

OF THE WHEEL AND PINION.

To find the figure of the teeth and leaves of a wheel and pinion, when that part of the teeth and leaves, which lies within their respective proportional circles are straight lines directed to the centres of these circles.

30. Having set off upon the proportional circles, the points G, Q, L, and O, *o*, H, &c., according to the thickness of the teeth and leaves, draw lines from these points, tending towards the centre of their respective circles, to serve as the sides of the spaces between the teeth and between the leaves, the depth of which spaces must be such as to give room for the action of the curved parts of the teeth and leaves.

FIG. 14.



Then describe upon the extremities of the sides of each tooth, epicycloids, such as QD, LD, with the generating circle Y, the diameter of which is equal to the proportional radius of the pinion, upon the circumference of the proportional circle of the wheel as a base.

The mode of forming the teeth being thus shown, that of the leaves will be plain, V being the generating circle of their epicycloid, upon the circumference of the circle of the proportional pinion as a base.

We have seen*, if the radius BH of the proportional pinion, be pushed by an epicycloid C P, generated by the circle Y, upon the pitch line of the wheel, and projecting therefrom, the pinion shall turn with the same velocity as the wheel.

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* Chap. I. Article 20.

In the same manner it may be proved, that the same effect will be produced, if the epicycloid O M *m*, attached to the pinion, be pushed towards the line of centres, by the radius, L F, of the wheel.

Lastly, the two opposite sides of the teeth, and those of the leaves, ought to have the same figure, for the ease of action, and to give the wheel and pinion the liberty of being moved in either direction.

From these principles it will be evident, that the figure here given to the teeth, will make the wheel and pinion move with perfect regularity.

31. To make that part of a tooth which is within the pitch line or proportional circle a straight line, as proposed by the Author, seems to be the most advantageous form, because it causes least pressure on the axes.

When the teeth are small, and do not begin to act till they arrive at the line of centres, the teeth of the wheel, when the wheel drives the pinion, or the leaves of the pinion, when the pinion drives the wheel, may be described by a circular arc, of which the radius is equal to the pitch; and of which the centre is in the pitch line of the wheel or pinion.

This method will always enable a workman to execute short teeth nearer to the true form than any pattern tooth will enable him to do. Pattern teeth and compound curves, are things that may on some occasions be very useful; where the teeth are long, and of considerable magnitude in respect to that of the wheel or pinion to which they belong. But in all the ordinary forms of wheel-work such operations must consume an immense quantity of valuable labour to attain even the same degree of accuracy that is at once obtained by means of circular arcs. When a pattern tooth is necessary, one of its adjustments should be the centre of the wheel; and not two points in its circumference, as proposed in Imison's Elements of Science and Art, (Vol. I. p. 103,) because the latter method at least doubles the risk of error in adjusting the pattern.

When part of the action takes place before the teeth arrive at the line of centres, the method of forming teeth proposed by our Author, (Art. 41,) seems to be equal, if not superior, to any other. And its practical application is shown in Art. 42.

REMARKS.

32. As it is the curved part of the teeth of the wheel, that should push the straight flank H K, of those of the pinion, in removing from the line of centres, and as the point E, where the flank is acted upon, is a perpendicular drawn from A, it shall be always that by which the wheel shall push, it is clear, that when the extremity P, of the epicycloid C P, reaches the point E, it shall cease to move the tooth H K; if the extremity P, arrive at the point E, before the flank O N, of the following tooth of the pinion has reached the line of centres, the curved part, O, M, *m*, of this tooth, must be pushed by the straight flank L I, of the following tooth of the wheel, till the flank O N, reaches that line: so that in this case, the wheel conducts the pinion, at one time before, and, at another, beyond the line of centres.

But were it so, that the extremity P, did not reach the point E, till after the flank, O N, had arrived at the line of centres; it would not be necessary, that the curved parts of the leaves should be pushed by the flanks of the teeth. Thus, in this case, the wheel would conduct the pinion, in pushing its leaves beyond the line of centres only.

33. It is the general opinion of those who are in the

practice of constructing wheel work, that teeth ought, if possible, never to begin to act before they reach the line of centres, as that mode of action is thought to occasion much unnecessary friction*. The cause of this great unnecessary friction, when the teeth are of wood, appears to be the following:

Friction depends not only upon the pressure made on moving bodies, but on the inequalities of the surface upon which they move; and as the surfaces even of the most highly polished bodies have some inequalities, whenever two of them are pressed together, the inequalities of the one must enter the other.

Suppose A and B to be a wheel and pinion, having wooden teeth, as they would appear through a microscope; it is impossible, though there be no other resistance than that arising from friction, to move them towards the line of centres, until either the centres, on which the wheels turn, give way, or some of the small inequalities, *c*, *d*, of the teeth be broken off†.

On the other hand, a very small force will move the teeth outwards from the line of centres, as the small inequalities, *c*, *d*, and *a*, *b*, may then slide over one another without being broken; for the teeth, when so working, are mutually receding from each other in their point of contact,

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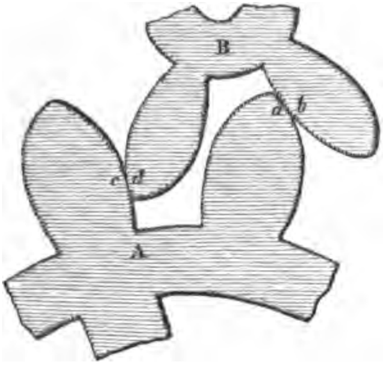
[*] The increase of friction is not the only disadvantage from part of the action taking place before the teeth arrive at the line of centres. For when a machine becomes worn, it causes considerable irregularity in its movements, in consequence of the action, in approaching the line of centres, tending to spread the axes of the wheels; while the action, in receding from that line, tends to draw these axes together; and hence occasions more irregular action, friction, and wear in the machine, in proportion to the wear of the parts.

We have an obvious remedy in making pinions sufficiently large to allow of the action being wholly on one side of the line of centres.

[†] See a further illustration of this subject in the Supplementary Observations. Art. 66, &c.

and the wheels move on their centres with ease; and whereas, in the first case, they must have a tendency to force the centres on which they turn outward, from their true position, in the second, they have no such tendency*.

FIG. 15.



Again, when the teeth are of metal, this unnecessary friction seems to arise principally after the teeth are in some degree worn. (See Fig. 18, in p. 36.) The teeth in that case have a kind of seat formed at their bottom, and the curve at the outward extremity is too much inclined to the radius, and very abrupt. In the action which takes place before the line of centres, the sliding of the teeth of the conducting wheel along those of the conducted, has a tendency to accumulate hardened grease, dust, sand, &c., at the bottom, which getting between the abrupt extremity of the tooth and the seat at the bottom, become like the key-stone of an arch, and must require often a considerable force to bruise the teeth in this situation past the line of centres.

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* The true cause of the increase of friction is shown in Art. 67. The motion against the grain of the wood will scarcely have a sensible effect, except when the teeth are new.

Thus it appears, that the friction of teeth, approaching the line of centres, is much greater than in receding from it. But in cases where the pinion is small, the action in approaching to the line of centres, cannot be altogether prevented. M. Camus, in his "Cours de Mathematique," has demonstrated, that a wheel of 50 teeth cannot conduct a pinion of 7 leaves, without their acting partly before they arrive in the line of centres. He also proves the same with regard to 57 teeth and 8 leaves, 64 and 9, 72 and 10*.

34. Since the calculations which Camus has given are confined to particular cases, and troublesome to apply to new ones, because his method is indirect; we will, in these additional articles, investigate a rule, which will be found somewhat more general, and of easier application.

Let B be the centre of the pinion, and C the centre of the wheel; then we have to ascertain the relation between the number of teeth on the pinion and the number of those on the wheel, so that any tooth D of the wheel may arrive at the line of centres before the preceding tooth of the wheel quits the tooth A of the pinion.

The last point of contact cannot be beyond the middle point of the tooth, and therefore the line AC will be in the middle of it, A being the point of contact. Also AD will be perpendicular to BA, the point D being in the pitch lines.

Now by the principles of trigonometry we have this proportion; $AB : BC :: \sin. C : \sin. A = \frac{BC. \sin. C}{AB}$. But $\sin. A - \sin. C = \sin. B$; therefore, $\sin. C \left(\frac{BC}{AB} - 1\right) = \sin. B$. The primitive radius of a wheel or pinion being proportional to the number of its teeth, it may always be represented by that number; and if N be the number of teeth in the wheel,

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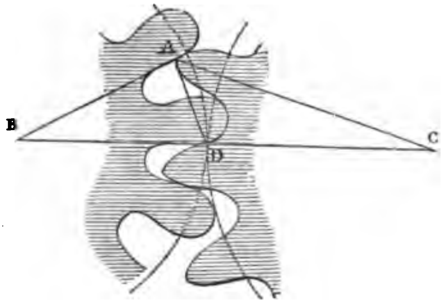
* See Camus on the Teeth of Wheels. English translation, p. 60–67.

Fig. *B.*



and n those of the pinion, we shall have $BC = N + n$. The line BA, being a side of a right-angled triangle, considering the radius unity, is found by this proportion; $1 : \cos. B :: n : AB = n. \cos. B$. And the angle B is $\frac{360°}{n}$; consequently, $\sin. C \left(\frac{BC}{AB} - 1\right) = \sin. B = \sin. C \left(\frac{N + n}{n \cos. \frac{360}{n}} - 1\right)$ $= \sin. \frac{360}{n}$.

35. As the sin. C is, in practical cases, sensibly equal to its corresponding arc, we may use the arc for the sine, which will render the equation much more simple. The arc is $\frac{2 \times 3·4161}{N} \times \frac{3}{4} = \frac{4·7124}{N}$; which being substituted for sin. C, we have

$$N = \frac{4·7124\, n \left(1 - \cos. \frac{360}{n}\right)}{n \sin. \frac{360}{n}. \cos. \frac{360}{n} - 4·7124}$$

Or, because $\sin. \frac{360}{n} \times \cos. \frac{360}{n} = \frac{1}{2} \sin. \frac{2 \times 360}{n}$.

(Gregory's Trigonometry, Chap. iv. Art. 20.)

$$\text{N} = \frac{9{\cdot}4248\,(1 - \cos.\frac{360}{n})}{\sin.\frac{720}{n} - \frac{9{\cdot}4248}{n}}$$

This equation gives a result nearer to the conditions required in the actual construction of wheels and pinions than when the sine is used instead of the arc; for it makes an allowance for the wearing away of the teeth. It supposes the teeth of the pinion to be the same size as the teeth of the wheel, which is the least thickness that can be given consistent with strength and durability.

36. The above equation shows, that when a pinion has less than 10 leaves, it cannot be conducted uniformly by a wheel with any number of teeth whatever, unless they act partly before they arrive at the line of centres; because the denominator of the second member of the equation becomes negative when n is less than 10.

This is manifest, for when $n = 10$, the angle in the numerator is 36 degrees, and that in the denominator is 72. But the natural sine of 72°, as shown by the Trigonometrical Tables, is ·95106 when estimated in parts of the radius, and the second term in the denominator when divided by 10, becomes ·94248, which being less than the sine of 72°, the difference between them is a positive quantity, and will continue so for any value of n that is greater than 10; but for any integral value of n that is less than 10, the second term in the denominator will exceed the first, and consequently, the difference must be negative, and a negative number of teeth in a wheel is obviously absurd.

But a pinion of 10 leaves may be moved uniformly by a wheel with 209 teeth, when the whole of the action is after they arrive at the line of centres; for in this case

$n = 10$, and by a table of sines, we find cos. $36° = \cdot 80902$; and sin. $72° = \cdot 95106$. Therefore $\frac{9 \cdot 4248 (1 - \cdot 80902)}{\cdot 95106 - \cdot 94248} = 209 = \text{N}$.

In like manner it may be calculated that a pinion of 11 leaves may be moved uniformly by a wheel having not less than 28 teeth; and a pinion of 12 leaves when the wheel has not less than 15 teeth; so that the whole of the action may be after the teeth arrive at the line of centres.

37. The same equation may easily be extended to the case of the wheel and trundle, and as it does not appear to have been investigated, we shall here show the result under the following conditions; 1. The extremity of each tooth of the wheel is to be one-third of its thickness at the root. 2. The centre of the stave is to be in the pitch line when the point of the tooth quits the preceding stave. When limited to these circumstances the equation in Art. 35. becomes

$$\text{N} = \frac{7 \cdot 33 \left(1 - \cos. \frac{270}{n}\right)}{\sin. \frac{540}{n} - \frac{7 \cdot 33}{n}}$$

Whence it appears that a trundle with less than eight staves cannot be moved uniformly by a wheel with any number of teeth whatever.

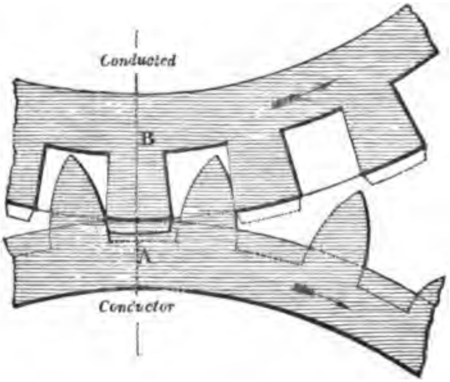
The same is true also of stave-formed teeth (see Art. 41.) when the wheel drives the pinion.

38. From what we have already said, it will be evident, when the pinion consists of such a number of teeth, as to be conducted uniformly by the wheel in *receding* only from the line of centres, that, except in small numbers, the epicycloid is necessary on the conductors only, whether it be a wheel or pinion. For instance, in Fig. 16, which represents two wheels of equal numbers, A is the conducting, and B the conducted wheel. But it is to be observed,

when of two wheels acting on each other, sometimes the one, and sometimes the other, is the conductor, the teeth of both should be epicycloidal, as in Fig. 14. p. 25.

When the teeth of the conducted wheel or pinion are acted upon by those of the conductor, in *receding* only, from the line of centres, it may be remarked, if they were perfectly made, and of durable materials, it would be unnecessary to extend the conducted teeth beyond their proportional circle. But these properties being unattainable, and as the angles which terminate their sides, would be apt to cut the conducting teeth, and occasion an irregular motion, it is proper to form the extremities of the teeth of the conductor, in the manner represented in the figure by the dotted lines.

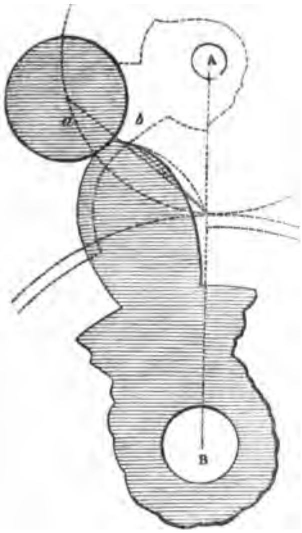
Fig. 16.



39. Sometimes it may be requisite to have but few teeth in the pinion. In such cases, in the *conducted*, whether wheel or pinion, Buchanan preferred staves to teeth, properly so called, or to leaves, because a trundle or wheel, whose staves are cylindric, will be less acted upon in approaching the line of centres, and consequently have less friction than a pinion or wheel, the sides of whose teeth tend to the centre.

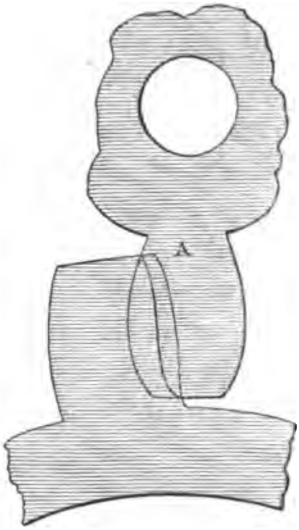
This will appear by Fig. 17, which represents a stave, *a*, of a trundle, and a leaf, *b*, of a pinion, turning round on the same centre, A, and a tooth adapted to each, turning on a common centre, B. The thickness of each of the teeth, and the proportional circle of both wheels, are the same, and the proportional circles of the pinions are also equal, and teeth are each made of the greatest length which the intersection of the curves will admit, which turns out considerably greater in the tooth adapted to the stave. The shaded parts represent the tooth adapted to, and acting upon, the stave; and the dotted lines represent the tooth adapted to, and acting upon, the leaf. The teeth, in both cases, are represented as just at the point where they would cease to move the leaves or staves uniformly; and it appears the stave is conducted considerably further beyond the line of centres than the leaf; hence the stave will be less acted upon in approaching the line of centres.

FIG. 17.



40. A trundle has besides another considerable advantage over a pinion; which is, that it wears much more

FIG. 18.



equally. Every one experienced in wheel work knows, that when a pinion comes to be considerably worn, the leaves take somewhat of the form represented at A, which is evidently the cause of a great deal of unnecessary friction, and strain in a machine. Whereas no such thing happens to the trundle*. The trundle has, however, a defect perhaps as bad, if not worse, that of weakness. Its staves being supported at the ends only, are not long in use before they become quite unable to bear any considerable strain, and for this reason, it is now in a great measure disused in machines. It however appears to Mr. Buchanan, that a wheel might be made, which would combine the advantages of both the pinion and the trundle, and he accordingly had some wheels made on that idea, and they appear to answer every expectation.

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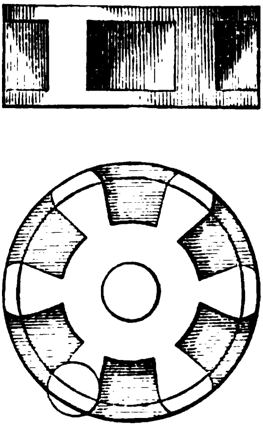
* This is a mistake, as trundles, in consequence of the surfaces of contact being small, become soon indented by pressure, and wear and cease to turn round in their sockets.

These wheels were made of cast iron. They were each cast of one solid mass. The upper figure represents the edge view, and the lower the section of one of them; whereby is shown the manner in which the teeth are supported, like the staves of a trundle at each end, and like the leaves of a pinion at the roots, but so very thin there, as to run no risk of having the common fault of pinions just now noticed. They were difficult to mould: but were they to come more into use, he had no doubt ingenious workmen would soon get over this obstacle*.

Fig. 19.



41. It is mentioned above, in cases where the pinion had few teeth, that in the *conducted*, whether *wheel* or pinion, staves should be preferred; but it is obvious, that the method just described, of making a small trundle of cast iron, would not apply to a *wheel* of a great number of staves. Nor is it in that case so necessary, as the greater the number of teeth are, the longer they will be in losing their proper figure. In such cases, therefore, *staves*, strictly speaking, should not be used, but *teeth* made so as

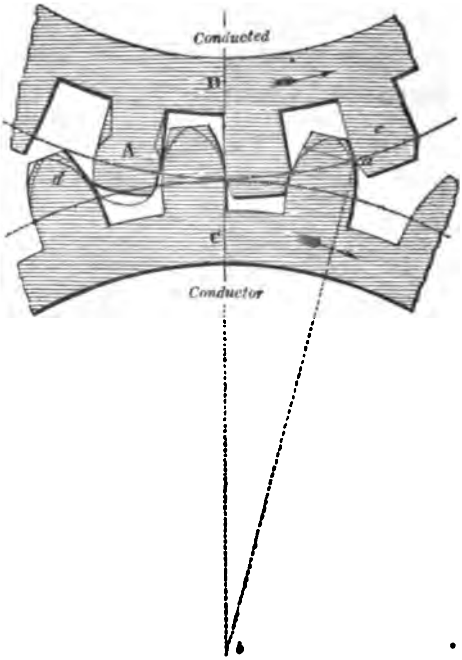
---

* By casting separate plates, with indents to fit the teeth, and bolting them together, the pinion might be made sufficiently strong; such a method indeed is used frequently in crane-work, where it has the important advantage of preventing the wheels getting out of gear.

to produce the same effect—that is, having their acting parts of the figure of a stave.

What is meant will be better understood by inspecting the figure, where the lines show the alteration necessary on the tooth A, in order to make it produce the effect of a stave; which stave is represented by the faint dots. The dotted lines on *d*, represent the alteration requisite to adapt it to the stave, it being necessary, as formerly proved, to have it a different epicycloid from what is required to adapt it to a tooth, whose acting part is a straight line, tending to the centre of its proportional circle.

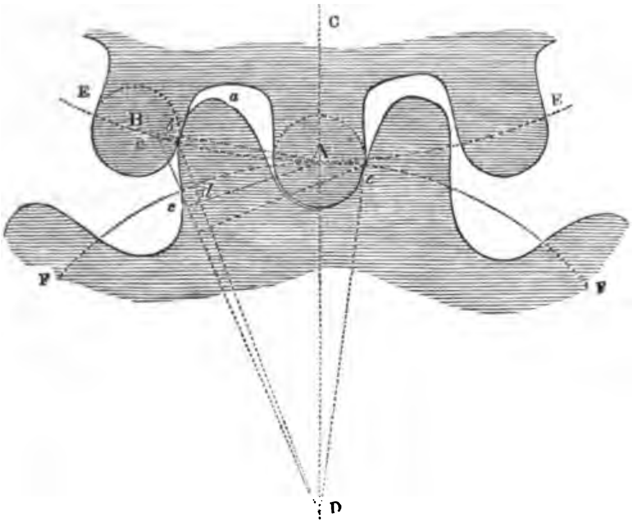
FIG. 20.



42. Teeth seem to be very well adapted for various purposes, when formed on the principle recommended in the preceding article. We therefore will endeavour to show a simple method of describing such teeth.

It must be observed that the teeth to resemble staves are to be always on the conducted wheel or pinion; thus affording the peculiar advantage of the wheel and trundle in either increasing or diminishing velocity.

FIG. *C.*



Let the teeth be divided as usual on the pitch lines, E E, F F; and on the conducted wheel C describe circles, as though there were to be staves. Conceive the centre of one of these stave teeth to be in the line of centres at A, and draw the line A B joining the centres of the stave teeth. Then the radius A *b*, from the centre A, will describe the curved side *b c* of the tooth of the conductor, and the curved part *b a* of the conducted wheel. And since this radius is equal to the pitch diminished by half the diameter of the circle of the stave teeth, and the centres will always be in the pitch lines of the wheels; all the other teeth may be easily described.

43. The real radius, when the wheel is the conductor, may be very easily calculated with sufficient accuracy in

this manner: Let $b\,d$ be drawn towards the centre, so that $d\,c$ is equal to $\frac{1}{3}$ of the tooth; and make $\mathrm{A}\,d$ perpendicular to $b\,d$; then we shall have $\sqrt{\mathrm{A}\,b^2 - \mathrm{A}\,d^2} = b\,d$. But $\mathrm{A}\,b$ is $\frac{3}{4}$ of the pitch, when the teeth of the pinion are of the same thickness as those of the wheel, and $\mathrm{A}\,d$ is sensibly equal to $\frac{7}{12}$ of the pitch; therefore if $\mathrm{P} =$ the pitch, $\mathrm{P} \times \sqrt{\frac{9}{16} - \frac{49}{144}} = b\,d. = {\cdot}4714\,\mathrm{P}.$ And as in practice we may always regard $b\,d$ as the difference between the real and proportional radius; it being only a very small quantity in excess*, we have this rule:

The real radius of a wheel, of the construction now described, should be equal to the proportional radius added to ·47 times the pitch: and ·47 times the pitch, is very little less than half the pitch. But when the teeth of the pinion are thicker than those of the wheel, the real radius may be less than is given by this rule.

The same rule applies to a wheel to drive a trundle.

This approximate method was chosen by Tredgold in preference to a more accurate one, because the result is exhibited in those terms which are most directly comparable with the proportions founded on practical experience.

When the pinion is the conductor, the real radius should be the secant of the angle contained by the pitch, when the proportional radius is considered the radius of that angle.

Thus, $c\,e$ is the angle contained by the pitch, and $e\,\mathrm{D}$ is sensibly equal to the proportional radius; but by the form of the teeth $r\,e$ is perpendicular to $e\,\mathrm{D}$, consequently $r\,\mathrm{D}$ is the secant to the arc or angle $c\,e$; and the real radius required to impel the wheel till the succeeding tooth begins to act.

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* The excess is the difference between the radius and the secant of the angle containing $\frac{7}{12}$ of the pitch.

44. Hitherto the conducting teeth have been considered, as being always made as long as the epicycloidic form of their sides would admit. This however is not always necessary, and in some cases may be improper.

It now remains to show, the smallest real radius a wheel adapted to a trundle can have, without destroying the uniformity of the motion.

When the stave E (Fig. 13, p. 22.) shall have been conducted to the situation in which it is represented, the point E of the following stave, shall be in the line of centres G F. The stave A, in its turn, may then be conducted by the following tooth, T Y V, and then it shall no longer be absolutely necessary, that the tooth R O S conducted the stave E. The tooth R O S may therefore be terminated in the point X, where it should touch the stave E, when the point T of the following stave shall be in the line of centres, and the distance X F of this touching point, from the centre of the wheel, shall be the least real radius which can be given to the wheel.

To determine the point X, draw from the centre of the stave E to the point T, the straight line E T, and where this line meets the circumference of the stave E, you have the point required*.

45. The smallest real radius which can be given to a wheel, adapted to the leaf of a pinion, or the teeth of a wheel, must evidently be terminated by that point *a* of its tooth, which is in contact with the tooth or leaf *e*, after it has conducted it just until the tooth following begins to act: thus *a b* is the smallest real radius of the wheel c†.

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* See Art. 29. and 43., where such proportions as are applicable in practice are given.

† When the wheel is conductor, if we pursue the same mode of calculation as in Art. 43, making $P =$ the pitch, we shall have $P \times \sqrt{1 - \frac{25}{36}} = \cdot 553\ P$. That is, when the teeth of the conductor do not begin to act till they arrive

But, in practice, perfect accuracy is not to be expected; and though it were even practicable to have wheels perfectly accurate when new, yet the moment they are put in motion they begin to wear, and deviate from the true figure of their teeth. It would therefore be attended with bad consequences, to make the real radius no greater than what we have here determined it to be, which is the least which can be given. How much greater it should be, may be determined by circumstances*.

46. But it appears to us, that when wheels are made with their conducting teeth only epicycloidic, and the motion is steady, there is not much danger of their being too long; for the longer the teeth are, the greater number of them will be in action at the same time, and consequently the strain will be more general, which will cause them to retain their true form longer. Besides, though a tooth, from any accident, should be broken, the wheel will continue to go very well for a long time; whereas, had its teeth been short, the wheel would, by such an accident, have been rendered useless. We are however aware, that very long teeth are less able to sustain any sudden stress upon their extremities†.

But even supposing the teeth, made in the manner above described, to be no longer than those formed epicycloidic, upon both the conductor and conducted, yet the former

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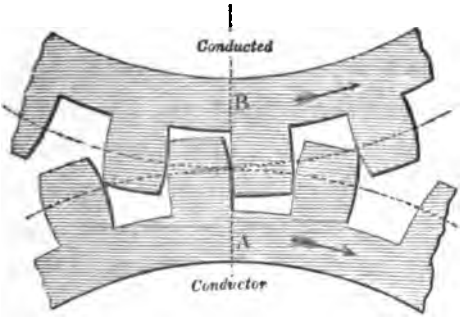
at the line of centres, the real radius should be equal to ·553 times the pitch added to the proportional radius. But when the pinion is conductor, the proportional radius is to the real radius, as the radius to the secant of the arc equal to the pitch.

* Buchanan had been informed, that Mr. Watt drew the figure of teeth with segments and points, what is below as well as what is above the pitch lines, and that for small strains and great velocities, he used the pitch line near the root of the driver, but upon other occasions, he used it just in the middle, as giving the *greatest number of touching points*, which certainly adds to the strength of the wheels.

† See the last paragraph of Art. 70.

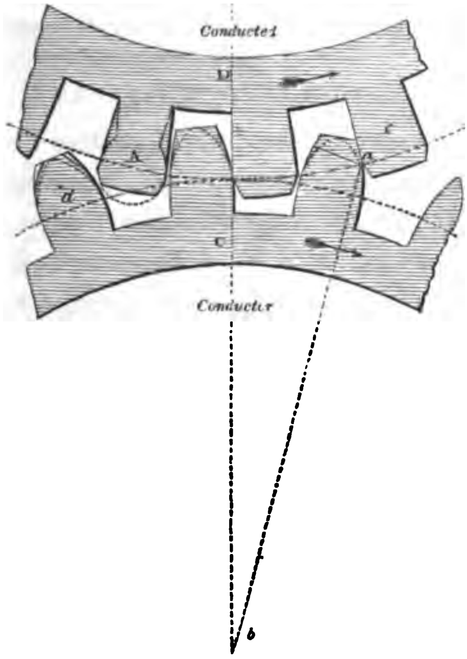
will have less friction, and consequently wear longer than the latter, and if they be of the same length, and the same

FIG. 21.



thickness at the roots, they must be equally strong. Let Fig. 21 represent wheels having the teeth of both con-

FIG. 22.



ductor and conducted epicycloidic. In Fig. 22, those of the conductor only are epicycloids, and by inspecting the figure, it will be made obvious, that the teeth of A B, even when the wheels are new, must act as much upon each other, in approaching the line of centres, as they do in receding from it. Whereas the teeth of C D, when new, do not begin to act until they arrive in the line of centres; and C conducts D much further beyond that line than A does B; and even when much worn, C D acts but very little before the line of centres. But it was formerly observed, that when pinions have few teeth, they must act before they arrive in the line of centres, and consequently in such cases, the teeth of both the conductor and conducted, ought to be epicycloidic. Hence arises one of the disadvantages of wheels and pinions having few teeth, a fault carefully avoided by every good mechanic.

It has been mentioned to me, that the following rule, in order to determine the length of the teeth of wheels, is employed by the ingenious Mr. Murray of *Leeds*[*].

RULE TO DETERMINE THE LENGTH OF THE TEETH OF WHEELS.

47. Perpendicular to the line of centres C D, draw the line A B, a tangent to the pitch lines. Take half the pitch, that is, half the distance between the centres of two adjoining teeth, within a pair of compasses, setting their points upon the pitch lines E and F, parallel with the line of centres C D, draw the line *a b*, and where that is cut by the line A B at *c*, gives the points of the teeth of wheel and pinion[†].

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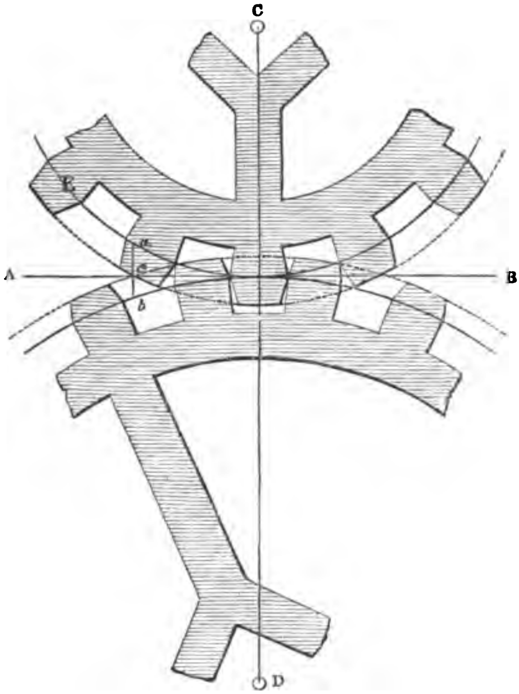
[*] This we are informed was communicated to him by the late Mr. Rupp, of Manchester, a native of Germany.

[†] This rule is founded on the properties of involute teeth; for it may easily be proved that the line *a b* will always be divided in the same ratio as the pitch line divides the distance between the centres of the wheels; and that the pitch line always divides the length of the teeth in that ratio, in involute teeth. Hence the observations in the following article of the text, are to be understood as if made on involute teeth; the properties of which our author has not been much acquainted with. See Chap. III. Art. 62.

FIG. 23.



OBSERVATIONS.

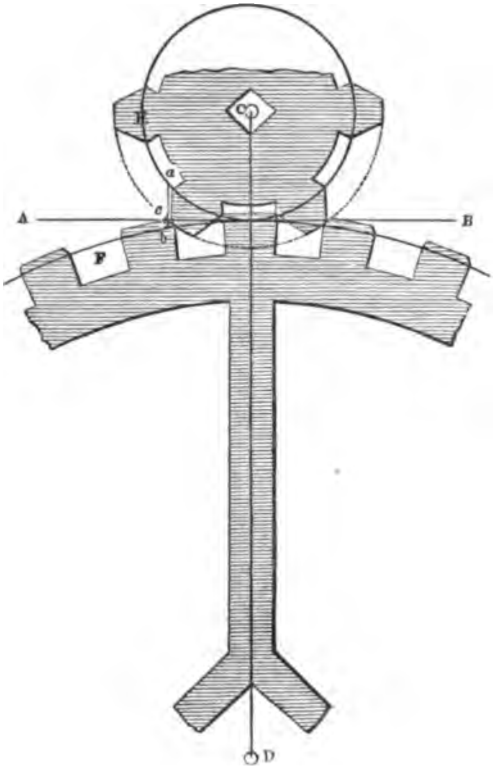
48. On this rule, Buchanan remarks that it does not seem to be founded on any satisfactory principle; were the pinion, at all times, the *conductor*, he should not perhaps differ from Mr. Murray, because the action of the teeth would be, in that case, generally *after* their arrival at the line of centres.

But in case the wheel were the conductor, the action of the teeth would generally be almost entirely *in approaching* the line of centres.

The evils arising from this mode of action, have already been clearly proved, (see Art. 37,) it is therefore unnecessary here to repeat them.

When the wheel and pinion are nearly of the same diameters, as in Fig. 23, the effects are not so obvious as when the pinion is much smaller than the wheel, as in Fig. 24.

Fig. 24.



OF THE INTERNAL PINION.

49. When a pinion is to act internally, as in Fig. 25, it is evident, that the teeth may be formed on the principles already laid down, with this difference only, that the epicycloid generated by the proportional circle of the pinion upon that of the wheel, should be an interior epicycloid.

The internal pinion may be adopted in many cases with advantage, as it has less friction than the external one*.

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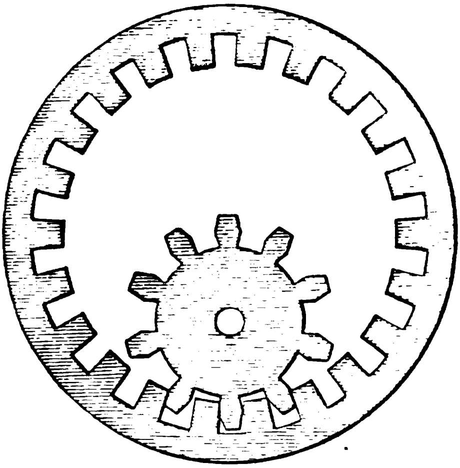
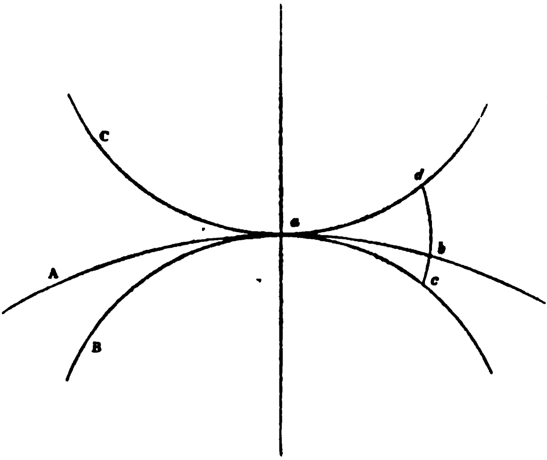
* A parallel motion upon this principle has recently been erected at the Bank of England.

FIG. 25.



50. To illustrate this, (See Fig. 26.) let A be the pitch line of a wheel, B that of an internal pinion, and C that of an external pinion.

FIG. 26.



Suppose the circle A to be moved till the point *a* arrives at *b*, and that the points *c d*, in the circles B C, have both moved over a space equal to *a b*. Now it is evident, that the distance from *c* to *b* is much less than that from *b* to *d*, and consequently had the circles moved one another by means of teeth, a tooth of the interior circle B, in the same part of a revolution, would have slid over a smaller part of a tooth of the circle A, than a tooth of the exterior circle C, and therefore would have had less velocity. But other things being equal, the less the velocity, the less the friction; an interior pinion has consequently less friction than an exterior one*.

It is upon this principle, that bevelled wheels have less friction than external spur wheels; bevelled wheels acting in a mean situation between external and internal spur wheels†.

OF THE RACK AND PINION.

51. What is called the rack and pinion, is used for various purposes in mechanics; as in jacks for raising great weights, and for the opening and shutting of sluices.

The rack and pinion should be made upon the principles of spur gears; with this difference only, that in forming the teeth, the *cycloid* is, for reasons obvious from its definition, used in place of the epicycloid‡.

Doctor Johnson gives this definition of the cycloid: "A geometrical curve, of which the genesis may be conceived by imagining a nail in the circumference of a wheel: the line which the head of the nail describes in the air, while the wheel revolves in a right line, is the cycloid."

Thus A B C, is a cycloid generated by the point *a*, in the circle D, while it revolves on the right line A C§.

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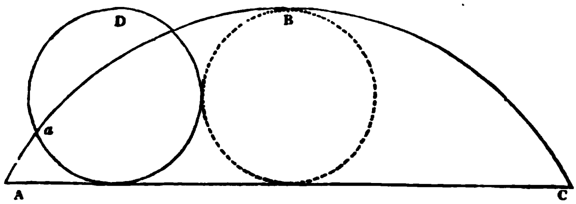
* See Art. 65.

† The notion that bevelled wheels have less friction is not correct; unless in the case where the teeth are in the concave surface of a cone, as in Fig. 41.

‡ See Art. 71.

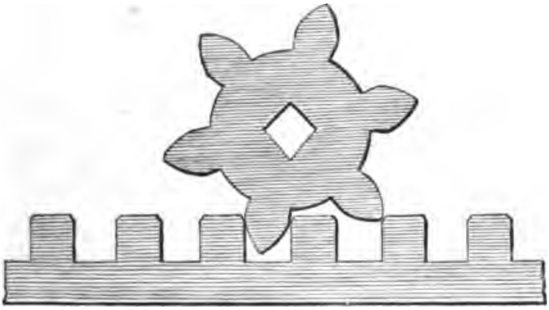
§ Velocity makes no difference in friction.

FIG. 27.



The subjoined figure represents the teeth of a rack and pinion, formed in what seems the best mode in cases where a great weight is attached to the rack.

FIG. 28.



The leaves of the pinion are made as long as the curve will admit, in order to prevent them from beginning to act before they arrive in the line passing through the centre of the pinion, perpendicular to the rack. Were they to act much before they arrived in that line, which may be considered as the line of centres, and against a very great weight, they would be apt to jam, and run the risk of their being broken, or, at least, very much increase the friction*.

52. The construction above proposed for the rack and pinion not being exactly correct, I will here endeavour to remedy that defect.

If a pinion move a rack, and the dotted line A B be the pitch line of the rack; and the dotted circle C B the pitch line of the pinion; then the curved side C D of the tooth of the pinion should be an involute of a circle. Or, it is

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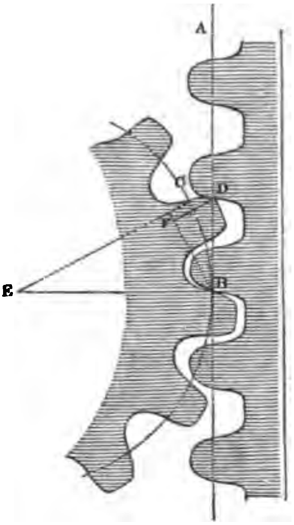
* See this Chap. Art. 33; also the Supplementary Observations.

FIG. *D.*



that curve which a point in a cord would describe on the pinion, were the pinion turned by drawing the cord constantly in the direction A B; the cord being supposed to be wound round the pitch circle of the pinion. Now as B D is the part of the cord which unwinds from the arc C B, it is obvious that the pitch lines move with equal velocities; and the force acting constantly at the same distance from the centre of motion, the force will be constant, except that variable part which is lost in friction.

53. In order that the teeth of the rack may be durable, and not liable to cut the face of the teeth of the pinion, they should extend beyond the pitch line of the rack; and the teeth of the pinion should be of sufficient length for one to move the rack, till the following tooth arrives at the point B. To determine the length that will fulfil the latter condition, or what amounts to the same thing, to

find the real radius of the pinion, we may suppose a line drawn from the point C, to the centre E of the pinion; also make F B perpendicular to C E, and F D perpendicular to F B. Then, by similar triangles we have F B : B D :: B E : the real radius $= \frac{\text{B D} \times \text{B E}}{\text{F B}}$. But B D is equal to the pitch, and F B to five sixths of the pitch, therefore the real radius $= \frac{6 \times \text{B E}}{5}$. That is, the real radius should be six fifths of the proportional radius.

In ordinary cases, the curved surfaces of the teeth of the pinion may be described from centres in the pitch circle, with the radius D B. And instead of making the teeth of the rack square to the pitch line, they may be described by the same radius B D, from points in the pitch line of the rack; and the ends of the teeth as well as the hollows to receive them, in the pinion may be semicircular. This mode of forming the teeth will make them very strong without affecting the motion.

54. When the rack impels the pinion, the curved face of each of the teeth of the rack, should be a portion of a cycloid, (as A *a*, Fig. 27,) and the leaves of the pinion straight lines radiating from the centre of the pinion; the diameter of the generating circle for describing the cycloidal teeth should be half the proportional diameter of the pinion.

SECTION II.

OF BEVEL GEAR.

55. Hitherto our inquiry has been confined to what is called spur gear, or the action of wheels and pinions whose axes are parallel: we come now to speak of what is called *bevel gear*, or the action of wheels of which the axes are angular to each other. As we formerly regarded the action of spur gear, with teeth indefinitely small, as the

rolling of cylinders upon the surface of each other, we may now regard the action of bevel gear with such teeth, as the rolling of cones in a similar manner.

In order to illustrate this, let us suppose it is required to make one wheel move another, the axes of which are not parallel.

Fig. 29.

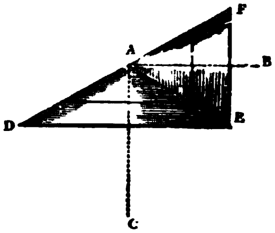


Fig. 30.

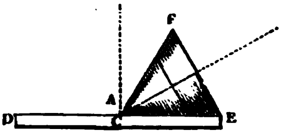


Fig. 31.

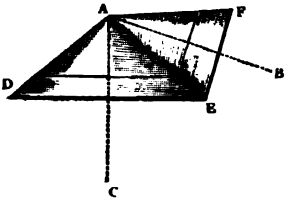
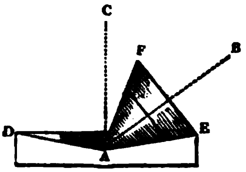


Fig. 32.



Let A B, A C, be their axes, and D E, E F, their proportional diameters or pitch lines.

To the point A, where the axes intersect, draw A E, A F, A D; then D A E, and E A F, shall be the outline of two cones[*], which rolling the one upon the surface of the other, will both revolve, so that, like two cylinders with their axes parallel[†], all the corresponding points in each, shall move in every part of their revolution with equal velocity.

For, suppose any touching point D, the diameters of the cones at that point shall bear exactly the same proportion to one another, that their bases do. The same may be said of every other point on their surfaces, and consequently they shall revolve in the same manner as two cylinders having their axes parallel, and the cones may be considered as bevel wheels with indefinitely small teeth.

But, in practice, we require finite and sensible teeth: in bevel gear, these are made similar to those of spur gear, with this difference, that in spur gear, they are parallel; but in bevel gear they must, as is evident, diminish in length and thickness, as they approach the summit of the cone.

The teeth may be made of any breadth, according to the strength required, and they are thereby enabled to overcome a much greater resistance, and work smoother, than is possible for a common face wheel and trundle, which, for that reason, are now superseded by bevel gear.

56. The epicycloid, which gives the true curve to the teeth of bevel gear, differs from that used in spur gear, in being generated by the rolling of one cone upon the surface of another, while their summits coincide.

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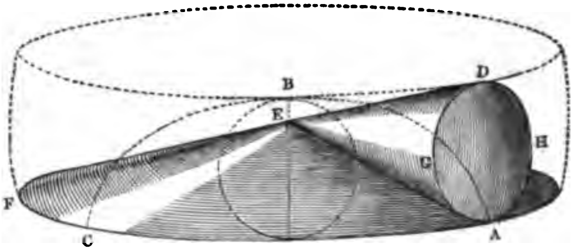
[*] These cones we shall call the proportional cones of the wheel and pinion.

[†] It is to be observed, when the axes are in certain inclinations, the surface of one of the cones becomes concave, as in Figure 32; in others, as in Figure 30, the point A is in the same plane with D E.

Thus for example, in the figure, the curve, A B C, is described by a supposed style, fixed in the point A, of the circumference of the base of the cone A D E, while it rolls upon the cone F C A E.

FIG. 33.



The style A, being always at the same distance from the point E, where the summit of the cone is fixed, all the points of the curve A B C, shall be equidistant from the point E, and consequently upon the surface of a sphere which shall have the point E for its centre.

Hence the curve is called a *spherical epicycloid*.

The circle A G D H, which in rolling describes the spherical epicycloid, is named the *generating circle* of that curve; and the part A C, of the circumference upon which it rolls, is called the base of the epicycloid.

When the sphere is given upon which the spherical epicycloid is required to be traced, and we know the size and position of the rolling cone which should generate this epicycloid, it will be easy, from what has been said relative to the plane epicycloid*, to find as many points of the curve, as may be necessary, and it will be evident, that what is said on spur gear, respecting the most advantageous figure of their teeth, is all applicable to bevel gear; with this difference, that the spherical is substituted for the plane epicycloid.

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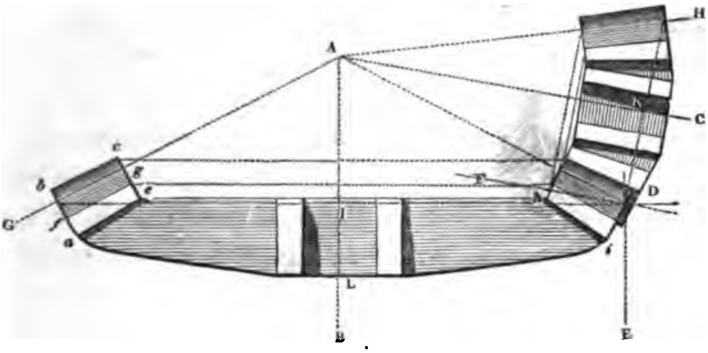
* See Chap. I. Art. 17.

In order therefore to avoid tedious repetitions, we shall immediately proceed to give some account of what seems the best practical method of laying down the lines necessary to the right construction of bevel gear.

57. Having calculated the proportional diameters or pitch lines of the wheel and pinion, draw their axes, A B, A C, in the proposed direction with respect to each other, when the wheels are in action. Parallel to A B, and at the distance of half the proportional diameter of the wheel, draw the line D E. In the same manner, draw F D at the distance of half the proportional diameter of the pinion from A C. From the point D, where these lines intersect, draw the line D G, perpendicular to A B, and also the line D H, perpendicular to C A. Make G I equal to I D, and K H equal to K D. Then D G is what we shall call the *principal diameter*, or the *diameter at the pitch line* of the wheel, and D H that of the pinion.

FIG. 34.



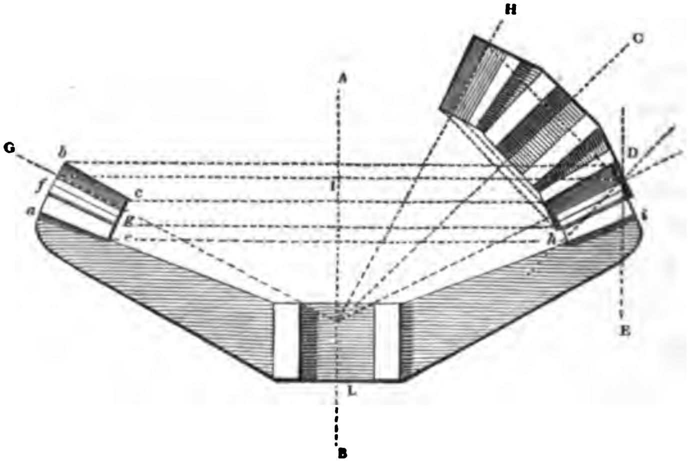
Join G A, D A, H A. Then G A D, is the outline of the proportional cone of the wheel, and D A H, that of the pinion.

Now proceed to draw the teeth of the wheel. With the distance G A, from A, as a centre, sweep a small arc, such as G *a*; at the principal diameter to their extremity, set off the length of the teeth, from G to *b*, and draw *bc* tending to A.

The line *b c* represents the breadth of the teeth, which, according to circumstances, may be more or less; only it is to be observed, that if continued to the point A, the teeth near that point would be so small as to be of little or no use.

Describe the arc *c e*, concentric to *b a*[*]; and from G to *f*, set off part of the required length of the tooth, from the principal diameter to the root: then draw *fg* tending to A, the line *fg* becomes the root of the tooth. Parallel to *fg*, draw *a e*, then *a f g e*, represent the section of the solid ring of the wheel. The particular direction of the line *a e* is no way essential; all that is necessary is, that the ring be of sufficient strength for the purpose to which it is to be applied: but patterns for cast iron wheels are usually made as represented in the plate.

FIG. 35.



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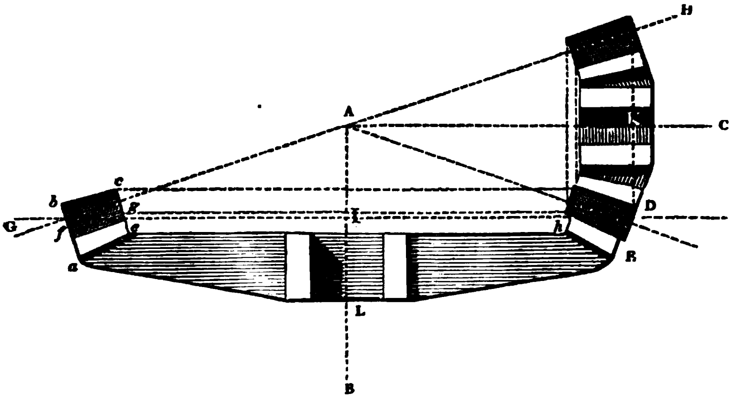
[*] In practice, it is found easier, and sufficiently accurate, to use, instead of these curves, straight lines, as near as may be, in the same direction with the curves.

FIG. 36.



Having thus drawn a section of a tooth at G, draw in the same manner one at D, then D, I, G, L, will be the section of the wheel, in which *e*, *h*, *i*, L, *a*, represent the space occupied by the arms. The dimensions of these, and their particular form, may however be varied according to circumstances.

The mode of drawing the section of this pinion will now be obvious, by inspecting the figure, where it will be observed, that the teeth of the pinion are made a little broader than those of the wheel. This is a practice generally followed, as the teeth by this means wear more equally than otherwise they would.

58. For the use of young mechanics, I will, in these additions, attempt to free the principles of constructing bevelled wheels of part of their intricacy, first briefly noticing the old principles.

The teeth of bevelled wheels, for moving one another uniformly, may be formed according to different principles. Those which have been delivered, are, first, When the wheel drives the pinion, the acting faces of the teeth of the wheel, should be portions of a spherical epicycloid, generated by the revolution of a cone, (as described in Art.

59,) of which the base is half the diameter of the pinion, and the teeth of the pinion plane surfaces directed to its centre. (Camus on the Teeth of Wheels, Art. 569. Brewster, Edin. Ency. vol. xiii. p. 575.) Second, If the teeth of the pinion be staves, or formed to act as staves, then, when the wheel drives the pinion, the acting faces of the teeth of the wheel should be portions of a curve parallel to a spherical epicycloid, generated by the revolution of a cone, of which the base is equal to the diameter of the pinion. (Camus, Art. 560. Brewster, Edin. Ency. vol. xiii. p. 575.) To these a third may be added, that is, When the pinion drives the wheel, the most advantageous form for the acting faces of the teeth of the pinion, will be a spherical involute of a circle, the teeth of the wheel being plane surfaces directed to its axis.

The description of these curves is not a very simple operation, nor yet adapted for application in practice; I shall therefore propose a new method, which appears to have escaped the notice of former inquirers, one which is general; spur wheels, racks, &c. being particular cases of its application. See Fig. *E*, p. 59.)

59. If A B be the axis of a pinion, and A C the axis of the wheel, D E the proportional diameter of pinion, and D F that of the wheel, if C B be made perpendicular to A D; then, the proper form for the acting surfaces of the teeth, may be described upon the surfaces of the cones D E B, and D F C. And since the surfaces of these cones can be spread out, or developed upon a plane surface, the form of the teeth proper for communicating equable motion, may be drawn upon a plane.

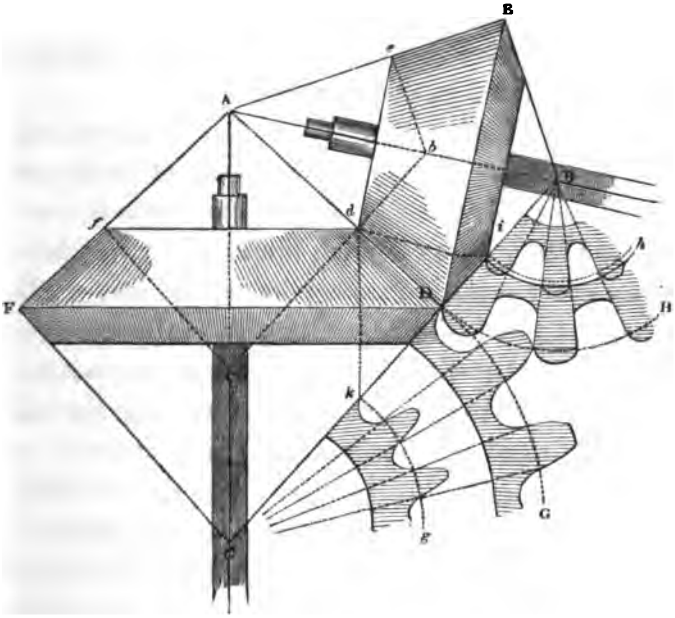
60. But the developement of a cone is a sector of a circle of which the radius is the slant height of the cone, and the arc equal to the circumference of the base of the cone. Thus, if the arc D G, be described with the radius C D; this arc D G will be the developement of part of the

circumference of the proportional circle or pitch line, of which the diameter is F D, then D C G is part of the developement of the cone, and in the same manner the developement D B H, of part of the cone D B E, may be described. For in practice, a small portion of the developement is sufficient.

Now, if the sectors that would cover part of the cones D B E, and D C F, be considered portions of spur wheels, and the teeth be formed, so that these sectors would move one another equably, by the methods described for spur wheels, (see Art. 29, 31, 42, 63,) on sheet copper, or any other flexible body that could be applied upon the surfaces of the cones, the outline of the teeth formed by these patterns, would be such as are adapted for bevelled wheels.

61. The breadth of the teeth *d* D, being settled by the

FIG. *E.*



same rules as for plane spur wheels, draw *c b* through *d*, parallel to C B. Then the form of the interior end of the teeth may be found by spreading out the cones *d c f*, and *d b e*. But this is more easily done by making *d k* parallel to the axis A C, and from the point *k*, where this line cuts C B, with the radius C *k*, describe the arc *k g*, which is part of the developement of the proportional circle at *d*. Now lines drawn from the teeth on the arc D, to the centre C, will determine the magnitude of the teeth at the arc *k*; and the teeth may be described there accordingly, as is shown in the figure. Also, when *d i* is drawn parallel to A B, we shall have B *i* equal the radius of the developement of an arc of the proportional circle of the pinion at *d*; and on this arc the pattern teeth should be described for the interior ends of the teeth of the pinion, as indicated by the lines in the figure.

Thin copper will be very well adapted for the patterns, and it will be desirable that there should be two teeth upon each pattern, but more will not be necessary.

The limit of the pitch will be the same as for spur wheels, (see Art. 138,) and the breadth of the teeth is also to be regulated by the same rules.

The length of the teeth, the friction of them, and the peculiar advantages of the different modes of forming them, may be considered on the developed pitch lines in the same manner as if they were the pitch lines of spur wheels; consequently every remark that applies to the one, applies to the other. Indeed, the only difficulty in this construction of the teeth of bevelled wheels, consists in applying the patterns correctly to the conic surface whereon the ends of the teeth are to be described; but it is a difficulty which is very easily overcome by having proper lines on that surface, to adjust the corresponding lines on the pattern by.

Perhaps it will be of use to make a few small models of wheels, in order to fully understand the process, and apply it with certainty of success. For though it is extremely simple to those accustomed to work by developed patterns, such as carpenters, joiners, and masons[*], it may not appear so, at first, to others.

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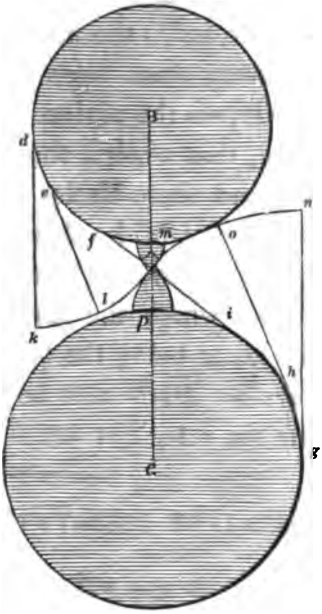
[*] See the Art. JOINERY, Supplement to the Encyclopædia Britannica, §. 11—17; or Nicholson's Carpenter's Guide, p. 18.

CHAPTER III.

62. We shall now proceed to describe a mode of forming the teeth of spur wheels, the first hint of which, we have been informed, was given by Professor Robison, of Edinburgh. In order to understand the description and demonstration, it will be necessary to recollect what was proved in Chapter I. Art. 11, viz.: That if a wheel move uniformly, it is necessary, in order to move another wheel uniformly, that the form of the teeth be such as that the perpendicular from the touching surfaces *in all situations*, cut the line of centres, in the same point A, which point divides the line of centres, so that the one part A B, shall be to the other A G, as the number of teeth in the one wheel B, is to the number of teeth in the other G.

This being understood, let B and G be the centres of the two wheels, and *def*, *ghi*, the rings upon which the teeth are placed; the diameters of which rings are to one another, as their number of teeth. If the thread *kd*, be lapped round the ring; as it folds up, its extremity *k*, will describe the curve *klm*. It is evident, that the thread in describing the curve, is perpendicular to it; and the thread, *e*, *l*, *f*, A, is therefore perpendicular to the curve at *l*, or A. In the same manner it may be shown, that the point *n*, of the thread *gn*, will describe a similar curve, *nop*, which is perpendicular to the thread *gn*, *ho*, *i* A, at the points *n*, *o* and A. If therefore *m*, A, *l*, *k*, be a curve formed by the evolution of the ring B, and *nop*, be a curve formed by the evolution of the ring of the wheel G, a line drawn through the point of contact A, perpendicular to the touching surfaces, will touch both rings in the points *f* and *i*, and the two curves, supposing them teeth, will act as if the one pulled the other by the thread *i f*. The line *i f*,

FIG. 37.



will be the line of action; that is, the teeth will always touch each other in a point of this line, and since this line always passes through the same point of the line of centres, B G, the action will be invariable; so that if the one move uniformly, the other will also move uniformly, and two weights which balance in *any one position*, will balance in all positions of the teeth.

It is obvious, that though these teeth must work, both before and after passing the line of centres, that they will work with equal truth, whether pitched deep or shallow; a quality peculiar to them, and of very great importance.

63. The following properties of involute teeth are of most importance in wheelwork.

1. When the wheel drives the pinion, the greater part

of the action will take place before the teeth arrive at the line of centres.

2. When the pinion drives the wheel, they act chiefly after passing the line of centres; and therefore involute teeth are most adapted for this case.

3. When the teeth are long, the action is very oblique, causing much unnecessary stress upon the axes; particularly when the wheel impels the pinion.

It may be remarked, that when a thread winds off from one circle to another, and these circles touch one another at the circumference, a point in the thread will describe an epicycloid; and the same epicycloid would be described by making the circle from which the thread winds off, the generating circle, the other being the base.

The length of involute teeth may be determined with sufficient accuracy by the rule, Art. 47.

SUPPLEMENTARY OBSERVATIONS.

64. The foregoing Essay was written several years before the publication of the Supplement to the Encyclopædia Britannica. Professor Robison has there (Vol. II. page 103, 106) described and recommended the mode which will be found, Chap. III. of forming teeth of wheels by involutes of circles. Dr. Brewster, however, in his second Edition of Ferguson's Lectures, Vol. II. page 227, observes, that this principle is not new; De la Hire having long ago considered the involute of a circle, as the last of the exterior epicycloids; which it may be proved to be, if we consider the generating straight line as a curve of infinite radius*.

Professor Robison says†, that "this form of teeth admits of several teeth to be acting at the same time, (twice the number that can be admitted in M. De la Hire's method.) This, by dividing the pressure among several teeth, diminishes its quantity on any one of them, and therefore diminishes the dents or impressions which they unavoidably make on each other. It is not altogether free from sliding and friction, but the whole of it can hardly be said to be sensible. The whole slide of a tooth, three inches long, belonging to a wheel of ten feet diameter, does not amount to one sixtieth of an inch, a quantity altogether insignificant*.

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* See Art. 71.

† Encyclopædia Britannica, Volume XX. page 104. See also Rees's Cyclopædia, Art. Clock Movement.

In the same article, this highly respectable philosopher was mistaken, in supposing, with other eminent authors, that *the mutual action of the teeth*, (when formed into epicycloids, by the method of M. Camus,) *is absolutely without friction*, and in saying, "*That one tooth only* APPLIES *itself to the other, and* ROLLS *on it, but does not* SLIDE *or* RUB *on it in the smallest degree. This makes them last long, or rather does not allow them to wear.*" A very slight examination of the figures given in various parts of the preceding Essay, will, I hope, show, that the point of contact must slide from the pitch line of the conducting tooth outwards. Dr. Young, in his Natural Philosophy, Vol. II. page 183, says, that "*a form* [of teeth] *without friction, is perfectly impracticable, although, for a single tooth, possible.*"

65. In the first volume of the same work, he makes the following judicious observations on our present subject:

"It has been supposed by some of the best authors that the epicycloidal tooth has also the advantage of completely avoiding friction; this is however by no means true, and it is even impracticable to invent any form for the teeth of a wheel, which will enable them to act on other teeth without friction.

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* See Art. 70.

"In order to diminish it as much as possible, the teeth must be as small and as numerous as is consistent with strength and durability; for the effect of friction always increases with the distance of the point of contact from the line joining the centres of the wheels.

"In calculating the quantity of the friction, the velocity with which the parts slide over each other has generally been taken for its measure: this is a slight inaccuracy of conception, for, as we have already seen, the actual resistance is not at all increased by increasing the relative velocity; but the effect of that resistance, in retarding the motion of the wheels, may be shown, from the general laws of mechanics, to be proportional to the relative velocity thus ascertained. When it is possible to make one wheel act on teeth fixed in the concave surface of another, the friction may be thus diminished in the proportion of the difference of the diameters to their sum.

"If the face of the teeth, where they are in contact, is too much inclined to the radius, their mutual friction is not much affected, but a great pressure on their axes is produced; and this occasions a strain on the machinery, as well as an increase of the friction on the axes."*

The concluding part of these observations appears to me peculiarly applicable to the figure of teeth described in our last chapter; for in wearing, they will be more liable than many other forms, to have *the face of the teeth, where they are in contact, too much inclined to the radius.*

REMARKS ON THE FRICTION OF WHEEL WORK, AND ON THE FORMS BEST SUITED FOR TEETH,

IN A LETTER FROM DR. YOUNG.

66. "I have been considering your observations on the difference of the friction, accordingly as the teeth

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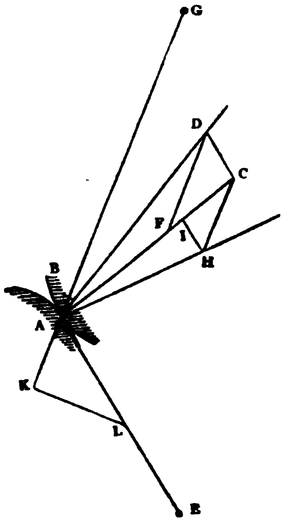
* Young's Lectures, Vol. I. p. 176.

touch before or after the line of centres; at first I was disposed to doubt of the fact; but, upon more mature examination, I found that, like many other practical observations, they went beyond the scope of the doctrines of theoretical writers. I cannot however perfectly agree with you as to the explanation of the fact; but I will state to you briefly my opinion on the subject, not having leisure at present to enter into a more ample discussion.

"The magnitude of the friction has usually been estimated by the relative velocity of the surfaces concerned; a mode of calculation, which, as I have observed in my lecture on machinery, is only so far correct, as it shows the comparative effect of a given friction in retarding the machine. But in fact the primitive friction itself is liable to variation, according to the obliquity of the surfaces; for since the friction is nearly proportional to the mutual pressure, it will be greater or less, as the direction of these surfaces is more or less inclined to the radii, the force of rotation being supposed to be given: and, what is of still more immediate importance to the resolution of the difficulty in question, the direction of the force, by which the one wheel acts on the other, is not to be considered as perpendicular to the surface of the teeth, but as oblique to it, being so situated as to oppose the joint result of the direct resistance and the friction; that is, as being inclined to the surface in a certain constant angle, which a late anonymous writer has called the angle of repose, and which is equal to the inclination of a plane, on which one of the substances concerned would begin to slide on the other by its gravitation.

67. "Let the tooth A impel the tooth B with the given force A C, perpendicular to the common surface of the teeth; make C A D equal to the angle of repose, then the force must act in the direction A D, and making C D parallel to the radius A E, A D, will be the actual pressure: then

FIG. 38.



drawing D F parallel to the radius A G, A F will be the effective force in the direction A C, and F C will be the loss by friction. Again, if B impel A, the angle of repose must lie on the other side of A C, and C H must be parallel to A G, and H I to A E, and the friction in this case will be I C, which is obviously less than F C.

" Hence we may easily calculate the magnitude of the resistance F C, or I C, produced by friction, calling the force A C unity; for C D becomes $\frac{s.\,CAD}{s.\,ADC}$, and F C $=$ C D. $\frac{s.\,CDF}{s.\,CFD} = \frac{s.\,CAD}{s.\,ADC} \cdot \frac{s.\,CDF}{s.\,CFD} = \frac{s.\,CAD}{s.\,(CAE + CAD)} \cdot \frac{s.\,GAE}{s.\,GAC}$; and in the same manner $IC = \frac{s.\,CAH}{s.\,AHC} \cdot \frac{s.\,CHI}{s.\,CIH} = \frac{s.\,CAD}{s.\,(GAC + CAD)} \cdot \frac{s.\,GAE}{s.\,EAC}$. Both these quantities vary ultimately as the angle formed by the radii, and vanish when the point of contact is in the line of the centres; and in this case the common theory agrees with this calculation. When C A E is always a right angle, as in the epicycloidal tooth commonly recommended, the friction F C varies, in the different positions of the teeth, as $\frac{s.\mathrm{GAE}}{s.\mathrm{GAC}}$, or as *cot.* G A C; that is, if A K be made constant, as K L.

68. "Since therefore it is demonstrable, that the friction or pressure is always greater in approaching the line of the centres, than at an equal distance beyond it, it must obviously be desirable that the contact should be rather after than before the passage of the teeth over that line, although it is better that it should be at a small distance before, than at a much greater distance beyond it. Hence the impelling teeth ought to be of such a form, as to accelerate the motion of the impelled a little before, and a little more after, the passage of the line of the centres, and then to retard it again, so that the next tooth may succeed to a similar operation.

69. "A wheel acting on a trundle, with cylindrical staves, has in this respect an advantage over two wheels with teeth, since the curve, fitted for impelling the trundle, is adapted only to act on it beyond the line of the centres. This curve may however be formed more easily, and at the same time more advantageously, than by the method which has hitherto been recommended: for if we employ an epicycloid described by the rolling of a circle, which would just touch the internal surface of all the staves of the trundle, on the circumference of the wheel, the trundle will at first be accelerated a very little, and will then be allowed to fall back from each tooth to the succeeding one, soon after its passage over the line of the centres. The same form will also answer very well, when the trundle is to impel the wheel, although this mode of action produces a greater friction than the former.

70. "A similar advantage may be obtained in teeth of any other form, by finishing them in such a manner as to project a very little beyond the regular outline, at the point

which is intended to come into contact a little beyond the line of the centres. Such a corrected outline may be described at once, if it be required. If the tooth is to be formed into an involute of a circle, having fitted a thread or fine wire to the circumference of the wheel, find the point of contact at the instant when the end of the wire is describing the part of the tooth which is to act at, or a little before, the line of the centres; cut off from the wheel, beyond this point, an arc equal to the distance of the centres of two adjoining teeth, and fix a pin in the tangent at the same point, that is, in the continuation of the part of the wire which is unrolled, at such a distance as just to stretch the part which is left loose by the removal of the arc: the pin thus fixed, and the remainder of the circle, will serve as bases for continuing the evolution of the wire, and the description of the tooth. The same position of the wire will show the outline of a basis proper for describing by means of a circle rolled on it, the curve which must be substituted for the form of any epicycloidal tooth, which might have been described by causing the same circle to roll on the simple circumference of the wheel as a basis; the curved part of the tooth beginning, in this case, at the point of contact first mentioned.

"If it be objected, that in such an arrangement, the equability of the motion would be lost, and a shake would be created; it may be answered, that the inequality would be utterly imperceptible in practice. But I do not know, that the form, thus determined, would have any material advantage over teeth made as short as possible, or so cut away as not to act before the passage of the line of centres, which may easily be done in all cases, nearly in the same way as you have shown with respect to epicycloidal teeth.

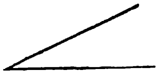
"The advantage of dividing the pressure among several teeth ought not to be purchased at the expense of an increase of friction, since the property of greater durability may be obtained, in an equal degree, by simply making the wheels thicker, without materially adding to the friction: and in fact, although the momentary pressure on each tooth may be lessened by dividing it, yet its duration is increased in the same proportion.

71. "I must beg leave to observe, that the form proper for the teeth of a pinion, acting on a rack, is the involute of a circle, and not a cycloid. The cycloid would be a proper form for the teeth of the rack, if they were intended to impel the pinion.

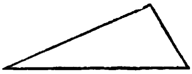
"It has been remarked that the form of the involute of a circle is not immediately deducible from the general principle of La Hire; and the remark is strictly true, since the curves, formed, according to that principle, from two contiguous circles as bases, could not act on each other without a further separation of the centres, which would render the demonstration inadequate. But I have observed in the Additions to my second volume, p. x. the principle may be extended to any other curves, as well as circles and straight lines: and if we employ an equiangular spiral, instead of a straight line, we shall have the involutes, exactly as they are recommended for practice."

SUPPLEMENTARY DEFINITIONS.

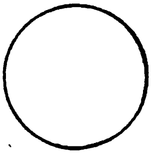
72. An angle is the inclination of two lines to one another which meeting do not lie in one line.



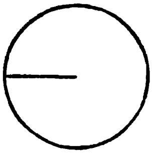
73. A triangle is a figure contained by three straight lines.



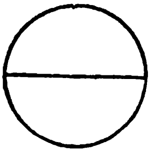
74. A circle is a plane figure contained by one line, which is called the circumference, and is such, that all straight lines drawn from a certain point within the figure to the circumference, are equal to one another, and this point is called the centre of the circle.



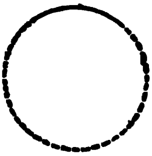
75. The radius of a circle, is a straight line drawn from the centre to the circumference. The word *radii* is used, when more than one such line is spoken of.



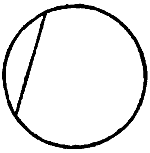
76. The diameter of a circle, is a straight line drawn through the centre, and terminated both ways by the circumference.



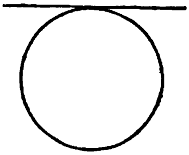
77. The arc of a circle is any part of its circumference.



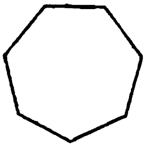
78. A chord of an arc, is a straight line joining the two extremities of the arc.



79. A tangent of a circle is a straight line, which passes through a point in the circumference without cutting it.



80. A polygon is a figure, having more than four sides. The term is seldom applied to figures that have less than five sides.

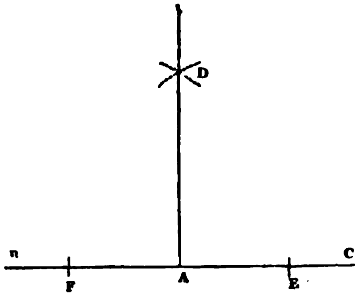


81. Parallel straight lines are such as are in the same plane, and which, being continued ever so far either way, never meet.

82. The word perpendicular is the same with square, as used by workmen.

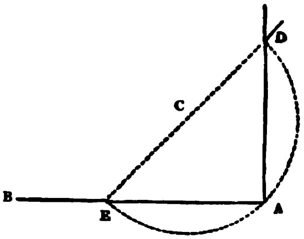
In order to draw from a given point, A, in a given line B C, another line perpendicular to it,

Take A E, equal to A F, and from the points F and E, with any radius greater than A B, make the intersection D; draw D A, which will be perpendicular to B C.



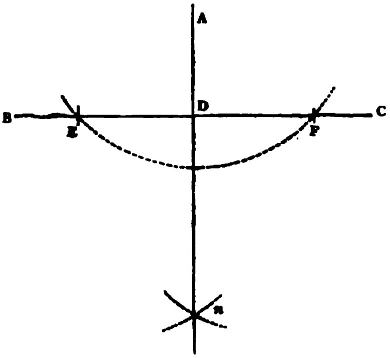
83. If it be required, from the end of a given straight line A B, to raise a perpendicular,

Take the point C nearer to A than B; about the centre C, with the radius C A, describe the circle, E A D; through the points E and C, draw the line E D, and join A D, which will be perpendicular to A B; and B A D, is called a right angle.

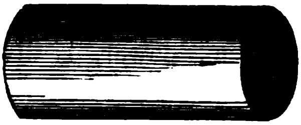


84. To let fall from a given point A, a perpendicular upon a given straight line B C.

About the given point, describe a circle cutting B C in E and F; from the points E and F, make the section *n*, and draw the line A D from A towards *n*, and A D is the perpendicular required.



85. A cylinder is a body having two flat surfaces, and one circular. For instance, a roller is a cylinder.



86. A cone is a solid body, of which the base is a circle, and which ends in a point.

87. Velocity is a term equivalent to speed.

88. Mr. Smeaton thus defines the term power: "The word power, as used in practical mechanics, I apprehend to signify the exertion of strength, gravitation, impulse, or pressure, compounded with motion, to be capable of producing an effect; and that no effect is properly mechanical, but what requires such a kind of power to produce it."*

89. A proposition is a sentence in which any thing is affirmed.

90. A corollary is an inference or deduction.

91. The extract from Smeaton's works in Art. 88, conveys very little information respecting power; and yet it is necessary that every mechanic should have correct ideas on this subject, which will be a sufficient reason for introducing a further explanation here.

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* Smeaton's Miscellaneous Papers, p. 30. See Art. 91.

92. *Power* is the general term for that which causes motion or rest. For bodies in nature are in a state of rest, only, when the opposing powers acting upon them are in equilibrium.

But this general term, power, is divided into several particular ones according to the circumstances under which it acts.

93. When power is, or can be, balanced at rest, it seems to be most proper to call it *force;* but, to distinguish more precisely the circumstances of its action, it is necessary to employ the simple terms, *weight*, *pressure*, and *stress*, and also the compound terms, force of attraction, force of gravity, cohesive force, centripetal force, centrifugal force, and others of a like nature.

94. But when a body is in motion, its power, at any instant, or at any point in its path, is usually termed *momentum*, or moving force, or quantity of motion. It is this species of power which Sir Isaac Newton makes the object of his second definition. (Mathematical Principles of Natural Philosophy, Book I.) Some writers propose to use the term energy instead of momentum, (See Edin. Rev. vol. xii. p. 130,) but there does not appear to be sufficient reason for adopting it, the other having been in a considerable degree restricted to this species of power*.

Here we take the liberty of remarking, that neither the measure of momentum nor that of any other kind of power, has any relation whatever to time; for momentum simply expresses the quantity of power in a moving body at a particular instant, without reference to the rate of accumulation, or to the effect it would produce; and similar remarks apply to other species of power.

95. It is further necessary, both for practical and scien-

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* The term energy has also been applied to the product of the mass of the body into the square of its velocity. See Dr. Young's Nat. Phil. Vol. II. Art. 347.

tific purposes, to have a term to designate that power which is equivalent to momentum, when the velocity is uniform. Smeaton employed the term *mechanical power* for this purpose; and since this term is sanctioned by the language of all writers on the first principles of mechanics; and the simple machines, by means of which such power is modified to produce the desired effect, have always been called the *mechanical powers**. We think it will be found desirable to use the term mechanical power in preference to any other that has been proposed. The term impetus is objectionable, because it indicates a degree of violence in the action of power, which does not agree with what takes place in the most common applications of mechanical power. And it is questionable, whether its proposer did not intend it to be a measure of effect.

We must now attempt to inform the reader, more particularly, of the circumstances to which these different modifications of power apply, and in so doing, we shall have occasion to place a most interesting department of mechanical science in a different light from what it has been regarded by my predecessors.

96. *Force* is immediately comparable with the weight of a quiescent body. Its intensity, direction, and equilibrium, are the proper objects of that part of mechanics called statics, or hydrostatics, and aerostatics, when the body exerting force is fluid.

97. *Momentum*, or the force of a moving body, is proportional to the quantity of matter in the body, multiplied by its velocity at that instant when the comparison is made. Its rate of increase and decrease, its direction, and equilibrium, are the proper objects of those parts of mechanics called dynamics and hydrodynamics. In fact, statics is

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* See Dr. Jamieson's Mechanics for Practical Men, comprising Treatises on the Composition and Resolution of Forces, the Centre of Gravity, and the Mechanical Powers.

only that particular case of dynamics when the velocity is nothing. In like manner we simplify an important part of the science of mechanics by separating all problems in which the velocity is uniform; because in that case the length of the line the body moves over, is proportional to the velocity.

98. *Mechanical power* then is, a particular name for the momentum of a body in uniform motion; in that case, it is proportional, to the quantity of matter in motion, multiplied by the length of the line through which it acts; consequently, in all problems where the motions are uniform, (and there can be no difficulty in distinguishing such problems,) this measure of power may be employed, and its motion, equilibrium, and direction, determined accordingly. Every person conversant with the management of such problems must be aware of the advantage of this mode of investigation; it applies to the motion of water-wheels, of wind-mills, of rivers, the resistance of fluids, &c. &c., and in general to the motion of machines. It has often been partially applied in considering the equilibrium of mechanical powers, (see Wood's Mechanics, prop. xxx. and xxxi.,) but we are not aware of its having been previously pointed out as a general principle, with the object of forming a distinct branch of mechanics. Some writers have confounded measure of power with measure of effect so far as to suppose, that mechanical power is identical with the quantity of matter multiplied into the square of its velocity; we hope the true nature of mechanical power is here so defined as to prevent a recurrence of a like mistake.

It is much to be regretted that power has not been made the basis of all mechanical science, in the place of motion; for motion is merely an affection or mode of matter acted upon by unbalanced force. For, in the practical application of mechanics, it would prevent error; and in the theory of mechanics, that interesting phenomenon, *the accumula-*

tion of power, must have been forced upon the attention of philosophers.

99. Buchanan had long employed himself in making a collection of facts respecting wheels actually in use in millwork, and, by arranging them agreeably to his own views, draw such useful practical inferences as might benefit workmen generally. All the facts he had been able to collect and arrange will be found in the following Chapter.

Time and other circumstances did not allow him to enter more minutely into the subject. Yet these hints, even in their present state, may lead to a fuller investigation, and they will not be altogether without some advantage, especially as nothing exactly of the same kind has hitherto been published in this country.

With respect to the elementary propositions which guide us in this inquiry into the proportional strength of the teeth of wheels, Buchanan did not enter into their demonstrations. To the artisan, unacquainted with mathematics, they would be unintelligible; and the mathematician can either demonstrate them himself, or have recourse to those elementary writings where the demonstrations may be found: of these last, as being more generally accessible, we refer to "Emerson's Mechanics," quarto edition; and to the volumes of the "Encyclopædia Metropolitana" comprising machinery, and edited by Professor Barlow, of Woolwich; also Dr. Robinson's "Mechanical Philosophy," as edited by Brewster, and the excellent paper of Mr. Willis, on the "Teeth of Wheels," published originally in the second volume of the Transactions of the Institution of Civil Engineers, London, 1838, and added as an Appendix to this work of Robertson Buchanan.

CHAPTER IV.

A PRACTICAL INQUIRY RESPECTING THE STRENGTH AND DURABILITY OF THE TEETH OF WHEELS USED IN MILL-WORK.

100. Having treated of the forms of the teeth of wheels, we come now to consider their proportional strength with relation to the resistance they have to overcome.

We are aware that, owing to a great variety of circumstances, this subject is involved in much difficulty, and that it is no easy task to form any general rule with regard to the pitches and breadths of the teeth of wheels. We do not pretend to more than a mere approximation towards general rules; yet, were this judiciously done, we are of opinion, that it might be useful to the millwright, who has not had leisure or opportunity for scientific inquiries. A rule, though not absolutely perfect, is better in all cases, than to have no guide whatever.

And it is too evident to require proof, that it is essential to the beauty and utility of any machine, that the strength and bulk of its several parts be duly proportioned to the stress or wear to which the parts may be subject.

Some general observations on the wheel work of mills, will serve greatly to simplify our inquiries on the subject.

GENERAL OBSERVATIONS ON THE WHEEL WORK OF MILLS.

101. Mistaken attempts at economy have often prompted the use of wheels of too small diameter. This is an evil which ought carefully to be avoided. Knowing the pres-

sure on the teeth, we cannot with propriety reduce the diameter of a wheel below a certain measure.

Suppose, for instance, a water wheel of 20 horses' power, moving at the pitch line with a velocity of 3½ feet per second. It is known, that a pinion of 4 feet diameter, might work into it, without impropriety; but we also know, that it would be exceedingly improper to substitute a pinion of only one foot diameter, although the pressure and velocity at the pitch lines in both cases would be, in a certain sense, the same. In the case of the small pinion, however, a much greater stress would be thrown on the *journeys* (or *journals*) of the shaft. Not, indeed, on account of torsion or twist, but on account of transverse strain, arising, as well from greater direct pressure, as from the tendency which the oblique action of the teeth, particularly when somewhat worn, would have to produce great friction, and to force the pinion from the wheel, and make it bear harder on the journals. The small pinion is also evidently liable to wear much faster, on account of the more frequent recurrence of the friction of each particular tooth.

That these observations are not without foundation, is known to millwrights of experience. They have found a great saving of power, by altering corn mills, for example, from the old plan of using only one wheel and pinion, (or *trundle*,) to the method of bringing up the motion, by means of more wheels and pinions, and of larger diameters and finer pitches.

The increase of power has often by these means been nearly doubled, while the tear and wear has been much lessened; although it is evident, the machinery, thus altered, was more complex.

The due consideration of the proper communication of the original power, is of great importance for the construction of mills on the best principles. It may easily be seen, that in many cases, a very great portion of the original power is expended, before any force is actually applied to the work intended to be performed.

Notwithstanding the modern improvements in this department, there is still much to be done. In the usual modes of constructing mills, due attention is seldom given to scientific principles. It is certain, however, that were these principles better attended to, much power, that is unnecessarily expended, would be saved. In general, this might be in a great measure obtained, by bringing on the desired motions in a gradual manner, beginning with the first very slow, and gradually bringing up the desired motions, by wheels and pinions of larger diameters. This is a subject which should be well considered before we can determine, in any particular case, what ought to be the pitch of the wheels. In the case above alluded to, where the supposition is a pinion of 4 feet diameter, or of 1 foot diameter; it is obvious, that the same pitch for both would not be prudent. That for the small pinion, ought to be much less than that which might be allowed in the case of the larger pinion. It is also equally obvious, that the breadth of the teeth, in the case of the small pinion, ought to be much greater than that in the case of the larger pinion.

102. It is evident, however, that although great advantage may often be derived from a fine pitch, that there is a limit in this respect, as also with regard to the breadth. We shall endeavour to find some trace of this limit in what follows; and that we may the better do this, we shall call in the aid of propositions, which are true with respect to pieces of timber, or metal, subjected to ordinary cases of pressure. It is allowed, that they cannot here, in strictness, be *demonstrated*, as applicable to wheelwork. Yet they will, for want of better light, serve at least to prevent any material

practical error with regard to the strength of the teeth of wheels. For it is to be remembered, that we are not so much here in search of truths of curious or profound mathematical speculation, as of that kind of evidence of which the subject admits, and which may be sufficiently satisfactory for any practical purpose.

It must, however, be understood, that we suppose the diameters made sufficiently great to prevent the evils which we have already noticed, and in the annexed table, (Art. 120,) are some examples in actual use, which have been found in practice sufficiently durable. We would particularly recommend attention to those of Boulton and Watt, whose most extensive practice, as well as scientific knowledge, renders their work a model well worthy the attention of millwrights.

As cast-iron pinions are now generally used, and as the teeth of the pinion are most subject to wear, I think we are safe, in the present inquiry, in considering them all as cast-iron.

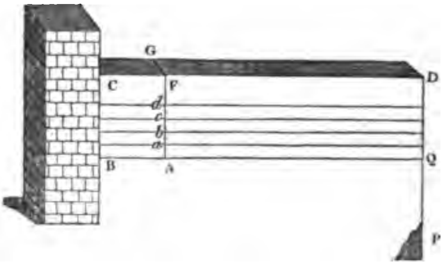
The laws to which we have alluded in this investigation are these :—

PRINCIPLES OF PROPORTIONING THE STRENGTH OF TEETH OF WHEELS.

PROPOSITION I.

103. *The strength of any piece of timber, or metal, whose section is a rectangle, is in direct proportion to the breadth, and as the square of the depth.*

Let BD be any beam, placed horizontally, and fixed at the end BC, and let AFG be the perpendicular section in which the fracture is supposed to take place. Divide the depth AF into an infinite number of equal parts at *a*, *b*, *c*, *d*, &c., whose aggregate is n = AF, and through each of those divisions suppose straight lines to be drawn parallel to FG the upper side of the beam. Then let any force be applied at P in the direction DP to break the beam at AF. Now,



since the strength of the timber is nothing but the force by which the particles cohere together, the breaking of the timber is nothing but overcoming this force and separating the parts from one another.

Let the force of cohesion of any one of the parts be denoted by unity, and imagine QAa, QAb, QAc, &c., to be so many bent levers whose fulcrum is at A; we have then to inquire what will be the sum of all the forces applied at Q the extremity of the levers, to break the beam at A.

Now by the property of the lever, the power applied at Q to equal or overcome the resistances at A, a, b, c, &c., will be $\frac{0}{AQ}$, $\frac{Aa}{AQ}$, $\frac{Ab}{AQ}$, $\frac{Ac}{AQ}$, &c., to $\frac{AF}{AQ}$; or because the cohesive force of any one of the filaments in the section is represented by unity, it will be

$$\frac{0}{AQ}, \frac{1}{AQ}, \frac{2}{AQ}, \frac{3}{AQ}, \text{ \&c., to } \frac{n}{AQ}.$$

Consequently, the effect of all the forces applied at Q to break the beam; that is, the whole strength of the beam will be, as

$$\frac{1}{AQ}(0 + 1 + 2 + 3 + \ldots . + n), \text{ or as } \frac{n^2}{AQ}.$$

Therefore, since AQ is given, the strength of the beam is as the square of the depth, or as $AF^2 = n^2$. Now, it is obvious, that if the breadth FG be increased in any proportion, the strength of the parts must be increased in the same proportion. So that the absolute lateral strength of the beam, will be

$$\text{as } FG \times AF^2.$$

104. Hence may be inferred, that the strength of teeth of wheels, moving at the same velocity, and under the same circumstances, is directly in proportion to their breadth, and as the square of their thickness. Thus, for example, if we double the breadth, we only double the strength; but if we double the thickness, in other words, double the pitch, keeping the original breadth, we increase the strength four times.

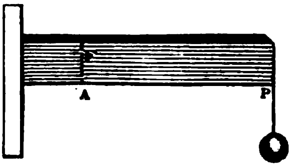
For although when wheels are working accurately, the strain is at the same time divided over several teeth, yet as a very small inaccuracy, or even the interposition of any small body, such as a chip of wood or stone, throws the whole stress upon a single tooth in practice; therefore, and in order to simplify this case, we may consider the strength of a single tooth, as resisting the pressure of the whole work.

But as the length of teeth commonly varies with the pitch, this circumstance must be taken into account, and the most simple view we can take of it seems to be, that of having the strain of each tooth, thrown all to the outward extremity; we have then the following proposition to guide this part of our inquiry.

PROPOSITION II.

105. *If any force be applied laterally to a lever or beam, the stress upon any place, is directly as the force and its distance from that place.*

For suppose PAF to be a bent lever; it is evident that the greater the power applied at P, the greater is the



force exerted at F to separate the particles of the beam in that place. Also the greater the distance AP, the greater effect has any power applied at P to overcome the cohesion of the wood at F; and therefore, the whole stress depends upon both.

PROPOSITION III.

106. *The pitch being the same, the stress is inversely as the velocity.*

This is obvious, for the teeth of wheels, and the wheels themselves, which act with greater force, must be proportionally stronger; and in any combination of wheels and axles, the strength must diminish gradually from the weight to the power, so that at every part it may be reciprocally as the velocity of that part.

For example—if the pitch lines of one pair of wheels be moving at the rate of 6 feet in a second, and another pair of wheels, in every other respect under the same circumstances, be moving at the rate of 3 feet in a second, the stress on the latter is double of that on the former.

107. This proposition is true only in the wheels of the same machine. To render it universal, the first movers of all machines must be reduced to the same standard; which may be done as follows, where the horse power is supposed to be the standard.

If P be the power of the first mover, in any machine, and V its velocity; also, let v be the velocity of any part on which it is necessary to determine the stress. Then, as $v : \mathrm{V} :: \mathrm{P} : \text{stress} = \frac{\mathrm{PV}}{v}$. (Wood's Mechanics, Prop. XXXI.)

Now taking the same value of the horse power as is used throughout this Work; that is 200 lbs. moved at the rate of $3\frac{2}{3}$ feet per second, and using H to represent the number of horses, we have $\frac{\mathrm{PV}}{v} = \frac{200\,\mathrm{H} \times 3\frac{2}{3}}{v} = \frac{733\frac{1}{3}\,\mathrm{H}}{v}$.

By using $\frac{740\,\mathrm{H}}{v}$ = the stress, we get rid of the fractions, and have a sufficiently accurate measure of the force. Hence, if the power of a machine be equal to any number H of horses, the stress at any pitch line of which the velocity is v feet per second will be $= \frac{740\,\mathrm{H}}{v}$.

In a like manner, the stress at the surface of any journal or shaft may be found.

But, in those cases where the same first mover gives motion to different trains of machinery, the stress, at any part of any one of them, should be measured by the greatest number of horses' power necessary to perform the work assigned to that train. That is, if H be the number of horses that could perform the work, then $\frac{740\,\mathrm{H}}{v}$ = the stress at any point in the train moving with the velocity v.

We shall confine our attention for the present to wheels having cast-iron teeth; and in order to take experience as our guide, several examples in the annexed tables, actually in use, are selected.

The pitch, velocity, and strain, are all stated; the strain is measured by the *horses' power*, at which the resistance is valued. Horses' power is a term now in general use, to express the force required, in order to drive any kind of mill, and it may be proper here to give some further account of it.

HORSES' POWER.

108. Although horses are not all of one strength, yet there is a certain force now generally agreed upon among those who construct steam engines, which force is denominated a *horse's power*, and hence, steam engines are distinguished, in size, by the number of horses' power to which they are said to be equal.

The measure of a mechanical effect equal to a horse's power, has been much disputed: this I believe to be a matter of little consequence, if the measure be generally understood, since there is no such thing as bringing this into any real measure. Some horses will work double of others, and horses of one country will work more than those of another. Desagulier's measure is, that a horse will walk at the rate of 2½ miles per hour, against a resistance of 200 lbs.* and which gives, as a number for comparisons, 44,000; that is, the raising of 1 lb. 44,000 feet in a minute; or, what amounts to the same, the raising of 44,000 lbs. 1 foot in a minute.

Emerson's measure is the same as Desaguliers's, (see Emerson's Mechanics, p. 178,) and Mr. Smeaton's result is 22916 lbs. under the same circumstances†.

James Watt found, from repeated experiments, that 33,000 lbs. 1 foot per minute, was the average value of a horse's power; but his engines were calculated to work equal to 44,000 lbs. 1 foot per minute.

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* When working 8 hours a day, (Desaguliers's Course of Experimental Philosophy, Vol. I. p. 241,) 2½ miles per hour is equal to 220 feet per minute, or 3⅔ feet per second.

† Reports, Vol. I. p. 229. But Desaguliers gives the immediate power of a horse, Smeaton the effect of that power applied to raise water: hence the friction of the machinery should be added to Smeaton's horse's power.

But, that he allows only 33,000 in his calculations, applied to mills, considering the difference as being lost in the friction of the engine itself."

109. It is common in practice, to reckon, that it requires one horse's power to drive 100 spindles with preparation of cotton water twist.

110. One thousand spindles with preparation cotton mule yarn.

111. Seventy-five spindles with preparation flax yarn.

We beg leave here to make the following extract, on the subject of animal force, from Dr. Young's Natural Philosophy, Vol. II. p. 165.

112. "In order to compare the different estimates of the force of moving powers, it will be convenient to take a unit which may be considered as the mean effect of the labour of an active man, working to the greatest possible advantage, and without impediment; this will be found, upon a moderate estimation, sufficient to raise 10 pounds 10 feet in a second, for 10 hours in a day; or to raise 100 pounds, which is the weight of 12 wine gallons of water, 1 foot in a second, or 36,000 feet in a day, or 3,600,000 pounds, or 432,000 gallons, 1 foot in a day; this we may call a force of 1, continued 36,000."

113. Immediate force of men and horses, without deduction for friction.

	Force.	Continuance.	Day's Work.
"A man of ordinary strength can turn a winch, with a force of 30 pounds, and with a velocity of 3½ feet in 1″ for 10 hours a day."—Desaguliers	1·05	10h.	1·05
"Two men working at a windlass, with handles at right angles, can raise 70 pounds more easily than one can raise 30."—Desaguliers......	1·22	...	1·22
"For a short time, a man may exert a force of 80 pounds, with a fly, when the motion is pretty quick."—Desaguliers........................	3·	1″	
"A horse can draw, with a force of 200 pounds, 2½ miles an hour for 8 hours in the day.........	7·33	8h.	5·87
"With a force of 240, only 6 hours."—Desaguliers...	8·8	6h.	5·28

114. Performance of men and horses by machines.

	Force.	Continuance.	Day's Work.
"A man can raise, by a good common pump, a hogshead of water 10 feet high in a minute, for a whole day."—Desaguliers....................	·875	...	·875
"By means of pumps, a horse can raise 250 hogsheads of water 10 feet high in an hour."—Smeaton's Reports[*]..............................	·364	8h.	

115. The power of men and horses to move machines, has very frequently been made a subject of investigation by writers on mechanics; but yet it appears possible to consider it in a different manner, which will furnish results from principles somewhat more strictly practical than those which have hitherto been made the basis of calculation.

116. It is almost always a necessary condition, that the moving power of a machine be sensibly uniform, and consequently the power which moves it, should be uniform, or of that kind termed mechanical power, see Art. 98.

117. Let PD be the mechanical power of a horse, which can be continued during a whole day, and also day after day, without exhausting its strength. Then, since the exertion of the muscles will be constant, the product PD will be a constant quantity, for the degree of exertion will not be altered by altering either D, the distance passed over in a second, or the force P. But, let D be the distance when the force P is the least possible corresponding to the manner of action the machine requires, or in other words, when the power to move the machine to produce useful effect would be nothing. And let $p\,d$ be the mechanical power, when the distance passed over in a second is d. Then, $PD = pd$, and, $d\,(p - P)$ = the mechanical power exerted on the machine, which is to be the greatest possible. But $P = \frac{pd}{D}$, therefore $p\left(d - \frac{d^2}{D}\right)$ = a maximum.

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[*] Vol. I. p. 229.

Now it may be shown by the principles of maxima, &c., that this quantity is a maximum when $d = \frac{1}{2}$ D. For putting the expression $p\,(d - \frac{d^2}{D})$ into fluxions, and equating the fluxion with zero, or 0, we get

$$d - \frac{2\dot{d}d}{D} = 0;$$

and consequently, by transposition it becomes

$$\frac{2\,d}{D} = 1, \text{ or } d = \tfrac{1}{2}\,D.$$

Therefore, a man or a horse acts with the greatest advantage on a machine when he moves with half the velocity he could continue at, were the effective resistance of the machine nothing.

That portion of the mechanical power which is efficient in impelling the machine will be $\frac{1}{2}$ P D. For since $d = \frac{1}{2}$ D, $P = \frac{pd}{D} = \frac{1}{2}\,p$, or $2\,P = p$; hence $d\,(p - P) = \frac{1}{2}\,D\,(2\,P - P) = \frac{1}{2}\,P\,D$.

The force P, and the distance moved through D, will each vary in the same man or horse according to the manner of applying the force; but their product will be nearly a constant quantity. In any case, *one* of these quantities may be determined from experience, and in many instances both of them, and that *one* may always be supplied by calculation which cannot be found by experience.

118. When the effective force is nothing, it must not be understood that a man or a horse is acting against a force equal to his own weight, in any case whatever; for as has been observed by Dr. Young, (Vol. I. p. 132, and 212,) in walking, the resistance overcome is not exactly comparable with mere weight, and the same may be remarked on other modes of exerting force.

When a man ascends vertically, his velocity is reduced to about one half of his horizontal velocity, indicating that he acts against a double resistance; therefore when a man ascending a ladder, carries a load, the maximum effect will take place when his ascending velocity is about one fourth of the velocity he can walk horizontally without a load.

A man of ordinary strength will not be able to walk, unloaded, at a quicker rate than 3½ miles an hour, if this exertion is to be continued for 10 hours every day. Indeed, those who examine the subject with a view to a fair average, will find this to be about the extreme velocity that can be continued, without injury, for any considerable time.

According, therefore, to our investigation, a man ought to move with half this velocity to produce a maximum effect; that is, at the rate of 1¾ mile an hour, which is about 2½ feet per second.

But this supposes the whole load to be the useful effect, whereas part of it must consist of the apparatus employed to carry it, or the friction of an intermediate machine, or other circumstances of a like nature. About one fifth of the velocity may be considered equivalent, at an average, to the force lost in friction, &c., in all cases; in many it will exceed one fifth. Hence the maximum of useful effect will take place when the velocity is 2 feet per second, or about 11 furlongs an hour, continued for 10 hours each day.

Smeaton is said to have made numerous comparisons, from which he concluded that the mechanical power of a man is equivalent to 3750 lbs. moving at the velocity of one foot per minute *; and taking this average to be near the true one, as I have reason to conclude it is, we have $\frac{3750}{2 \times 60} = 31{\cdot}25$ lbs. Therefore, we make the average mechanical power of a man 31·25 lbs. moving at the velocity of 2 feet per second, when the useful effect is the greatest possible; or half a cubic foot of water raised two feet per

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* Art. Water, Rees's Cyclopædia.

second; a very convenient expression for hydrodynamical inquiries.

If a man ascend a vertical ladder, according to a preceding remark, (p. 91,) the velocity which corresponds to the maximum of useful effect will be 1 foot per second, and the load double that which he carries horizontally; consequently the average of useful effect is 62·5 lbs. raised one foot per second.

Bricklayers' labourers in London ascend ladders with a load of about 80 lbs. besides the hod; sometimes at the rate of one foot per second, but more frequently about 9 inches per second.

Ascending stairs is more fatiguing to the muscles of the legs than ascending a ladder; and therefore the useful effect is less, till a person has become accustomed to this kind of labour. And it is also to be observed that the space moved over is increased, unnecessarily, except where the horizontal distance is part of the path over which the load is to be moved.

We ought not to be surprised at the opposite conclusions here obtained, from those of other theoretical inquirers, when the data they have proceeded from are considered. For their data have been the extremes of force and velocity, without any knowledge of the true laws which connect them.

119. The force of a horse is, at an average, about equal to that of six men, according to various estimates; and the rate of travelling about the same, perhaps rather less than that of a man, when his exertion is continued for 8 hours; consequently the velocity corresponding to the maximum effect, will be about 2½ feet per second. Whence, the average mechanical power of a horse may be estimated at 187½ lbs. moving with a velocity of 2½ feet per second, or 3 cubic feet of water raised 2½ feet per second. The day's work being 8 hours.

This estimate of the power of a horse is equal to 28,125 lbs. raised one foot per minute; nearly a mean between Watt's and Smeaton's. But since our author has employed that of Desaguliers, it was not very easy to make a change in this work; and still more objectionable to employ two measures of different values.

French writers use as a dynamical unit a given measure of water raised through a given space; and the Americans use a similar measure. In my opinion, it is preferable to make a horse's power the dynamical unit; because, a practical man has a more correct idea of the quantity of mechanical power expressed by this unit, than he can have of any one less frequently under his observation; besides, it is a power often employed to move machines; and therefore is a familiar measure of comparison.

120. TABLE OF PITCHES OF WHEELS IN ACTUAL USE IN MILLWORK.

Kind of Machine.	Horses' power.	Pitch in inches.	Breadth of teeth in inches.	Wheel.				Pinion.				Breadth proportionate to 10 horses' power, and present velocity.	Present velocity per second, in feet.	Breadth in inches proportionate to 10 horses' power, at 3 feet per second, that is, reducing all the examples to the same denom.
				Teeth.	Revolutions per minute.	Diameter. Ft.	In.	Teeth.	Revolutions per minute.	Diameter. Ft.	In.			
Water-wheel[1], A	10	4⅛	5½	..	..	..		..	..		..	..	..	5·5
Water-wheel[2], B	30	3	10¼	304	3½	24		3	318·47		..	3·41	3·95	4·489
Water-wheel, C	15	3	6	204	4½	16	2	44	20	3	6	4·	3·8	5·06
Water-wheel[3], D	5½	3	4	207	..	16	5¼	50	..	3	11⅝	7·27	3·	7·27
Horse-mill, E	1	2¼	4	91	3	'6	0⅛	22	12·9	1	5½	40·0	·949	12·65
Horse-mill, F	1	2¼	4½	91	3	6	0½	20	13·13	1	4	45·0	·949	14·23
Steam Engine, by B. & W. G.	24	3¼	6	96	19	8	0	42	43·32	3	6	2·5	7·95	6·625
Steam Engine, by B. & W. H.	46	3	8	152	17½	..		54	50		..	1·7	11·	6·2
Steam Engine, by B. & W. I.	32	3	6	116	19	8	10	..	..		..	1·87	8·78	5·47
Steam Engine, by B. & W. K.	14	3	5	64	25	5	1	29	55	2	4	3·57	6·65	7·91
Steam Engine, L	20	2½	5	90	18	5	11	38	42·63	2	7	2·5	5·57	4·64
Ditto, ditto[4], M	10	2¼	5¾	77	25	4	7⅛	40	48·5	2	4⅝	5·75	6·2	11·88
Ditto, ditto[5], N	6	2¼	5¼	60	28	3	7	27	62·6	1	7⅝	8·75	5·25	15·31
Ditto, ditto[6], O	4	2¼	4¾	48	32	2	10⅛	25	61·11	1	6	11·87	4·8	18·99
Ditto, ditto, P	2	2	4½	62	..	3	6	..	..	2		3·75	8·8	11·
Ditto, ditto[7], Q	10	1⅞	6	77	25	3	10	40	48·5	1	11⅛	6·	5·	10·
Ditto, ditto, R	12	1⅝	3	66	44	2	8	48	60·5	1	9	2·5	5·99	4·95

[1] Has been 16 years at work. Teeth much worn.

[2] Has been 16 years at work. This gearing was found rather too narrow for the strain, as it is wearing much faster than the rest of the wheels in the same mill.

[3] The only defect in this gearing, which has been 16 years at work, is the want of breadth in the spur-wheel and pinion: they ought to have been 6 inches or more, as they will not last half so long as the bevel-wheels and pinions connected with them.

[4] This is a better pitch for the power than Q.

[5] This wheel has wooden teeth, and has been working for three years past.

[6] Ditto.

[7] This pitch has been found to be too fine.

EXPLANATION OF THE TABLE OF WHEELS IN ACTUAL USE IN MILLWORK.

The wheels are all reduced to what may be called one denomination.

First—By proportioning their breadths all to what they should be to have the same strength, if the resistances were equal to the work of a steam engine of ten horses' power.

Secondly—By supposing their pitch lines all brought to the same velocity of three feet per second, and proportioning their breadths accordingly. I have chosen this particular velocity of 3 feet per second, because it is the velocity very common for overshot water wheels.

Such cases as appear to have worn too rapidly, are marked, which may tend to discover the limit in point of breadth.

Column 1 contains the horses' power.
2 . . . . pitch in inches.
3 . . . . breadth of teeth in inches.
4 . . . . number of teeth of wheel.
5 . . . . revolutions of wheel per minute.
6 . . . . diameter of wheel.
7 . . . . number of teeth of pinion.
8 . . . . revolution of pinion per minute.
9 . . . . diameter of pinion.
10 . . . . breadth proportionate to 10 horses' power, and at the present velocity.
11 . . . . present velocity per second in feet.
12 . . . . breadth in inches proportionate to 10 horses' power, at 3 feet per second. That is, all the cases reduced to the same denomination.

OBSERVATIONS ON THE TABLE OF WHEELS IN ACTUAL USE IN MILLWORK.

121. Having reduced the examples in the table, in the manner already described, to one denomination, the results approach nearer, considering all circumstances, than could have been expected.

1st. In two of the cases, viz. B and D, it appears, that the wheels were rather too narrow for their work. These, however, have been working about sixteen years, and may yet continue for a long time.—B is 4·489 inches in breadth, when reduced to 10 horses' power, at 3 feet per second.—D is 7·27 inches in breadth. The pitch of both is three inches.

Three inches being *a pitch* in very general use for the first motion of mills, could the proper breadth for this pitch be ascertained, it would serve as a very useful standard.

Rules, to be of practical use, must be easy of remembrance, as well as easy of application.

Let us, therefore, assume a simple standard, and try how far it will bear the test of experience; for, as Du Buat justly observes, "It is an excellent method, in the research of obscure difficult truths, to suppose a theory preexistent, founded upon the most probable principles, from which may be determined, the choice of some direct experiments, proper to evince the fallacy or accuracy of the principles proposed."

The actual cases in the table may be considered as satisfactory experiments. Let us, therefore, try the following simple rule, and compare some of its results with those cases.

Rule I. for pitch of three inches, when the velocity is three feet per second, at the pitch line.

Make the teeth as many inches broad as the number of horses' power which it has to resist.

For example, for nine horses' power, make the teeth nine inches broad.

122. Taking this example then as a point from which to set off, and supposing the same breadth of *nine inches* constant, we shall, in the first following table, state various pitches, and (by Proposition I.) shall first square these pitches, to find the number of horses' power, equal to the strength and durability of the pitch, when the teeth are all of one length. The strength, thus found, will be inserted in column Y.

But, as the lengths generally vary as the pitches, taking the same point (three inches pitch, nine inches broad) from which to set off, we shall diminish the value of the strength, as we ascend, and increase as we descend, agreeably to Proposition II. The results will be found in column Z.

But as in this investigation durability is of equal importance with strength, perhaps the true proportion may be somewhere between the results in the columns Y and Z.

DESCRIPTION OF THE SIX FOLLOWING TABLES OF PITCHES.

In all the following tables, the column W contains the pitch in inches.

Column X contains the breadth of the teeth also in inches.

Column Y is formed upon the supposition, that the teeth of all the pitches were of the same length, and contains the strength and durability of the teeth valued in horses' power.

Column Z contains the strength and durability of the teeth, also valued in horses' power, upon the supposition, that the lengths of the teeth are in the same proportion to one another, as the pitches; and that the strength (by Prop. II.) is inversely as the length. Having taken a three inch pitch as our standard, the two last columns, Y and Z, exactly coincide for that pitch; the column Z, decreasing upwards from that point, and increasing downwards in an inverse ratio of the lengths.

It is evident, that the tables upon these principles already laid down, might be greatly extended. But it is hoped, that these will be sufficient for our present purpose.

I. TABLE OF PITCHES

123. The velocity of the pitch line being three feet per second, and the breadth of the teeth nine inches.

W	Y	Z
Pitch in inches.	Value of strength in horses' power.	Value of strength in horses' power.
4	16·	12·
3½	12·25	10·5
3	9·	9·
2½	6·65	7·5
2	4·	6·
1½	2·25	4·5
1	1·	3·

Supposing again, that the pitches were the same as in the first table, and that the breadths were made, in each particular case, just double the pitch, then the horses' power would vary as in the following table, which is calculated by taking the above table, and by direct proportion, finding the horses' power equal to each particular breadth, when the breadth is just double the pitch.

II. TABLE OF PITCHES

124. The velocity being three feet per second, and the breadth of the teeth double each pitch.

W	X	Y	Z
Pitch in inches.	Twice the pitch in breadth.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	14·22	10·66
3½	7	9·53	8·17
3	6	6·	6·
2½	5	3·47	4·16
2	4	1·77	2·65
1½	3	·75	1·5
1	2	·22	·66

The strength being directly as the breadth, (by Prop. I.) it is easy from this to find, by the Rule of Three direct, the horses' power equal to any given breadth of these pitches.

The stress being inversely as the velocity, (by Prop. II.) the horses' power equal to any of these pitches, and breadths, may be easily found for any other velocity.

But, perhaps, it will be of advantage to take a different view of the subject, by fixing upon a case in the table of wheels in actual use, and proportioning breadths and pitches from it for various velocities, resistances, &c.

For this purpose we shall select the case H, erected by Boulton and Watt, being a pitch of three inches, the teeth 8 inches broad, moving with a velocity of 11 feet per second, and having a resistance valued at the power of 46 horses.

Then by following a similar procedure, with the two foregoing tables, we have the Tables III. and IV. proportionate to the case H at a velocity of 11 feet per second.

III.—TABLE OF PITCHES

125. Proportionate to H in the Table of Wheels. The breadth of teeth (8 inches) and velocity (eleven feet per second) being constant.

W	Y	Z
Pitch in inches.	Horses' power.	Value of strength in horses' power.
4	81·77	61·33
3½	62·61	53·66
3	46·	46·
2½	31·94	38·33
2	20·44	30·66
1½	11·5	23·
1	5·11	15·33

IV.—TABLE OF PITCHES

126. Proportionate to H, the breadth being in this case double the pitch. The velocity being constant, (eleven feet per second,) but the breadths double.

W	X	Y	Z
Pitch in inches.	Breadths double the pitch in inches.	Horses' power.	Value of strength in horses' power.
4	8	81·77	61·33
3½	7	54·78	46·95
3	6	34·5	34·5
2½	5	19·95	23·94
2	4	10·22	15·33
1½	3	4·31	8·62
1	2	1·28	3·84

But it may be satisfactory, in order to compare with the tables, first and second, to reduce the velocity to three feet per second.

The two following tables, therefore, are calculated accordingly at that velocity.

V.—TABLE OF PITCHES

127. Proportionate to H, at a velocity of three feet per second; the breadth being constantly eight inches.

W	Y	Z
Pitch in inches.	Horses' power.	Value of strength in horses' power.
4	22·30	16·72
3½	17·07	14·63
3	12·54	12·54
2½	8·71	10·45
2	5·57	8·36
1½	3·13	6·26
1	1·39	4·17

VI.—TABLE OF PITCHES

128. Proportionate to H, at a velocity of three feet; the breadths being double each pitch.

W	X	Y	Z
Pitch in inches.	Breadth of teeth in inches.	Horses' power.	Value of strength in horses' power.
4	8	22·30	16·72
3½	7	14·93	12·79
3	6	9·4	9·4
2½	5	5·44	6·53
2	4	2·78	4·17
1½	3	1·17	2·34
1	2	0·34	1·02

From a comparison of the second table with the sixth, it appears, that the rule we have annexed, is at least safe in point of strength; for by that rule, a *three inch pitch, six inches broad, is equal to a strain of six horses.* Whereas in the sixth table, the same pitch and velocity, at six inches breadth, has strength valued at nine and nearly a half horses' power.

But when we consider how much more liable, from sand, &c., teeth attached to water wheels are to wear, than those which are properly greased and free from sand, the results correspond as nearly as could be expected.

129. Rule II. So that, taking H as a standard, we may conclude, that, *for a pitch of three inches*, with a velocity of three feet per second, every inch of breadth may be valued at one and a half horses' power.

The first rule (Art. 121.) I think therefore may safely be followed for teeth attached to water wheels, and the above conclusion for wheels in all situations where they are properly greased and free from sand.

The conclusions here drawn, will, I think, give teeth sufficiently durable and strong for the work which they may have to perform. This first conclusion gives, perhaps, too great a result: how much may with prudence be deducted from the results of either, those of experience will determine. But to the young millwright, I would advise, of the two extremes, rather to err in making his work too strong. Durability ought not for a moment to be out of sight in the arrangements of wheelwork; there are many parts of machines subjected to greater stress, but not liable to wear; whereas the teeth of wheels, the moment they begin to act, begin to change from their original form, and to become progressively less strong.

130. In millwork, at present, the breadth of the teeth, as commonly executed by the best masters, seems to be from about twice to thrice the pitch.

It is, perhaps, not easy to determine what proportion is on the whole the most advantageous for the breadth of teeth. A fine pitch, on the one hand, gives a smooth motion, and the teeth will rub less on each other; but an increase of breadth increases in some degree the friction*. The durability, as well as the strength of teeth, is perhaps nearly in direct proportion to their breadth.

131. After sending the foregoing "*Inquiry respecting the Strength of the Teeth of Wheels*" to the press, Mr. John Roberton, engineer, perused a manuscript copy of it, and was so obliging as to communicate to me the substance of what follows. His rule, it will be readily perceived, is founded on the principles laid down in the "Inquiry;" but it is more simple, and perhaps more accurate than the mode of approximation which occurred to me. It is however satisfactory to find, that the table formed on his rule, (which from his experience he is of opinion cannot be far from the truth,) very nearly coincides in its results with tables fifth and sixth. It may be observed, that he founds

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* See Art. 67 and 68.

his calculations upon *the thickness of the teeth*, which in all cases he supposes a little less than half the pitch, which proportion is very common in practice. When both wheel and pinion, however, are of cast iron, it is evident, that, being more liable to wear, the teeth of the pinion ought to be thicker than those of the wheel.

CONSTRUCTION OF THE FOLLOWING TABLE.

132. *The thickness of the teeth*, in each of the lines, is varied one-tenth of an inch. *The breadth of the teeth* is always four times as much as their thickness. *The strength* of the teeth is ascertained *by multiplying the square of their thickness into their breadth*, taken in inches and tenths, &c. The *pitch* is found *by multiplying the thickness of the teeth by 2·1*. The number that represents the strength of the teeth, will also represent the number of horses' power, at a velocity of about four feet per second. Thus in the table where the *pitch is 3·15 inches, the thickness* of the teeth 1·5 *inches*, and the *breadth* 6· *inches*, the strength is valued at 13½ *horses' power*, with a *velocity of four feet per second* at the pitch line.

A TABLE OF PITCHES OF WHEELS,

133. With the *breadth* and *thickness* of the teeth, and the corresponding number of horses' power, moving at the pitch line at the rate of three feet, of four feet, of six feet, and of eight feet per second.

Pitch in inches.	Thickness of teeth in inches.	Breadth of teeth in inches.	Strength of teeth, or number of horses' power at four feet per second.	Horses' power at three feet per second.	Horses' power at six feet per second.	Horses' power at eight feet per second.
3·99	1·9	7·6	27·43	20·57	41·14	54·85
3·78	1·8	7·2	23·32	17·49	34·98	46·64
3·57	1·7	6·8	19·65	14·73	29·46	39·28
3·36	1·6	6·4	16·38	12·28	24·56	32·74
3·15	1·5	6·	13·5	10·12	20·24	26·98
2·94	1·4	5·6	10·97	8·22	16·44	21·92
2·73	1·3	5·2	8·78	6·58	13·16	17·54
2·52	1·2	4·8	6·91	5·18	10·36	13·81
2·31	1·1	4·4	5·32	3·99	7·98	10·64
2·1	1·0	4·	4·0	3·0	6·0	8·0
1·89	·9	3·6	2·91	2·18	4·36	5·81
1·68	·8	3·2	2·04	1·53	3·06	3·08
1·47	·7	2·8	1·37	1.027	2·04	2·72
1·26	·6	2·4	·86	·64	1·38	1·84
1·05	·5	2·	·5	·375	·75	1·

James Carmichael, millwright, (of Dundee,) made the following remarks to Buchanan on the strength, &c., of wheelwork.

134. "Sir—It is a corroboration of the truth of the tables of pitches, that Mr. Roberton's table coincides very nearly with columns marked Y in your tables; but he seems to have overlooked the propriety of taking the length of the teeth into his calculations. I am, therefore, still of opinion, that the true value is in the columns marked Z in your tables.

"Admitting the truth of the fundamental propositions, and, from a comparison of the tables of pitches, I would propose the following rule, which is on the same principle as that of columns Z in your tables, for calculating the proportionate strength of the teeth of wheels.

" *Rule.*—Multiply the breadth of the teeth by the square of the thickness, and divide the product by the length. The quotient will be the proportionate strength in horses' power, with a velocity of 2·27 feet per second.

" By that rule I have calculated the following table; and, for the sake of comparison, I have taken three cases from Mr. Roberton's table, and three from your Tables 3d and 5th.

" EXPLANATION OF THE TABLE.

" Column 2 contains the thickness of the teeth. The pitch is found by multiplying the thickness by 2·1*; and the length is found by multiplying the thickness by 1·2†.

" Column 5 contains the proportionate strength, and also the number of horses' power (proportionate to the case H, see p. 100) which the teeth are equal to, with a velocity of 2·27 feet per second.

1	2	3	4	5	6	7	8
Pitch in inches.	Thickness of teeth in inches.	Breadth of teeth in inches.	Length of teeth in inches.	Strength of teeth, or number of horses' power, at 2·27 feet per second.	Horses' power at three feet per second.	Horses' power at six feet per second.	Horses' power at eleven feet per second.
3·9	1·9	7·6	2·28	11·73	15·46	30·92	56·84
2·9	1·4	5·6	1·68	6·53	8·63	17·26	31·64
2·1	1·	4·	1·2	3·33	4·4	8·8	16·1
4·	1·904	8·	2·285	12·698	16·78	33·56	61·52
3·	1·428	8·	1·714	9·523	12·58	25·16	46·
1½	·714	8·	·857	4·752	6·27	12·54	23·02

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* That is in order to make the space between the teeth a little wider than the thickness of a tooth. See Art. 132.

† Respecting the length of teeth, see Art. 145.

" REMARKS.

" 1st. The last three cases in the table are taken from the Tables 3d and 5th, and the results coincide so well with the columns marked z, that I presume the rule is just.

" 2dly. The first three cases are from Roberton's table; the second case is very near the same as in his table; but the first is considerably less, and the third considerably more. Hence I infer that Roberton has taken his data from a pitch about three inches.

" 3dly. If any two wheels have the length and thickness of their teeth in the same proportion to their respective pitches, the breadth of the teeth and the velocity being the same, *the strength will be directly as the pitches.* The truth of this is deduced from the columns marked z in Tables 1st, 3d, and 5th."

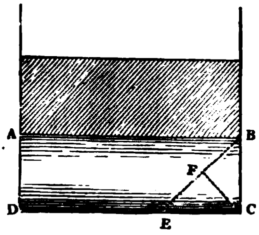
135. Table of pitches of wheelwork, with the *breadth* and *thickness* of the teeth, and the corresponding strength in horses' power, calculated by Carmichael's rule.

Pitch in inches.	Thickness of teeth in inches.	Breadth of teeth in inches.	Length of teeth in inches.	Strength of teeth in horses' power at 2·27 feet per second.	Horses' power at three feet per second.	Horses' power at six feet per second.	Horses' power at eleven feet per second.
3·99	1·9	7·6	2·28	12·03	15·90	31·80	58·30
3·78	1·8	7·2	2·16	10·80	14·27	28·54	52·32
3·57	1·7	6·8	2·04	9·63	12·72	25·54	46·68
3·36	1·6	6·4	1·92	8·53	11·27	22·54	41·32
3·15	1·5	6·0	1·80	7·50	9·91	19·82	36·33
2·94	1·4	5·6	1·68	6·53	8·63	17·26	31·64
2·73	1·3	5·2	1·56	5·63	7·44	14·88	27·28
2·52	1·2	4·8	1·44	4·80	6·34	12·68	23·24
2·31	1·1	4·4	1·32	4·03	5·32	10·64	19·54
2·10	1·0	4·0	1·20	3·33	4·40	8·81	16·15
1·89	0·9	3·6	1·08	2·70	3·57	7·14	13·09
1·68	0·8	3·2	0·96	2·13	2·81	5·62	10·33
1·47	0·7	2·8	0·84	1·63	2·15	4·30	7·88
1·26	0·6	2·4	0·72	1·20	1·59	3·18	5·83
1·05	0·5	2·0	0·60	0·83	1·10	2·20	4·03

136. It is not perhaps quite so difficult, as our author imagined, to determine, from first principles, the strength proper for teeth of wheels, and such a method must always be preferred to empirical rules. We shall here show how to apply those principles which will give the reader an opportunity of comparing the two methods.

In the first place let us consider under what circumstances the strain on a tooth will be the greatest possible.

Let ABCD be the side of a tooth, then it will be evident, that the strain will be greatest, when the stress is thrown upon one corner of the tooth, as at C, whether it be from irregular action or from any substance getting between the teeth.



In such a case, it may be shown by the rules of *maxima* and *minima*, that EC being equal to CB, the strain will be greatest in the line EB; and, in the case of fracture, it would take place according to that line.

Since the thickness of a tooth is not regular, we shall have a result, sufficiently near for this purpose, if we express the relation between the stress and strain by $\frac{740\,\text{H}}{v} = \frac{\cdot 7854 f 2 (\text{FC}) d^2}{8 (\text{FC})}$. (*Essay on Cast Iron*, Art. 81, and Art. 107 of this Essay,) which, when $f =$ 15,300 lbs. on a square inch, reduces to $\frac{\cdot 247\,\text{H}}{v} = d^2 =$ the square of the thickness of the tooth in inches. But, a tooth should be capable of resisting this stress, when it is considerably worn by friction; and an allowance fully equal to that which ought to take place before renewing the wearing parts of the machine, will be one third of the thickness of a tooth. Now to allow of this degree of wear in the tooth, and that it should remain equal to the stress, it may be easily shown that the tooth should be capable of resisting 2¼ times the power at the first; therefore $\frac{\cdot 556\,\text{H}}{v} = d^2$; or $\frac{3}{4}\sqrt{\frac{\text{H}}{v}} = d$. Where H is the number of horses which are equal to the power of the first mover; v the velocity of the pitch line of the wheel in feet per second, and d the thickness of a tooth in inches.

This investigation furnishes an easy practical rule for the thickness of teeth; and, consequently, for the pitch of wheels and pinions; we shall give it in words at length, with an example, and then proceed to determine a rule for the breadth of teeth.

RULE FOR THE THICKNESS OF CAST IRON TEETH FOR WHEELS.

137. Find the number of horses which are equivalent to the power of the first mover of the train of machinery, and divide that number by the velocity, in feet per second, of the pitch line of the pinion or wheel; extract the square root of the quotient, and three fourths of this root will be the least thickness of the tooth for the wheel or pinion, in inches.

RULE FOR THE LEAST QUANTITY OF PITCH FOR A WHEEL OR PINION WITH TEETH OF CAST IRON.

138. If the thickness of the teeth of the pinion be intended to be the same as those of the wheel, multiply the thickness above determined by 2·1, the product will be the pitch required.

But we may observe, that if a pinion makes three turns, for example, while the wheel makes one, the teeth of the pinion will be worn three times the quantity of those of the wheel. Hence, to provide against such excess of wear, if the pinion makes n revolutions, while the wheel makes one, the pitch should be $\frac{5{\cdot}3+n}{3}d$ inches, and the thickness of the teeth of the pinion $\frac{2+n}{3}d$ inches, when d is the thickness of the teeth of the wheel.

EXAMPLE.

139. Let the force of the first mover be equivalent to ten horses, and the velocity of the pitch line three feet per second. Dividing 10 by 3, we have $3\frac{1}{3}$; and the square root of $3\frac{1}{3}$ (by the Table of Powers, Art. 479.) is 1·83 nearly; and $\frac{4}{5} \times 1{\cdot}83 = 1{\cdot}45$ inches for the thickness of the teeth of the wheel.

Again, suppose the pinion to turn twice while the wheel turns once, then $n = 2$, and $\frac{5{\cdot}3+2}{3} \times 1{\cdot}45 = 3{\cdot}5$ inches the pitch.

And $\frac{2+n}{3} \times 1{\cdot}45 = 1{\cdot}93$ inches, the thickness of the teeth of the pinion.

THICKNESS OF WOODEN TEETH.

140. The kind of wood employed for teeth, is usually about one fourth of the strength of cast iron, and since the thickness of the teeth should vary inversely as the square root of the power of the material, the square root of $\frac{1}{4}$ being $\frac{1}{2}$, wooden teeth should be twice the thickness of cast iron teeth. The pitch of course will be greater in the same proportion.

TO DETERMINE THE BREADTH OF CAST IRON TEETH.

141. That case where a beam is fixed at one end, and the load acts at the other, applies to the teeth of wheels, in their general state of action, and the stress is $\frac{740\,\text{H}}{v}$. (Art. 107.) Hence, when l = the length, and b = the breadth of a tooth, $\frac{740\,\text{H}\,l}{12v} = 212\ b\ d^2$. (*Essay on Cast Iron*, Art. 116.) And to allow one third of the thickness of the tooth for wear, the equation becomes $\frac{2\frac{1}{4} \times 740\,\text{H}\,l}{12v} = 212\ b\ d^2$. But we have already seen, that $d^2 = \frac{\cdot 556\,\text{H}}{v}$; therefore $\frac{2\frac{1}{4} \times 740 \times l}{12} = \cdot 556\ b \times 212$, or $1\cdot 2\ l = b$.

This calculation informs us what breadth is essential for strength; that is, the breadth should never be less than 1·2 multiplied by the length of the tooth; but, it may be proved that the durability is nearly in direct proportion to the breadth, and inversely as the pressure. Some of the maxims of our author, as far as regards the breadths of teeth, agree well with the theory of durability; but the conclusions respecting strength and the limits of pitch are not so much to be relied upon; indeed the facts drawn from practical construction are not so well adapted for the latter object.

142. If we suppose that teeth six inches in breadth are sufficiently durable for a power equivalent to 10 horses, when the pitch line moves at the rate of 3 feet per second; and the Table of Wheels in p. 95, Art. 120. seems to indicate that this supposition is near the truth; therefore $\frac{10}{3} : 6 :: \frac{\text{H}}{v} : b = \frac{3 \times 6 \times \text{H}}{10v} = \frac{1\cdot 8\,\text{H}}{v}$. That is, multiply the horses' power by 1·8, and divide by the velocity of the

pitch line in feet per second, the quotient will be the breadth in inches.

EXAMPLE.

Taking the case H in the table Art. 120, we have $\frac{1\cdot 8\,\text{H}}{v} = \frac{1\cdot 8 \times 46}{11} = 7\cdot 53$ inches: Messrs. Boulton and Watt in this case made the breadth 8 inches.

In the case K, by the same makers, $\frac{1\cdot 8\,\text{H}}{v} = \frac{1\cdot 8 \times 14}{6\cdot 65} =$ 3·8 inches. The breadth actually employed was 5 inches. Hence it appears, that, in a considerable range of power and velocity, our formula gives results below those actually employed, but the breadth assigned by the table calculated by Mr. Roberton, Art. 133, is always vastly below ours in the greater powers; indeed it is manifestly erroneous in the breadths, for where the moving power is doubled, the breadth is increased only one third. In the thickness of teeth, it nearly agrees with our rule.

BREADTH OF WOODEN TEETH.

143. For wooden teeth we may take, as the basis of a practical rule, the case N in the table, Art. 120; which, expressed in the nearest whole number, is $\frac{5\,\text{H}}{v} = b$ in inches.

EXAMPLE.

In the case O; $\text{H} = 4$ horses, and $v = 4\cdot 8$ feet per second; therefore, $\frac{5\,\text{H}}{v} = \frac{5 \times 4}{4\cdot 8} = 4\cdot 17$ inches the breadth; the actual breadth used was $4\frac{3}{4}$ inches.

OF THE STRENGTH OF STAVES FOR TRUNDLES.

144. This is a subject our author has not touched upon, but we consider it necessary to examine the strength of staves, because trundles seem capable of improvement, and they have some advantages which toothed pinions have not.

If the length of a stave in feet be l, its diameter in inches d, and the stress upon it $\frac{740\,\text{H}}{v}$; which is supposed to act at the weakest part of the stave; that is, in the middle of its length; then, by the rule for the strength of cylinders, (*Essay on Cast Iron* *, Art. 129,) $\frac{740\,\text{H}\,l}{500\,v} = d^3$. Now, if it be made to resist $2\frac{1}{4}$ times the power when first made, in order to allow for wear; we shall have $\frac{2\frac{1}{4} \times 740\,\text{H}\,l}{500\,v} = d^3$, or $\left(\frac{3{\cdot}33\,\text{H}\,l}{v}\right)^{\frac{1}{3}} = d$; or with sufficient accuracy, it is $2\left(\frac{\text{H}\,l}{v}\right)^{\frac{1}{3}} = d$.

EXAMPLE.

Let the power of the first mover be equal to 10 horses, the velocity of the pitch line 3 feet per second, and the length of the stave ·9 of a foot; then $2\left(\frac{\text{H}\,l}{v}\right)^{\frac{1}{3}} = 2\left(\frac{10 \times .9}{3}\right)^{\frac{1}{3}} = 2 \times 1{\cdot}442 = 2{\cdot}884$ inches the diameter of the stave.

145. We take the liberty of inserting the following table from a respectable periodical publication, as we purpose to show its application, knowing it may be of use to millwrights †.

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* Tredgold's work, which we occasionally quote.

† This very useful table was printed in a small pamphlet, price one shilling, in 1803, but is at present out of print. When the number of teeth is less than 10, look for the radius of double the number of teeth, and half of it will be the radius required. When the number of teeth exceeds 300, look for the radius of half the number of teeth, the double of which will be the one required.

TABLE OF THE RADII OF WHEELS, FROM TEN TO THREE HUNDRED TEETH, THE PITCH* BEING TWO INCHES.

BY B. DONKIN, ESQ., CIVIL ENGINEER, LONDON.

No. of teeth.	Radius in inches.	No. of teeth.	Radius in inches.	No. of teeth.	Radius in inches.	No. of teeth.	Radius in inches.
10	3·236	47	14·972	84	26·741	121	38·520
11	3·549	48	15·290	85	27·063	122	38·838
12	3·864	49	15·608	86	27·381	123	39·156
13	4·179	50	15·926	87	27·699	124	39·475
14	4·494	51	16·244	88	28·017	125	39·793
15	4·810	52	16·562	89	28·336	126	40·111
16	5·126	53	16·880	90	28·654	127	40·429
17	5·442	54	17·198	91	28·972	128	40·748
18	5·759	55	17·517	92	29·290	129	41·066
19	6·076	56	17·835	93	29·608	130	41·384
20	6·392	57	18·153	94	29·927	131	41·703
21	6·710	58	18·471	95	30·245	132	42·021
22	7·027	59	18·789	96	30·563	133	42·339
23	7·344	60	19·107	97	30·881	134	42·657
24	7·661	61	19·425	98	31·200	135	42·976
25	7·979	62	19·744	99	31·518	136	43·294
26	8·296	63	20·062	100	31·836	137	43·612
27	8·614	64	20·380	101	32·155	138	43·931
28	8·931	65	20·698	102	32·473	139	44·249
29	9·249	66	21·016	103	32·791	140	44·567
30	9·567	67	21·335	104	33·109	141	44·885
31	9·885	68	21·653	105	33·427	142	45·204
32	10·202	69	21·971	106	33·746	143	45·522
33	10·520	70	22·289	107	34·064	144	45·840
34	10·838	71	22·607	108	34·382	145	46·158
35	11·156	72	22·926	109	34·700	146	46·477
36	11·474	73	23·244	110	35·018	147	46·795
37	11·792	74	23·562	111	35·337	148	47·113
38	12·110	75	23·880	112	35·655	149	47·432
39	12·428	76	24·198	113	35·974	150	47·750
40	12·746	77	24·517	114	36·292	151	48·068
41	13·064	78	24·835	115	36·611	152	48·387
42	13·382	79	25·153	116	36·929	153	48·705
43	13·700	80	25·471	117	37·247	154	49·023
44	14·018	81	25·790	118	37·565	155	49·341
45	14·336	82	26·108	119	37·883	156	49·660
46	14·654	83	26·426	120	38·202	157	49·978

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* By the pitch is understood the distance between the centres of two contiguous teeth; and by the radius is understood the distance between the centre of the wheel and the centre of each tooth. For any other pitch, say, as two inches is to the radius in the table, so is the given pitch to the radius required.

No. of teeth.	Radius in inches.	No. of teeth.	Radius in inches.	No. of teeth.	Radius in inches.	No. of teeth.	Radius in inches.
158	50·296	194	61·755	230	73·214	266	84·673
159	50·615	195	62·073	231	73·532	267	84·991
160	50·933	196	62·392	232	73·850	268	85·309
161	51·251	197	62·710	233	74·168	269	85·627
162	51·569	198	63·028	234	74·487	270	85·946
163	51·888	199	63·346	235	74·805	271	86·264
164	52·206	200	63·665	236	75·123	272	86·582
165	52·524	201	63·983	237	75·441	273	86·900
166	52·843	202	64·301	238	75·760	274	87·219
167	53·161	203	64·620	239	76·078	275	87·537
168	53·479	204	64·938	240	76·397	276	87·855
169	53·798	205	65·256	241	76·715	277	88·174
170	54·116	206	65·574	242	77·033	278	88·492
171	54·434	207	65·893	243	77·351	279	88·810
172	54·752	208	66·211	244	77·670	280	89·129
173	55·071	209	66·529	245	77·988	281	89·447
174	55·389	210	66·848	246	78·306	282	89·765
175	55·707	211	67·166	247	78·625	283	90·084
176	56·026	212	67·484	248	78·943	284	90·402
177	56·344	213	67·803	249	79·261	285	90·720
178	56·662	214	68·121	250	79·580	286	91·038
179	56·980	215	68·439	251	79·898	287	91·357
180	57·299	216	68·757	252	80·216	288	91·675
181	57·617	217	69·075	253	80·534	289	91·993
182	57·935	218	69·394	254	80·853	290	92·312
183	58·253	219	69·712	255	81·171	291	92·630
184	58·572	220	70·031	256	81·489	292	92·948
185	58·890	221	70·349	257	81·808	293	93·267
186	59·209	222	70·667	258	82·126	294	93·585
187	59·527	223	70·985	259	82·444	295	93·903
188	59·845	224	71·304	260	82·763	296	94·222
189	60·163	225	71·622	261	83·081	297	94·540
190	60·482	226	71·941	262	83·399	298	94·858
191	60·800	227	72·258	263	83·717	299	95·177
192	61·118	228	72·577	264	84·036	300	95·495
193	61·436	229	72·895	265	84·354		

OF ARRANGING THE NUMBERS OF WHEEL-WORK.

146. In a machine, the velocity of the impelled point should be to that of the working point in the ratio which is adapted to the maximum effect of the moving power on the one part, and the best working effect on the other part. Any other arrangement of the relative motions of the parts

of a machine must clearly be attended with a loss of power, or the work will not be done properly. But when the best working velocity is known, and also that which enables the first mover to produce the greatest effect; the proper arrangement of the numbers of the teeth of the wheels and pinions is a very simple operation. The subject has been treated of for particular machines, by several writers; but since it has been chiefly under a somewhat erroneous view of the real nature of the maximum effect of machines, it will be perhaps of use to give a general formula, and a few particular examples, to save the trouble of reference, and render the work somewhat more complete*.

It will be an advantage to advertise the young mechanic of one or two essential particulars, before proceeding to the principal object.

147. In the first place, when the wheels drive the pinions, the number of teeth in any one pinion should not be less than 8; but rather let there be 11 or 12 if it can be done conveniently. And in the particular form of teeth described in Art. 30, the number of teeth in a pinion should not be less than 10; but it would be better to have 13 or 14. (See Art. 34 to 37.)

148. Secondly, when the pinions drive the wheels, the number of teeth on a pinion may be less; but it will not in any case be desirable to have fewer than 6 teeth on a pinion; and give the preference to 8 or 9, where it can be done with convenience.

149. Thirdly, the number of teeth in a wheel should be prime to the number of teeth in its pinion; that is, the

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* The methods of adjusting the numbers of wheel-work, so that the contemporary revolutions may be always in a given ratio, is a distinct branch of this subject, chiefly useful in clock and watch-work, planetary machines, and the like; and since our plan does not include the construction of such machines, the reader, desirous of such information, may consult Camus on the Teeth of Wheels.

number representing the teeth in the wheel should not be divisible by the number of teeth in the pinion without a remainder. And as the numbers of pinions will in general be first settled, it will be an advantage to take a prime number for each pinion, as 7, 11, 13, 17, 19, 23, &c., because such numbers are seldomer factors than others. But when it happens that a prime number can be directly fixed upon for the wheel, any whole number which approaches near to the required ratio will answer for the pinion; as minute accuracy is not required. A prime number for the wheel, or one which is not divisible by the number of the pinion, is esteemed the best, because the same teeth will not always come together, and the wear will be more uniform.

150. Fourthly, if it be desired that a given increase or decrease of velocity should be communicated with the least quantity of wheel-work, it has been shown that the number of teeth on each pinion should be to the number on its wheel, as 1 : 3·59. (Dr. Young's Nat. Phil. Vol. II. Art. 366.) But, on account of the space required for several wheels, and the expense of them, it will often be necessary to have 5 or 6 times the number of teeth on the wheel that there is on the pinion. The ratio of 1 : 6 should however not be exceeded, unless there be some other important reason for a higher ratio.

151. *Of calculating the Numbers for Wheel-Work.* Let n be the number of revolutions per minute for the first axis, to which the moving power gives motion; and N the number of revolutions per minute of the last axis, where the resistance or working point is. Then, $n : N :: 1 : \frac{N}{n}$, which is the ratio the velocity is to be increased or diminished.

If this ratio should not exceed 1 : 6 a single wheel and pinion will be sufficient; but when it exceeds that ratio, more will be necessary.

When each of the pinions has the same number of teeth, and each wheel the same number; then, the ratio of the number on a pinion, will be to the number on a wheel

$$\text{as } 1 : \frac{N^{\frac{1}{a}}}{n^{\frac{1}{a}}}; \text{ or as } 1 : \left(\frac{N}{n}\right)^{\frac{1}{a}} \qquad (1.)$$

The number of pinions being a.

But it is sometimes necessary to adapt the trains of machinery to produce different velocities at the working points, and it is on this account often desirable to vary the size of the wheels. Then, the ratio $1 : \frac{N}{n}$ must be decomposed into factors suitable to the nature of the work to be done; if those factors be any numbers a, b, c, &c. the ratio will be

$$1 : a \times b \times c, \&c. : \frac{N}{n}. \qquad (2.)$$

The first mover should be as near as possible to the resistance. But, when it is absolutely necessary to perform operations at a considerable distance from the first mover, the velocity of the communicating shafts should be brought up, as near as possible, to the first mover, to that which is most advantageous for the different species of work.

But the velocities of the parts of machines are often given in feet per second; let v be the velocity in feet per second; then $60\,v$ is the feet described in a minute. Also let $d \times 3{\cdot}1416$ be the circumference which moves with the velocity v; then $\frac{60\,v}{3{\cdot}1416\,d} = \frac{19{\cdot}09\,v}{d}$ being the revolutions which the axis makes in a minute.

When the first mover acts with a velocity v at the distance $\frac{1}{2}\,d$ from the axis; then $\frac{10{\cdot}09\,v}{d} = n$; and the ratio of the teeth on the pinions should be to those on the wheels

as $1 : \left(\frac{\mathrm{N}d}{99{\cdot}09\,v}\right)^{\frac{1}{a}}$ (3.)

And, when the velocity V of the working point is also given,

then $\frac{19{\cdot}09\,\mathrm{V}}{\mathrm{D}} = \mathrm{N}$; and the ratio of the teeth on the

pinions should be to those on the wheels as $1 : \left(\frac{\mathrm{V}d}{v\mathrm{D}}\right)^{\frac{1}{a}}$. (4.)

If the velocity is to be decreased, then it will be as the wheels are to the pinions, instead of the pinions to the wheels. The use of these proportions will be best illustrated by examples.

152. EXAMPLE I. Let it be required to calculate the numbers for a corn mill moved by an overshot water-wheel.

This case comes under Proportion (4). where a, the number of pinions, will never exceed 2, and not often more than 1. And in order that the grain may not be too much heated in grinding, the velocity of the circumference of the millstone, should not be greater than 23 feet per second, hence $\mathrm{V} = 23$; and D will be the diameter of the millstone in feet.

153. If we suppose the wheel and its concomitants to offer no resistance to the impulse of the water, it is manifest that the velocity of the circumference will then be the same as that with which the water strikes it, or that which is due to the whole height of the fall, and will therefore be expressed by $v = \sqrt{2gh}$; where h denotes the height of the fall in feet; v the velocity in feet per second, and $g = 32\frac{1}{6}$ feet, the velocity generated by gravity.

But it is not consistent with the laws of nature that a machine can be put in motion without offering some resistance to the moving power; for, in the first place, the friction of the parts has to be overcome, and it must be a machine of a very simple construction indeed, if the accomplishment of this alone does not expend one-half of the force applied: but in the generality of combinations, it will be found to balance nearly two-thirds of the power applied; we will therefore be pretty near the truth by assuming the friction as equivalent to two-thirds of the moving power.

In the next place, there is the resistance to be overcome at the working point, and this is equivalent to the quantity of work to be performed, which must therefore expend the remaining third of the moving power. It thence appears, that, in the case of an overshot water-wheel, the height of the fall which produces the velocity of the circumference, must be divided into *two* parts; *one* part to overcome the friction, and the *other* to produce the useful effect.

Let e be that part of the fall which is competent to overcome the friction of the loaded machine only; or that which corresponds to the velocity when the useful effect is nothing; and let x be that part of the fall producing the velocity corresponding to the maximum of useful effect. It is therefore manifest, that the effective force of the water on the wheel, when the work done is the *greatest* possible, will always be proportional to $h - x$; and when the work done is *nothing*, the effective force will always be proportional to $h - e$. Now, the difference of these two quantities drawn into the velocity must be a maximum when the greatest effect is obtained; hence we have $v\,(h - x - h + e) = v\,(e - x)$ a maximum. But, by the laws of falling bodies, we have $v = \sqrt{2gx}$; let this be substituted for v in the above expression, and it becomes $\sqrt{2gx}\,(e - x)$, a maximum; or by dropping the constant factor $2g$, it is $e\,x^{\frac{1}{2}} - x^{\frac{3}{2}} =$ a maximum. Let this expression be thrown into fluxions and equated with *zero* or nothing, and it becomes

$$\tfrac{1}{2}\, e\, x^{-\frac{1}{2}}\; x - \tfrac{3}{2}\, x^{\frac{1}{2}}\; x = 0.$$

Consequently, by transposition and reduction, we get

$$e = 3\, x.$$

But we have stated above that two-thirds of the moving power is expended in overcoming the friction; hence we have $e = h - \frac{2}{3} h = \frac{1}{3} h$; therefore, $x = \frac{1}{9} h$, and the velocity corresponding to x is

$$v = \sqrt{64\tfrac{1}{3} \times \tfrac{1}{9} h} = 2{\cdot}673 \sqrt{h}.$$

We have next to determine the diameter of the wheel in relation to the height of the fall when the effect is a maximum; and for this purpose, let ϕ be equal to that portion of the circumference which is loaded with water, and x equal the arc comprehended between the point of impact and the horizontal radius; then, by the principles of mensuration, the magnitude of the solid which represents the effective force, is $\frac{1}{2}\, b\, \left(\frac{\phi^2 - 2\, x^2}{\phi - x}\right)$, where b is the section of the stream supplying the buckets. But this, by the question, is to be a maximum. Let it therefore be thrown into fluxions, and put equal to *zero*, and we get

$$-4\, x\, \dot{x}\, (\phi - x) + \dot{x}\, (\phi^2 - 2\, x^2) = 0;$$

from which by transposition we get

$$x^2 - 2\, \phi\, x = -\tfrac{1}{2}\, \phi^2,$$

and reducing the quadratic by the rules of algebra, it becomes

$$x = \phi\, (1 - \sqrt{\tfrac{1}{2}}).$$

But $1 - \sqrt{\frac{1}{2}} = 1 - {\cdot}70711 = {\cdot}29289$; hence we get

$$x = {\cdot}29289\, \phi; \text{ and } \phi = \frac{x}{{\cdot}29289}.$$

It is, however, obvious, that the difference between the two arcs ϕ and x, must be equal to a quadrant; that is,

$$\frac{x}{\cdot 29289} - x = 90^\circ, \text{ and this gives } x = 37^\circ\ 16'.$$

Let r denote the radius of the wheel, estimated from the centre to the remote point of the bucket; then will $r\ (1 + \sin.\ 37^\circ\ 16')$ express the effective height of the fall. But the natural sine of 37° 16′, is ·60553; hence we have $1\cdot 60553\ r$ for the effective height; and if the absolute height of the fall be equivalent to nine-eighths of the effective height, we shall have $h = 1\cdot 8062\ r$; consequently, $r = \cdot 554\ h$ and $d = 2\ r = 1\cdot 108\ h$. Having thus determined the values of v and d, let them be substituted in the appropriate Proportion number (4) preceding, and we get

$$1 : \frac{9\cdot 54\ \sqrt{h}}{\text{D}}, \text{ when there is only one pinion;}$$

but when there are two pinions, it becomes

$$\text{as } 1 : \left\{\frac{9\cdot 54\ \sqrt{h}}{\text{D}}\right\}^{\frac{1}{2}}.$$

In practice, the height of the fall and the diameter of the millstone will always be known; in the present instance let $\text{D} = 5$ feet, and $h = 16$ feet; then we have $\sqrt{16} = 4$, and therefore it is

$$1 : \frac{9\cdot 54 \times 4}{5},$$

or as 1 : 7·632. Now it will be better, in this case, to have two pinions, since the ratio is greater than 1 : 6; therefore as $1 : 7\cdot 632^{\frac{1}{2}}$ or as 1 : 2·763, so is the teeth in each pinion to the teeth in its wheel. And making the prime number 11 the number of teeth of each pinion, we shall have 11 × 2·763 = 30, the nearest whole number for the teeth of each wheel. The thickness of the teeth being found by the rules for that purpose, (see Art. 137,) the radius of the wheels and pinions will be found by the table of radii, (Art. 145.)

Again, let the fall be 4·84 feet, the diameter of the millstone being 5 feet as before; then, $\sqrt{h} = 2{\cdot}2$, and $1 : \frac{9{\cdot}54\sqrt{h}}{D}$ becomes as 1 : 4·1976. Here one pinion or trundle will be best; and making the teeth on the pinion 11 those on the wheel will be $11 \times 4{\cdot}1976 = 46$ in the nearest whole number.

The young millwright will find it useful to calculate a table by these rules, which might be extended to undershot wheels, and exhibit at one view the whole construction of mills.

154. Example II. Let it be required to arrange the numbers for a machine for raising water, where the pumps are to make N strokes per minute, the velocity of the moving power being v feet per second, and the diameter of the circle described by the power d feet.

This case is an example of the use of Proportion (3), or $1 : \left(\frac{N d}{19{\cdot}09\, v}\right)^{\frac{1}{a}}$. Now let the moving power be a horse, where v is $2\frac{1}{2}$ feet per second, (Art. 119,) and d the diameter of his track, 30 feet; the pump to make 20 strokes per minute, or $N = 20$, then the ratio is $1 : \frac{20 \times 30}{19{\cdot}09 \times 2\frac{1}{2}}$, or as 1 : 12·6 nearly. Therefore if the pinion have 13 teeth, the wheel should have $13 \times 12{\cdot}6 = 164$ teeth in the nearest whole numbers.

But we should prefer making two pinions, and then the ratio will be $1 : \sqrt{12{\cdot}6}$ or as 1 : 3·549 nearly, and each pinion having 11 teeth, the wheels should each have $11 \times 3{\cdot}549 = 39$ teeth.

These examples will perhaps be sufficient to explain the mode of calculation; and when once it is understood, it will very easily be applied to other cases; and will be found somewhat more convenient than the methods usually followed.

PRACTICAL OBSERVATIONS WITH REGARD TO THE MAKING OF PATTERNS FOR CAST IRON WHEELS.

155. Having determined the pitch of the wheel strong enough for the purpose to which it is to be applied, the thickness of the tooth serves to regulate the proportionate strength of the other parts.

A very respectable millwright informs me, that he has for a considerable time adopted the following rule for determining the length of the teeth of wheels, the practical efficacy of which he has found quite satisfactory.

Rule.—*Make the length of the teeth equal to the pitch, deducting freedom,* (by the freedom is meant the distance at the top of one tooth, and the root of another measured at the line of centres,) in other words, the distance from root to root of the teeth, at the line of teeth when the wheels are in action, exactly equal to the pitch.

For example—he makes the teeth of two inches' pitch, 1 inch and $\frac{13}{16}$ in length, which is allowing $\frac{3}{16}$ of freedom.

Another respectable millwright, who has had much experience, particularly in mills moved by horses, has for a considerable time past made the teeth of his wheels in length only one half of the pitch, and works them as deep as possible without the point touching the bottoms. Before he fell on this expedient, he found the teeth exceedingly liable to be broken from any sudden motion of the horses*.

Indeed, upon reflection, it will be found there is no occasion for more freedom, than that the point of the tooth of the one wheel shall just clear the ring of the other;

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* Respecting the length of teeth, see Art. 29, 42, and 47.

more than this must only serve to weaken the teeth. The mode of gearing, however, above alluded to, is more necessary in horse mills than where the moving power is steady and regular.

Hatton (on Clock-work) recommends making the distance of the pitch line $\frac{3}{4}$ of what we call the thickness of the tooth. Thus, suppose the rule applied to a two inch pitch and that the tooth and space were exactly equal, then the tooth would project $\frac{3}{4}$ of an inch beyond the pitch line, and its root would be as far within the pitch line, as to receive freely the tooth intended to act on it: suppose it also $\frac{3}{4}$, then the tooth would be $1\frac{1}{2}$ inch long, besides the freedom, which, making as above, $\frac{3}{16}$, the tooth would be in all $1\frac{11}{16}$ inch long.

156. But it is to be remarked, that the millwright, in making his pattern for a cast iron wheel, has to attend to a circumstance arising from the nature of that material. The pattern must not only be of such a form as to be sufficiently strong, calculating by the bulk of the parts, but also proportioned, so that when the fluid metal is poured in the mould, it may cool in every part nearly at the same time.

When due attention is not paid to this circumstance, as the metal is cooling, if it contract faster in one part than in another, it will be apt to break somewhere, just as a drinking glass is broken by suddenly cooling or heating in any particular part of it. In all patterns for cast iron, about $\frac{1}{8}$ of an inch to the foot, should be allowed for the contraction of the metal in cooling.

Attention must also be paid to taper the several parts, so that they may rise freely without injuring the mould, when the founder is drawing them out of the sand. A little observation of the operations of a common foundry, will better instruct on this part of the subject than many words. We may observe, however, that about $\frac{1}{16}$ of an inch, in a depth of 6 inches, is commonly a sufficient taper.

157. Attending to those circumstances, we offer the following proportions as having been found to answer in practice.

Make the thickness of the ring *a e* equal to the thickness of the tooth *a c* near its root. When the ring is made thinner than the root of the tooth, the ring commonly gives way to a strain, which would not break the tooth.

Make the arm, at the part where it proceeds from the ring, of the same breadth and thickness as the ring; and at the junction *e f*, let it be so formed as to take off any acute angle which would be apt to break off in sand.

Fig. 1.

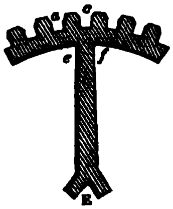


Fig. 2.

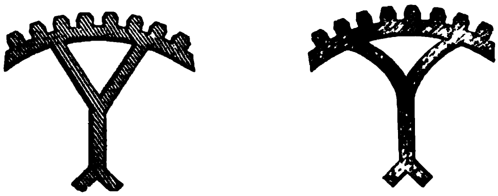


Fig. 3.

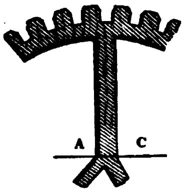
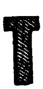


Fig. 4.



The arms should become larger as they approach the centre of the wheel, (see Emerson, Prop. 119, Rule 8,) and the eye E, should be sufficiently strong to resist the driving of the wedges, by means of which it is to be fixed on the shaft. This cannot be brought easily to calculation.

On the other hand, care must be taken not to make the eye so thick as to endanger unequal cooling.

It should be somewhat broader than the breadth of the teeth, in order that it may be the firmer on the shaft: this breadth must be greater in proportion as the wheel is large.

When the ring *a e* is about an inch thick, it is common to make the eye about an inch and a quarter thickness, and about one-fifth broader than the ring, when the wheel is about four feet diameter.

Small wheels have generally but four arms, but it being improper to have a great space of the ring unsupported, the number of arms should be increased in large wheels.

In order to strengthen the arms with little increase of metal, it is not unusual to make them feathered, which is done by adding a thin plate to the metal at right angles to the arm, as represented by figure third. Fig. 4 is a section of Fig. 3, at A B.

The same rules apply to bevelled wheels; of the practical mode of laying down the working drawings of which we have already spoken. But it is proper to observe, that the eye of a bevelled wheel should be placed more on that side which is furthest from the centre of the ideal cone of which the wheel forms a part.

158. When wheels are beyond a certain size, it becomes necessary to have patterns sometimes made for them, cast in parts, which are afterwards united by means of bolts.

To prevent the bad effects of unequal contraction, the arms may be forked or curved, as in the second figure; the forked or curved parts are commonly of the same radius as the wheel, and spring from the half length of the arms.

MATERIALS OF PATTERNS.

159. The patterns should be made of well-seasoned wood. The most proper is clean mahogany*, but that being now very expensive, white deal is most commonly used. Beech is very often used for the teeth, and being a close grained wood, it may be made very smooth.

It is almost superfluous to say, that the workmanship of wheel patterns should be such as to produce great accuracy, and a smooth surface, the former being essential to the good movement of the wheels, and the latter to make the patterns produce a good and clean impression in the sand.

It is a common practice, in many places, to make teeth very large in the pattern, and after fixing the wheels on their shafts, to chip and file the teeth to the proper size; but we doubt whether this practice be really advantageous; for besides the great time which it occupies thus to dress the iron teeth, and the consequent expense, there is the loss of the outer skin (if we may use the expression) of the cast iron, which is by far its most smooth and durable part. In cotton mills, therefore, this absurd method is now but seldom practised †.

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* The common chestnut tree is equal to any wood that can be used; and its dimensions adapt it equally well for moulds with Honduras mahogany.

† Messrs. Peel, Williams, and Co., have, after great time, trouble, and expense, made and arranged a very great number of patterns of wheels, so as to suit almost every case that can in practice occur. They have published a complete list of them, which they intend inserting also in the "Repertory of Arts." In my opinion, what they have done is a material national benefit; their expense, I am informed, for patterns, has not been less than four thousand pounds. There is, however, every reason to think, that it will be an excellent thing ultimately for themselves, as well as of great practical utility to the public.—*Buchanan.*

CHAPTER V.

ON THE USE OF CHARTS, AND SOME FURTHER EXPLANATION OF THE CONSTRUCTION OF THE TABLES OF PITCHES OF WHEEL-WORK.

160. WHEN quantities of any kind, such as time, space, money, &c., expressed in numbers, are mentioned, it often requires a painful exertion of the mind to recollect and compare them. Hence the utility of bringing them into one view in tables. But there is another mode of comparing quantities not so generally practised, though, in many cases, much more easy and satisfactory to the mind. I allude to charts, in which, instead of using figures, as in tables, the quantities are geometrically represented. This is done by dividing the sides of a square or rectangle into equal parts, and drawing parallel lines at right angles from the divisions. The quantities are pointed off at certain intersections of these scales.

161. When the quantities increase or decrease in arithmetical proportion, as 1, 2, 3, 4, &c., that proportion will be represented by a straight line, which will pass through these points.

162. But supposing the quantities to increase in geometrical proportion, as 1, 4, 9, 16, &c., the line passing through the points of intersection will form a curve.

These two cases will be best explained by examples.

163. *First,* Suppose the value of any thing to increase as its weight,—the scale on the one side of the square will then represent the value, and that on another the weight.

Let A C, Fig. 1, Plate I., represent weight, (say ounces,) and A D value, (say shillings.) Now, suppose we mark the price of four ounces, it is done by placing a dot opposite to four, on the line of value, and opposite to four on the line of weight, at the intersections of the perpendiculars from these points, which intersection is marked by *d* on the figure. In the same manner, we may mark the value of 1 2, 3, 5, 6 ounces. These points are marked *a b c e f*, and the straight line A B passes through them all, and shews the regular progress of the proportion.

It is of no consequence whether the divisions on the line A C be greater or less than those on A D, provided the lines be divided into equal parts.

164. *Second*, Suppose an accelerating motion, such as that of a falling body, is to be laid down on a chart,—this motion increases as the squares of the times; that is, the body falls a certain distance in the first second of time, four times that distance in the next second, and nine times in the third second, &c.

These points are accordingly marked in Figure 2, by *a* opposite to 1 on both scales, by *b* opposite 2 on the scale of time, A D and 4 on that of motion A C; by *c*, opposite 3, on A D, and 9 on A C, &c., the line A B passing through these points forms a curve.

165. When the proportion of any kind is regular, the curve has a regular easy sweep; if otherwise, the curve will undulate, or have irregular windings. Hence, it is a good mode of proving many kinds of tables, to lay down the quantities thus geometrically; for if there be any material error, when the proportion ought to be regular, an elbow will appear in the line A B.

166. Much calculation, too, may often be saved, for when a few of the principal points at some distance from each other are obtained in the curve, the rest of it may be easily found, by drawing the curve between them with a slip of thin wood, or any other such means of producing an easy curve.

167. Charts, on similar principles, are used for many purposes; for example, there are biographical charts, showing the periods when eminent men appeared, and the relative length of their lives. They are also used for representing revenue of any kind, which generally forms an undulating line, as does also the charts of the heights of the barometer, or the temperature indicated by the thermometer. The heights of mountains, the tides—in short, they may be considered as merely scales of equal parts, and, of course, are applicable to all subjects capable of being represented by numbers.

168. In order to give a more distinct comparative view of the tables of pitches, in the "Essay on the Teeth of Wheels," we shall lay their contents down in one chart, but previously collect all these tables, and give some further explanation of their mode of construction.

TABLES OF PITCHES OF WHEEL-WORK.

(See Chap. IV., Art. 123—129.)

TABLE I*.

Velocity of the pitch line being 3 feet per second, and breadth of teeth 9 inches.

W	X	Y	Z
Pitch in inches.	Breadth of teeth in inches.	Value of strength in horses' power.	Value of strength in horses' power.
4	9	16·	12·
3½	9	12·25	10·5
3	9	9·	9·
2½	9	6·25	7·5
2	9	4·	6·
1½	9	2·25	4·5
1	9	1·	3·

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* Tables I. and II. are for teeth attached to water-wheels, where liable to be worn by sand and water.

TABLE II.

The velocity being 3 feet per second, and the breadth of the teeth double each pitch.

W	X	Y	Z
Pitch in inches.	Twice the pitch in breadth.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	14·22	10·66
3½	7	9·53	8·17
3	6	6·	6·
2½	5	3·47	4·16
2	4	1·77	2·65
1½	3	·75	1·5
1	2	·22	·66

TABLE III*.

The velocity being 11 feet per second, and the breadth of teeth 8 inches.

W	X	Y	Z
Pitch in inches.	Breadth of teeth in inches.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	81·77	61·33
3½	8	62·61	53·66
3	8	46·	46·
2½	8	31·94	38·33
2	8	20·44	30·66
1½	8	11·5	23·
1	8	5·11	15·33

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* Tables III., IV., V., and VI. are for teeth properly greased, and free from sand.

TABLE IV.

The velocity being 11 feet per second, and the breadth double each pitch.

W	X	Y	Z
Pitch in inches.	Twice the pitch in breadth.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	81·77	61·33
3½	7	54·78	46·95
3	6	34·5	34·5
2½	5	19·95	23·94
2	4	10·22	15·33
1½	3	4·31	8·62
1	2	1·28	3·84

TABLE V.

The velocity being 3 feet per second, and breadth of teeth 8 inches.

W	X	Y	Z
Pitch in inches.	Breadth of teeth in inches.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	22·30	16·72
3½	8	17·07	14·63
3	8	12·54	12·54
2½	8	8·71	10·45
2	8	5·57	8·36
1½	8	3·13	6·26
1	8	1·39	4·17

TABLE VI.

The velocity being 3 feet per second, and breadth double each pitch.

W	X	Y	Z
Pitch in inches.	Twice the pitch in breadth.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	22·30	16·72
3½	7	14·93	12·79
3	6	9·4	9·4
2½	5	5·44	6·53
2	4	2·78	4·17
1½	3	1·17	2·34
1	2	0·34	1·02

REFERENCE TO TABLE I., ART. 123, IN THE ESSAY ON THE TEETH OF WHEELS.

169. Column X is omitted in Tables I., III., and V., being only a repetition of the same breadth for all the pitches of each table, but as being perhaps plainer, they are inserted here.

The numbers in column Y are found by squaring the pitch in column W.—(See Proposition I. p. 83.)

EXAMPLE.

The square of 4, (the pitch in inches) = 16, the value of strength in horses' power.—(See the first line of table.)

The column Z is found by inverse proportion.—(See Prop. II. p. 85.) taking three inches, (the standard pitch,) always as the first term, the pitch column, W, as the second, and the horses' power, in column Y, as the third term.

EXAMPLE.

In. In. Horses' power. Horses' power.
3 : 4 : : 16 : 12 (See first line of table.)
3 : 1 : : 1 : 3 (See last line of table.)

REFERENCE TO TABLE II.

170. The numbers in column Y are found here by direct proportion from Table I., nine inches (the breadth in Table I.) being always the first term of the proportion; the horses' power in Y, Table I. the second, and the breadth in X, Table II. the third term.

EXAMPLE.

In. Horses' power. In. Horses power.
9 : 16 : : 8 : 14·22 (See Table II. line first). Column Z is found, as in Table I. by inverse proportion, 3 inches (the standard pitch) being always the first term. Thus,
In. In. Horses' power. Horses' power.
3 : 4 : : 14·22 : 10·66.—(See first line of table.)

REFERENCE TO TABLE III.

171. The numbers in column Y are found by direct proportion, taking 9 (the square of the standard pitch of three inches) as the first term, and the square of the pitch in W as the second term.

EXAMPLE.

As 9 (the square of 3 inch pitch)
Is to 16 (the square of 4 inch pitch),
So is 46 horses' power (the value in column Y of 3 inch pitch)
To 81·77.—(See first line of table.)

Column Z is found by inverse proportion, as in former tables.

EXAMPLE.

$\overset{\text{In.}}{3} : \overset{\text{In.}}{4} :: \overset{\text{Horses' power.}}{81{\cdot}77} : \overset{\text{Horses' power.}}{61{\cdot}33}$.—(See 1st line of table.)

REFERENCE TO TABLE IV.

172. The numbers in column Y are found here in a manner similar to Table II. by direct proportion.

EXAMPLE.

$\overset{\text{In.}}{8} : \overset{\text{In.}}{7} :: \overset{\text{Horses' power.}}{62{\cdot}61} : \overset{\text{Horses' power.}}{54{\cdot}78}$.—(See 2d line of table.)

The numbers in column Z are found, as in the former tables, by inverse proportion,

$\overset{\text{In.}}{3} : \overset{\text{In.}}{4} :: \overset{\text{Horses' power.}}{81{\cdot}77} : \overset{\text{Horses' power.}}{61{\cdot}33}$.—(See 1st line of table.)

REFERENCE TO TABLE V.

173. The numbers in column Y of this table are found by direct proportion from column Y of Table IV., 11 feet (velocity per second) being always the first term, and 3 feet (velocity) the second term.

Thus, $\overset{\text{Ft.}}{11} : \overset{\text{Ft.}}{3} :: \overset{\text{Horses' power.}}{81{\cdot}77} : \overset{\text{Horses' power}}{22{\cdot}30}$.—(See 1st line of table.)

Column Z is found by inverse proportion, as in all the former tables:

Thus, $\overset{\text{In.}}{3} : \overset{\text{In.}}{4} :: \overset{\text{Horses' power.}}{22{\cdot}30} : \overset{\text{Horses' power.}}{16{\cdot}72}$.—(See 1st line of table.)

REFERENCE TO TABLE VI.

174. The numbers in column Y and Z in this table, are

formed from Table V. in the same manner as those columns in Table II. are formed from Table I. The only difference in Table VI. from Table V. is that which arises from the difference of the breadth of the teeth.

EXPLANATION OF THE CHART.

175. The scale on the line A B represents the pitch in inches.

The scale on the line A C the horses' power; a single example will probably be sufficient to illustrate the use of the chart.

Suppose the pitch to be 3½ inches, let it be required to find the horses' power to which that pitch is equal when moving at 3 feet per second, in situations where properly greased and free from sand—observe, where the line from the pitch 3½ intersects the curve z of Table VI. perpendicular to the point of intersection, on the line A C, will be found 12·79 on the scale or the horses' power to which 3½ pitch, when the teeth are 7 inches broad, is equal, after making allowance for the length of the teeth.

OBSERVATIONS.

176. 1st, It will be observed, that the curves Y and Z intersect each other, for all the tables on the pitch line marked 3 inches, because that is the standard. (See 1st Essay, p. 96, 97.)

177. 2d, The curves, continued from 1 inch pitch downward, unite in the points marked 0, being the commencement of the scale of pitches, and this part of the curve gives the horses' power equal to any fraction of an inch.

178. 3d, It has been already observed, p. 98, that durability as well as strength should be considered in this investigation. The true proportion, therefore, may be somewhere between the curves Y and Z, but nearer to Z than Y. For

although long teeth will be more easily broken than short ones, yet while they do not break, the strain being generally diffused over a greater number of teeth, they will wear longer*.

179. The pitch is laid down on A B, real measure, so that if the pitch should happen to be fractional, it may be taken by a pair of compasses and applied to the chart, which will at once indicate the power to which it may be equal at certain velocities.

Among the writers who have turned their attention to the forms of the Teeth of Wheels, Professor Willis, of Cambridge, stands pre-eminent; and we are greatly indebted to that gentleman for his liberality in permitting us to insert the following appendix, his Essay on the Teeth of Wheels, and which appeared originally in the second volume of the Transactions of the Institution of Civil Engineers, and for the additions he has since made to that paper.

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* See Art. 70.

APPENDIX A.

ON THE TEETH OF WHEELS. BY R. WILLIS, M.A., F.R.S., H.M.INST.C.E., JACKSONIAN PROFESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF CAMBRIDGE.

180. The investigation of the proper curves to be given to the teeth of wheels, has been a favourite occupation with mathematicians of the highest eminence, and the geometry of the subject may be considered to be very nearly complete.

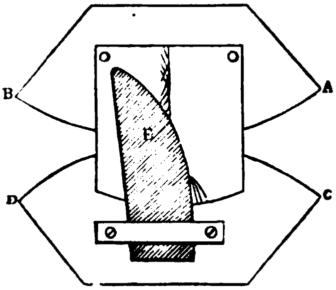
Its application to the requirements of modern construction appeared to me to be susceptible of improvement, and I therefore ventured to lay before the Institution of Civil Engineers the following suggestions, in which I endeavoured to point out forms possessing properties more general than those hitherto adopted, as well as some practical methods of tracing readily the outlines of the teeth.

SECTION I.

ON THE CURVES ADAPTED TO PRACTICE.

There are an infinite number of forms which will answer the conditions of enabling the teeth of one wheel to communicate equable motion to those of another, for it can be shewn that, under certain limitations, if *any* form of tooth be given, another may be determined which will work correctly with it*. A simple instrument which furnishes a practical solution of this problem, will probably carry more conviction to the minds of practical men than the demonstrations which have been given by the writers referred to below. Let a pair of boards be prepared, having their edges, A B, C D, Fig. 1, formed truly circular. Attach to one of them by any simple clamp the shape of the given tooth E, cut out in pasteboard, and to the other a piece of stiff paper secured by means of drawing pins; the shape E must be raised slightly above the surface of its board, so as to allow the paper which is appended to the other to slide under it, as is shewn in the figure. Make the circular edges of the two boards roll together, and in each successive position draw the outline of the shape E upon the paper below it. The result of all these intersecting lines will be a bounding curve, which from the very mode of its description will touch the shape E at some point of its edge in every one of the successive positions. But as these positions were all obtained by making one circular edge roll upon the other, so it is clear, that if the new curve be cut out and made to touch E, the motion produced by the mere contact of these two curves will be exactly the

FIG. 1.



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* Vide De la Hire, Traité des Epicycloides. Young's Natural Philosophy, Vol. I. page 176. Airy, Cambridge Philosophical Transactions, Vol. II. page 277.

same as that caused by the rolling of the circular edges, and therefore perfectly uniform.

Many forms of E, tried in this manner, will prove untractable, for some of the successive portions of its edge may cover up and obliterate parts of the curve that have been previously drawn. These are forms that fall under the limitations alluded to, but it is unnecessary here to investigate the general reasons for this effect, as the proposition in question is well known and recognized by mathematicians, although not so well understood by practical men.

From among the infinity of curves that may be offered, the epicycloids and involutes have been universally preferred, on account of the facility with which they can be mechanically described, and perhaps because they admit of ready and independent demonstrations of their possessing the properties required. But the practice has hitherto been confined to that class of epicycloids which work correctly with straight lines or circles. Teeth formed upon these principles possess this inconvenience: a wheel of a given pitch and number of teeth, say 40, if it be made to work correctly with a wheel of 50 teeth of the same pitch, will not work correctly with a wheel of 100 teeth of the same pitch. This is obvious, for the diameter of the describing circle by which the epicycloid is formed must be made equal to the radius of the pitch circle of the wheel with which the teeth are to work, and will therefore be twice as large in the second case as in the first.

In the old style of mill-work, in which the teeth of wheels always consisted of wooden cogs, this property offered no very serious impediment, although, as we shall see, it introduced some complication of method; but in the modern practice of making cast iron wheels, the objection is a very serious one. A founder must make a new pattern of a wheel of 40 teeth for every combination that it may be required to make of this wheel with others, and the same for a wheel of any other number. Besides, it often happens in machinery, that one wheel is required to drive two or more whose number of teeth are different, and in this case the teeth cannot be correctly formed at all on the common principles; and again, the perfection of machinery is impaired from the temptation to employ in one combination patterns that have been formed for some other combination very nearly the same; for example, to make a wheel of 40 teeth that has been formed to work with one of 80, serve for a required combination of 40 with 85.

It is essential, therefore, that the teeth of wheels should, if possible, be so formed as to allow a given wheel to work correctly with any other wheel of the same pitch. Now it has long been known that involute teeth have this very property, but the objections to these teeth on the score of the obliquity of their action have operated fatally against their introduction*. I shall now, therefore, explain a method of imparting to epicycloidal teeth this property, and that without making them deviate very much from the general form that has been established by practice.

To effect this, it is merely necessary to employ a proposition well known and stated by almost every writer on the subject, namely, If there be two pitch circles touching each other, then an epicycloidal tooth formed by causing a given describing circle to roll on the exterior circumference of the one, will work correctly with an interior epicycloid, formed by causing the same describing circle to roll on the interior circumference of the other†.

This proposition having been already demonstrated, it is unnecessary for me to dwell upon it longer than to remark, that our author, like all the other writers on the subject, has passed from it, to recommend for practice that particular case of it in which the describing circle being made equal in diameter to the radius of the pitch line, the interior epicycloid becomes a radial straight line, the inconveniences of which practice I have shewn*.

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* Vide Hawkins's Notes to Camus, page 161.

† Vide Art. 19.

The following corollary is, I believe, new, and constitutes the basis of the system I propose to explain.

Corollary. If for a set of wheels of the same pitch, a constant describing circle be taken, and employed to trace those portions of the teeth which project beyond each pitch line by rolling on the exterior circumference, and those which lie within it by rolling on its interior circumference: then any two wheels of this set will work correctly together.

For, in the first place, it is well known and can be shewn from general principles, that the portion of tooth *within* the pitch line of a driving wheel, works only with the portion that lies *beyond* the pitch line of its follower, and that its action is confined to the approach of the point of contact to the line of centres. After the point of contact of the teeth has passed that line, then the case is reversed, and the portion of the driving tooth which lies *beyond* the pitch line is in contact only with some part of the follower's tooth which lies *within* its pitch line.

Now as a constant describing circle is used for the whole set, it is clear that the proposition will apply to any pair of wheels both before and after the teeth have passed the line of centres, for in each case we have an exterior epicycloid working with an interior epicycloid, and both have been drawn by the same describing circle, that is, by the constant circle of the set.

To carry this scheme into practice, it only remains to settle the proper diameter to be given to this constant de-

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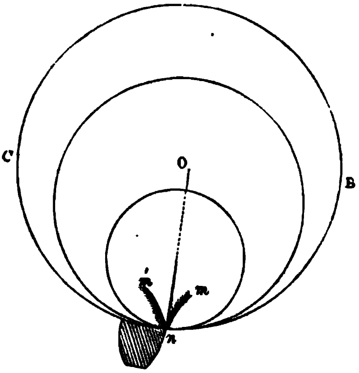
† Vide Brewster's Ferguson, Vol. II. p. 223. Camus, p. 27, or 25 new edition.

scribing circle, which may be done by considering the effect this diameter has upon the form of the tooth.

Let B C *n*, Fig. 2, be a pitch circle whose centre is O, then upon this system the flank of the tooth, or that portion which lies within the pitch circle, will be an arc of an

FIG. 2.



interior epicycloid (or hypocycloid) *m'n* or *mn*. Now if the describing circle be of half the diameter of the pitch line, the flank will become a straight line coinciding with the radius O *n*. If the describing circle be of less than half the diameter of the pitch line, the flank *m n* will be concave, and the base of the tooth will spread; but if the describing circle be of more than half the diameter, the flank *m'n* will be convex, and the base of the tooth lessen inwards, a form manifestly unpractical and useless. Hence the describing circle must not be greater than half the diameter of the pitch line.

On the other hand, if the diameter be too small, the base of the tooth will spread inconveniently, and the curvature of the exterior epicycloids be injuriously increased, therefore, on these grounds, it should be made as large as it can consistently with the limitation just stated, so that we finally obtain this rule for finding the diameter of the constant describing circle for a set of wheels.

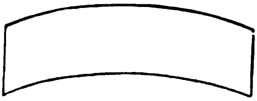
Make it equal to the radius of the least pitch circle of the set.

And as pinions should never have less than 12 or 14 teeth, it would be well to establish one of these numbers for that least pitch circle.

The proposition and corollary being perfectly general, will apply to racks, which must be considered as very large wheels, and also to annular or internal wheels. Accordingly, if the constant describing circle be employed in tracing their teeth, they will work correctly with any wheel of the set.

It will be seen that this system is more easy of practice for the workman than the old one. Every epicycloid requires two circular or rather segmental templets, which are usually cut out of thin board. One of these, which may be termed the pitch templet, has its edge formed into an arc of the pitch line of the wheel; the other, which represents the describing circle, and may be called the describing templet, has its circular edge formed accordingly. The tracing point is fixed upon the circumference of the latter, and the workman having previously described an arc of the pitch circle of the wheel upon his drawing board, fixes the pitch templet, so that its edge may coincide with this arc, and then causing the describing templet to roll upon the pitch templet, he traces the arc of the required epicycloid.

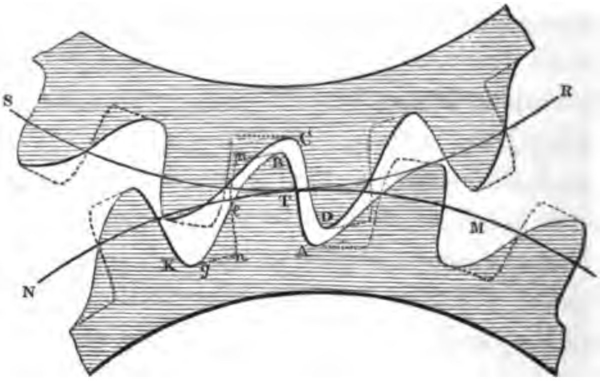
Now on the old system, a set of wheels requires as many templets as there are pitch circles in the set, and also as many describing templets, but on the system just explained, only one describing templet is needed. As, however, the flanks of the teeth within the pitch circles become curves instead of straight lines, it is necessary to have concave templets adapted to the pitch circles, upon whose edges the describing templet may be made to roll for the purpose of obtaining the proper interior epicycloid. The best way is to make each pitch templet with two edges, one convex and the other concave, as in the figure, and to write the diameter upon each of them.



ON A FORM OF INCREASED STRENGTH.

In a large class of machinery, the wheels constantly move in the same direction, and whenever this is the case, it is possible to increase the strength of the teeth in a very great degree, by an alteration of the common form represented in Figure 3.

FIG. 3.



Let A B, C D, be the acting faces of the teeth of a pair of wheels, of which M N, R S, are parts of the pitch lines. Now, according to the ordinary practice, the backs of the teeth would be formed exactly in the same manner as the acting faces, as shewn by the dotted lines, and this enables the teeth to work backwards or forwards at pleasure if required. If, however, the back is never required to act, the strength of the tooth will be nearly doubled by making it of the form B *eg* K, that is, by taking off the portion B *me*, and filling up the nook *eng*. Teeth so formed will clear each other quite as well as those formed in the usual manner, with the advantage of a root of nearly double extent, and as the acting faces remain of the usual form, they will work together just as the ordinary teeth do. Strictly speaking, the back B*eg* should be an arc of an involute so proportioned as to work correctly with the corresponding back of the tooth of the other wheel. For then, as the backs of the teeth would drive each other truly, they are sure to clear each other; and besides, if the machinery be made accidentally to run backwards, the teeth will still work, although with a considerable divergent pressure upon the axes*. It will be quite near enough, however, to make the back an arc of a circle described through three points B*eg*, the first of which, B, should be taken a little way from the point of the tooth in order to blunt it slightly; the second, *e*, on the pitch circle is set off in the usual manner, so that T*e* may be about $\frac{6}{11}$ths of the pitch; and the third point, *g*, may be found by dividing AK into five parts, and taking *g*K equal to one of them. The space *g*K is required to enable the point of the corresponding tooth to clear itself.

This form resembles the saw-shaped teeth which have been employed occasionally by mechanists, for example, (according to Mr. Reid, in his Horology, p. 100,) Lepine, of Paris, had in some of his watches the teeth and pinion-leaves of a saw-teeth form, but I am not aware that the advantage of this shape has ever been systematically shewn, or any principle of its formation given.

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* This divergent pressure will do no harm, because the kind of machinery to which I propose to adapt this form, never drives backwards, while the working pressure is upon it, but only during some previous adjustments, when the only pressure to be overcome is that produced by inertia or by the friction of the parts of the engine upon each other.

SECTION II.

ON A PRACTICAL APPROXIMATION TO THE TRUE FORM BY ARCS OF CIRCLES.

Although the practice in the best workshops is to describe the shape of a tooth carefully with templets in the manner just described, yet this is not done for every tooth in the wheel or pattern; on the contrary, having traced the shape of a single tooth in this manner, the workman next finds with his compasses, by trial, a centre and small radius by which an arc of a circle can be described that will coincide as nearly as he can manage to make it with the templet-traced epicycloid. Then having struck upon the face of the rough cogs a circle concentric with the pitch circle, and whose distance from it is equal to that of the centre of his arc, he adjusts his compasses to the small radius, and always keeping one point in the circle just described, he steps with the other to each cog in succession, they having been previously divided into equal parts corresponding to the pitch and breadth of the teeth. Upon each cog he describes two arcs, one to the right and the other to the left, which serve him as guides in shaping and finishing the acting faces.

The portion of curve employed in a tooth is so short, that a circular arc would be quite sufficiently accurate, if its centre and radius were determined more correctly than by this coarse mode of trial. This consideration induced me to investigate the method I am about to describe, in which the examination of the nature and properties of the curves made use of for teeth is entirely dispensed with. I have deduced a simple construction by which a pair of centres may at once be assigned for a given pair of wheels, from whence, if arcs of circles be struck and employed for the working faces of teeth, they will answer the purpose of enabling these wheels to work correctly together*.

I shall first explain the methods that arise from their construction for the use of practical men, and then add the theory upon which they are founded.

The working face of each tooth may be formed of one arc of a circle, or of two. In Fig. 8, Plate 20, each face is formed of a single arc *op*, and the resulting tooth has considerable analogy to the involute tooth, and like it has the fault of acting with rather too much obliquity; but the mode of describing it, on the other hand, is exceedingly simple, and any two wheels whose teeth are thus formed, will work correctly together. For small teeth, I am inclined to think this method would answer the purpose very well. It only remains to show how the centre P and radius PT of the arc *o*T*p* is to be determined.

Let AT be the radius of the pitch circle of the proposed wheel. Upon AT describe a semicircle TPA, and from T set off TP equal to one quarter of the radius; then will P be the centre from which, if an arc *op* be described through T, the required side of the tooth will be obtained.

Or, construct a bevil in brass, of which the angle at T shall be exactly equal to 75° 30′, and graduate the side TP into a scale of quarter inches, as in the figure. Apply the plain side of this bevil to the radius A T of the proposed wheel, and its point T to the pitch circle; read off the length of the radius A T in inches upon the reduced scale T P, and the point P so indicated will be the centre of the tooth as before. Thus in the figure, A T is four inches, and P is found at 4 upon the scale.

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* Euler, in his second paper on the teeth of wheels, (N. C. Pet. XI. 209,) has with his usual ability investigated the proper curves, by examining the relation between their radii of curvature at every point. This method has naturally conducted him to results of a similar nature to those which I have given in the following pages, and he suggests that a small arc of the circle of curvature would suffice in practice for the forms of teeth. He has given some geometrical constructions for this purpose, and has then passed on finally to recommend the involute as the best curve, this paper being, in fact, the first in which that curve is pointed out as possessing the required properties. To Euler, then, belongs the merit of first suggesting the substitution of an arc of the circle of curvature for the real curve, a hint which has been, as far as I know, neglected by every succeeding writer. This may perhaps be attributed to the abstruse manner in which he has treated the subject.

When the side of the tooth is formed of two arcs of circles, the forms shown in Figs. 9 and 10 are obtained: these represent the same teeth in different relative positions. In Fig. 9 the tooth *abc* is approaching the line of centres A B, and in Fig. 10 the same tooth *abc* is retiring from it. The portion of tooth *ab* which lies within the pitch circle, is described from a centre P, Fig. 9; and the portion *bc* which lies beyond the pitch circle is described from a centre P, Fig. 10. The resulting form is a very strong one, possessing the property that any two wheels of a set will work together. Any practical man may convince himself of the degree of accuracy with which this is effected, by describing according to this method, on a large scale, (say six inches pitch,) a pinion of twelve or fourteen teeth, and a few teeth both of a wheel of fifty and of a rack. These teeth may be cut out of thin board, and it will be found that any two of them will work correctly together with a degree of precision amply sufficient for practice. To facilitate the description of teeth as much as possible, I have thrown the system into the form of an instrument, which I have termed an Odontagraph, and which any one may make for themselves out of a sheet of card paper, by observing the following instructions*.

F E D *t*, Fig. 11. represents this instrument on a scale of one quarter of the original. The angle D *t* F is exactly 75°, and the side K *t* F is graduated into a scale of half inches, each half inch being divided into ten parts. The half inches are numbered from zero at *t*, both ways towards the extremity of the scale, 0, 10, 20, ... up to about 200 upwards, and 40 downwards.

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* Those who are not disposed to take the trouble, may obtain it complete of Messrs. Holtzapfel of Charing Cross.

Upon the plain surface of the card are placed the tables which follow:

TABLES SHEWING THE PLACE OF THE CENTRES UPON THE SCALE.

CENTRES FOR TEETH WITHIN THE PITCH CIRCLE.

PITCH IN INCHES AND PARTS.

Number of Teeth.	¼	⅜	½	⅝	¾	1	1¼	1½	1¾	2	2¼	2½	3	3½
13	32	48	64	80	96	129	160	193	225	257	289	321	386	450
14	17	26	35	43	52	69	87	104	121	139	156	173	208	242
15	12	18	25	31	37	49	62	74	86	99	111	123	148	173
16	10	15	20	25	30	40	50	59	69	79	89	99	191	138
17	8	13	17	21	25	34	42	50	59	67	75	84	101	117
18	7	11	15	19	22	30	37	45	52	59	67	74	89	104
19	...	10	13	17	20	27	35	40	47	54	60	67	80	94
20	6	9	12	16	19	25	31	37	43	49	56	62	74	86
22	5	8	11	14	16	22	27	33	39	43	49	54	65	76
24	...	7	10	12	15	20	25	30	35	40	45	49	59	69
26	...	...	9	11	14	18	23	27	32	37	41	46	55	64
28	4	6	...	...	13	...	22	26	30	35	40	43	52	60
30	...	...	8	10	12	17	21	25	29	33	37	41	49	58
35	...	...	...	9	11	16	19	23	26	30	34	38	45	53
40	...	5	7	...	...	15	18	21	25	28	32	35	42	49
60	3	...	6	8	9	13	15	19	22	25	28	31	37	43
80	...	4	...	7	...	12	...	17	20	23	26	29	35	41
100	...	...	...	...	8	11	14	...	...	22	25	28	34	39
150	...	...	5	...	...	...	13	16	19	21	24	27	32	38
Rack.	2	...	...	6	7	10	12	15	17	20	22	25	30	34

CENTRES FOR TEETH OUTSIDE THE PITCH CIRCLE.

PITCH IN INCHES AND PARTS.

Number of Teeth.	¼	⅜	½	⅝	¾	1	1¼	1½	1¾	2	2¼	2½	3	3½
12	1	2	2	3	4	5	6	7	9	10	11	12	15	17
15	...	...	3	...	...	...	7	8	10	11	12	14	17	19
20	2	...	...	4	5	6	8	9	11	12	14	15	18	21
30	...	3	4	...	...	7	9	10	12	14	16	18	21	25
40	...	...	...	...	6	8	...	11	13	15	17	19	23	26
60	...	...	...	5	...	...	10	12	14	16	18	20	25	29
80	...	...	...	...	...	9	11	13	15	17	19	21	26	30
100	...	...	...	...	7	...	...	...	...	18	20	22	...	31
150	...	...	5	6	...	...	...	14	16	19	21	23	27	32
Rack.	...	4	...	...	...	10	12	15	17	20	22	25	30	34

One example will explain the mode of using this instrument. Let it be required to describe the form of a tooth for a wheel of 29 teeth of 3 inches pitch.

Describe an arc T *t* of the required pitch circle, and set off upon it T *t*, equal to the pitch, and bisected in *e*; draw radial lines B T, B *t*. For the arc within the pitch circle apply the slant edge of the scale to the radial line B *t*, placing its extremity *t* on the pitch circle, as in the figure. In the table, headed *Centres for teeth within the pitch circle*, look down the column of 3 inch pitch, and opposite to 30 teeth, which is the nearest number to that required, will be found the number 49. The point *r*, indicated on the drawing board by the position of this number on the scale of equal parts marked, *Scale of centres of teeth within pitch circle*, is the centre required, from which the arc *ef* must be drawn with a radius *re*.

The centre for the arc *de*, which lies outside the pitch circle, is formed in a manner precisely similar, by applying the slant edge of the scale to the radial line B T. The number 21 obtained from the table of *Centres for teeth outside the pitch circle* will indicate the position of this centre upon the *Scale of centres for teeth outside the pitch circle*, namely at R.

The radius of the wheel may be found, by help of the following table and rule. Multiply the number corresponding to the given pitch in this table by the number of teeth required, the product will be the radius of the pitch circle in inches and decimals. Thus, for a wheel of 29 teeth of 3 inches pitch, multiply ·4774 by 29, and the radius is 13·84 inches.

Pitch.	Factors.	Pitch.	Factors.
3½	·5570	1¼	·1989
3	·4774	1	·1591
2½	·3979	¾	·1193
2¼	·3581	⅝	·0994
2	·3183	½	·0795
1¾	·2785	⅜	·0597
1½	·2387	¼	·0398

The curve *def*, Fig. 11, is also true for an *annular wheel* of the same number of teeth, *f* becoming, of course, the point of the tooth, and *d* its root. For a *Rack*, the pitch line T*t* will be a straight line, and B*t*, BT be drawn perpendicular to it, at a distance from each other equal to the pitch. The numbers for pitches not inserted in the table, may be obtained from the column of some other pitch, by direct proportion. Thus for 4 inch pitch, by doubling the numbers in the column of the 2 inch pitch, for 4½ by doubling 2¼, and so on; or if the difference be small, the column belonging to the nearest pitch may be employed, without a serious error; or more accurately a number may be taken half way between those given in the two nearest columns.

No tabular numbers are given for twelve teeth, for within the pitch circle such teeth are bounded by radial lines.

But without using the Odontagraph, the geometrical construction shown in Figs. 9 and 10 may be employed. This must be of course drawn to the real size of the wheels in question.

Let AB be the centres of a pair of wheels, T the point of contingence of their pitch circles; through T draw KTK, making an angle of 15° with the line of centres, and bisected in T; also draw PT perpendicular to KTK, TK may be of any length less than the least radius of the pitch circles. There are thus obtained two points K, one near to the right hand centre B, and the other to the left hand centre A. The first is thus employed in Fig. 10, to obtain the arcs *bc*, *ef*. Join BK and produce it to Q; join AK intersecting QT in P. Set off T*n* equal to half the pitch, and with centre P and radius P*n* describe the arc *bc* outside the pitch circle of the left hand wheel, and with centre Q and radius Q*n* describe the arc *ef* within the pitch circle of the opposite wheel, whose centre of motion is B; then will these arcs work truly together. In like manner Fig. 9. Join AK and produce it to meet the line TQP in P; join BK intersecting the same line in Q. Set off T*m* equal to half the pitch, and with centre P and radius P*m* describe that portion of the tooth *ab* which lies within the pitch circle *b*T, and with centre Q and radius Q*m* describe the tooth *ed*, which lies beyond the pitch circle *e*T.

These rules must be observed in both cases, namely, that the half pitch T*n*, T*m* is always set off on the opposite side of the line of centres to the centres PQ of the teeth; and that the centres of the *concave* flanks *within* the pitch circle are obtained by joining the centre of the pitch circle with that K, which lies *nearest* to it, and producing the line to meet TQP: but that the centres of the *convex* teeth *beyond* the pitch circle are obtained by joining the centre of the pitch circle with that K, which is most *remote* from it.

Any two wheels in which the length of KT is the same, will work truly together.

ON TEETH WORKING WITH TRUNDLES OR RADIAL FLANKS.

The particular applications of the general construction which I have given, apply only to complete sets of wheels working together, and it may be as well to shew its use in obtaining teeth adapted to work with trundles or pin wheels, as well as teeth in which the flank is a radial line as in the common form. The diagram of Fig. 12, Plate 20, must be drawn of the full size for any required wheel, A and B are the centres as usual, *fg* and *hk* arcs of the pitch circles. Upon the radius of the trundle AT describe a semicircle, upon which set off from T, TP equal to the pitch. Draw PTQ and let fall a perpendicular BQ upon it from B, intersecting it in Q. If the point P be taken for the centre of the stave or pin, an arc *mn* described from Q and touching the stave in *m*, will be the side of the tooth required.

If the flanks of the teeth are to be radial lines, then the portions lying without the pitch circle may be arcs of circles found thus. (Fig. 14.) A and B are the centres; *fg*, *hk*, arcs of the pitch circles as before. Upon AT describe an entire circle, and upon its circumference from A and T set off A*z*, T*m* equal to each other and to about three quarters of the pitch; join B*z* and through *m*T draw *m*TQ intersecting B*z* in Q; then an arc described from centre Q and struck through *m*, will be the curved face of the tooth for BT, and this will work with the radial flank of the tooth of AT. To find the curved face of the latter tooth make a similar diagram, in which AT and BT exchange places.

ON CUTTERS.

The Odontagraph is also applicable to the obtaining a correct form for the cutters used in shaping the teeth of metal wheels. The form of the cutter is that of the space between two teeth, and in order to shew the nature of the change of form required for different teeth as well as the general form itself, I have in Figure 13, Plate 20, drawn with accuracy, and on a large scale, the teeth proper to the two extreme cases of a pinion of 12 on the one hand, and a rack on the other, and have applied these two together, so that the central line of the spaces shall coincide, and thus bring the shapes of the cutters into direct comparison.

Now between these two lie all the forms that are required for any number of teeth from 12 to a rack, or the largest possible wheel; but in making a set of cutters, for small pitches especially, it is by no means necessary to make one for every number, as the forms for numbers that lie close together are so nearly alike that the errors of workmanship would entirely destroy the difference.

The variation of form however is much less among high numbers than in low ones. For example, the difference of form between a cutter for 150 teeth, and one for 300, is not greater than that between cutters for 16 and 17 teeth.

This being the case, it appeared worth while to investigate some rule by which the necessary cutters could be determined for a set of wheels, so as to incur the least possible chance of error. To this effect I have calculated, by a method sufficiently accurate for the purpose, the following series of what may be termed equidistant values of cutters; that is, a table of cutters so arranged, that the same difference of form exists between any two consecutive numbers.

TABLE OF EQUIDISTANT VALUES FOR CUTTERS.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
No. of Teeth.	Rack.	300	150	100	76	60	50	43	38	34	30	27	25	23	21	20	19	17	16	×	15	14	13	×	12

This will be a guide in the selection of the wheel to which each cutter shall be accurately adapted after it has been determined how many are necessary in a set. For example, if a single cutter were thought sufficient for a set of very small wheels, it had better be accurately adapted to teeth of 25, for that value is intermediate between the two extremes. If three cutters are to suffice for the whole set, then 76, 25, and 15 must be selected, of which the cutter 76 may be used for all teeth from a rack to 38, the cutter 25 from 38 to 19, and the cutter 15 from 19 to 12, and so on.

It appears from the figure that the greatest difference of form is at the apex of the tooth, (that is, at the base of the cutter,) and amounts to ·25 inch in 2 inch pitch; from this the difference may be ascertained for any smaller pitch, and as many cutters interposed as the workman's notion of his own powers of accuracy may induce him to think necessary.

Thus if the hundredth of an inch be his limit of accuracy in forming cutters, and he is making a set for half inch pitch, where the difference of form is $\frac{1}{4} \times \cdot 25$ or ·06 nearly, then half a dozen cutters will be sufficient, and these must be made as nearly as possible to suit the wheels of 150, 50, 30, 21, 16, 13.

The following table contains a selection of numbers for different cases, which may save trouble.

TABLE OF CUTTERS.

No. of Cutters in the set.	Numbers of Teeth to be selected.																							
2	50	16																						
3	75	25	15																					
4	100	34	20	14																				
6	150	50	30	21	16	13																		
8	200	67	40	29	22	18	15	13																
10	200	77	50	35	27	22	19	16	14	13														
12	300	100	60	43	34	27	23	20	17	15	14	13												
18	300	150	100	70	50	40	35	30	26	24	22	20	18	16	15	14	13	12						
24	Rack.	300	150	100	75	60	50	43	38	34	30	27	25	23	21	20	19	18	17	16	15	14	13	12

When the numbers have been selected, the Odontagraph may be employed to draw the figure of the cutter corresponding to each wheel, either on the same scale as the proposed cutter, or on a much larger scale, which may be afterwards reduced proportionally.

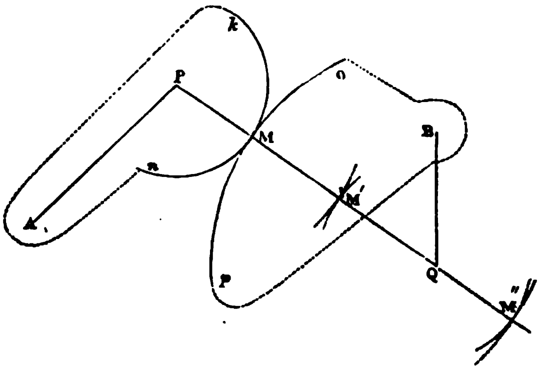
SECTION III.

THEORY OF THE PRECEDING CONSTRUCTIONS.

We must first examine the nature of the motion which is produced by the pressure of one circular arc upon another when disposed so as to work in the manner of teeth.

Let A B, Fig. 4, be two centres of motion, *k* M *n* a piece formed into a circular arc described from a centre P, and capable of revolving round A; *o* M *p* in like manner a circular arc described from Q, and capable of revolving round B; now if the arc *k* M *n* be made to press against *o* M *p*, so as to communicate rotation to it round B, the line P Q join-

Fig. 4.



ing the centres of the arc will necessarily always pass through the point of contact M, and will be of a constant length equal to the sum of the radii, so that in fact the motion will be exactly the same, if for the circular arcs a link PQ be substituted, which length is equal to the sum of the radii PM, QM, and which is jointed to the revolving pieces at P and Q, the places of the centres.

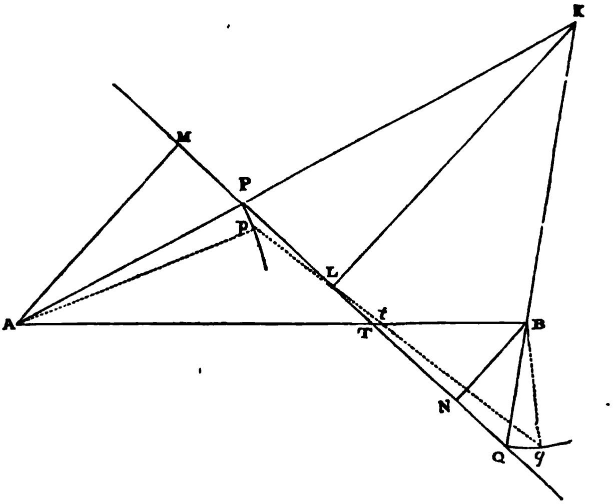
This also shews that a change of the actual lengths of the radii PM, QM, will not affect the motion, so long as the distance of the centres is constant, for that whether the circular arcs had been struck through M or M′, or even through a point M″ beyond the centre Q, the system would still have been equivalent to the link PQ, jointed to the arms AP, BQ.

It is only necessary then to examine the motion of this simple system of rods, and then to explain how it may be employed in forming the teeth of wheels.

Let the rod AP, Fig. 5, be moved into a new position A*p*, its extremity will carry with it the end of the link PQ, and communicate through it a motion to the arm BQ, by which it will be driven into the new position B*q*; and it is necessary to know the relative value of this motion to that of AP, which produced it.

Now this relation is continually changing, but its value

FIG. 5.



at any instant may be thus determined. The rod PQ during its motion may be considered as always turning round some centre or other in space, although the relative position of that centre to it is continually shifting. Produce the arms AP, BQ in the requisite directions to meet in K, then will this point K be the momentary centre. For as the extremity P moves round the centre A, the direction of its motion at starting from P must be perpendicular to AP, therefore the momentary centre will lie somewhere in AP produced. In like manner the initial motion of the other extremity Q must be perpendicular to BQ, and the momentary centre must also lie somewhere in the direction of BQ: therefore it must be in the intersection K of the two lines AP and BQ produced. But since the rod PQ turns on the momentary centre K, the direct motion of P and Q are to each other at any given instant as their radial distances from K, that is, as PK to QK, which is true, whether we consider them as the extremities of the rod PQ or of the radii AP, BQ; also the angular motions of the latter will be found by dividing these direct motions by their respective radii; therefore we have, Angular motion of P round A : angular motion of Q round B :: $\frac{PK}{AP} : \frac{QK}{BQ}$.

Draw KL, AM, BN, perpendicular to PQ. Then we have

PK : AP :: KL : AM by similar triangles KPL; APM
BQ : QK :: BN : KL . . . . BQN; KLQ
AT : BT :: AM : BN . . . . ATM; TBN

and compounding these three proportions we obtain

$$\frac{PK}{AP} : \frac{QK}{BQ} :: BT : AT$$

that is to say, the angular motion of the arms are to each other at any moment inversely as the segments into which the direction of the link divides the line joining the centres of motion, or *line of centres*, as it is usually termed. If now it happens that when the link PQ moves into its new position pq, very near to the first, this second position intersects the first in a point L above (or below) the line of centres, as in the figure; then the ratio of the segments AT, BT will be altered into that of At, Bt, consequently the ratio of the angular motion will be an increasing or decreasing ratio, as the case may be. But if the point L coincide with the line of centres, this ratio will for the moment remain constant.

Now a little consideration will show that the point of intersection between two successive positions PQ, pq of the link must be at the place where the perpendicular from K falls upon it. For as K is the momentary centre of motion of this link, the extremity L of the perpendicular will begin to move in a line at right angles with it, and consequently will remain in the direction of the first position PQ when the link has passed into the second pq, that is to say, it will be the point of intersection of the two positions; when, therefore, the rods are in such a position that the perpendicular from K meets the link PQ in the line of centres, the ratio of the angular motions of AP and BQ is constant: and if in this state of the system the points P and Q be employed (as in Fig. 4) as centres from whence short arcs are drawn through any common point M, and applied as teeth, these arcs will manifestly drive each other correctly when in the exact relative position described, and very nearly so when removed to a short distance on each side of it, which is the thing required*.

Now these relative positions of P and Q may be determined by a simple construction founded upon the necessary coincidence of L with T.

Let A and B, Fig. 6, be the given centres of motion, AB the line of centres divided in T, so that the segments AT, BT shall have the ratio of the required motions; or in other words, let T be the point of contact of the pitch lines. Draw PTQ, making any angle with AB, and through T draw TK perpendicular to it. Upon PTQ assume a point P as a centre, from whence the circular arc or tooth belonging to A is to be drawn. Then, to find the corresponding centre for B, join AP, and produce it to meet TK in K, join KB and produce it to meet PTQ in Q. Then will Q be the point required, which will appear by comparing this diagram with Fig. 5.

If the point P had been taken at P′, so that the angle AP′T were less than a right angle, then the line P′A would have intersected TK in a point K′ on the other side of T, and this would have thrown Q to Q′ nearer to T.

Again, P might have been assumed on the other side of AB as at P″, but then the driving arc struck through *m* would have been concave. It is not worth while to examine all the cases that arise from the different relative positions of the points; I shall merely show those that are applicable to practice.

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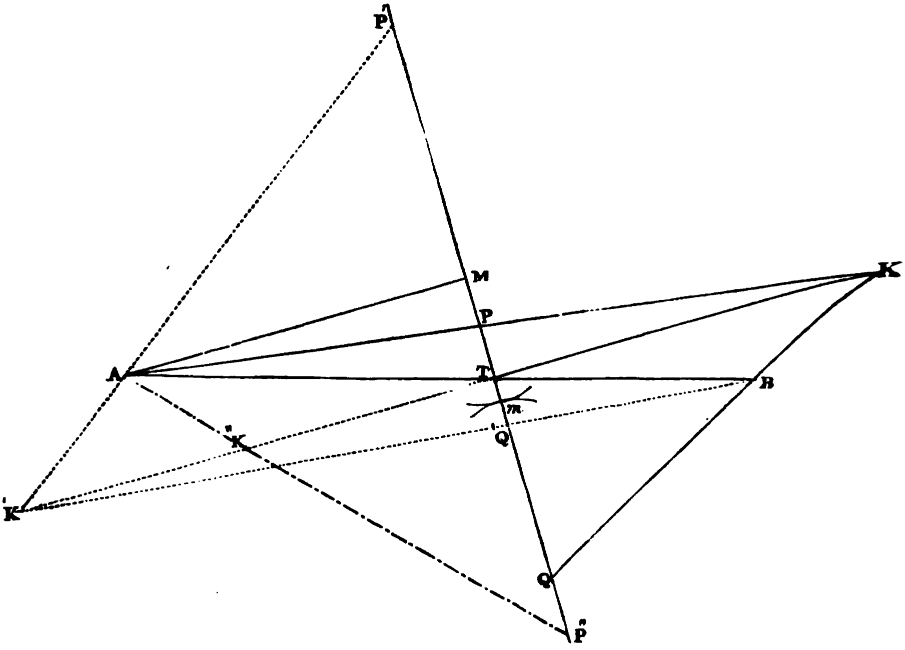
* It is hardly necessary to remark, that a more direct and simple demonstration of this construction might have been given by employing infinitesimals, which I was desirous of avoiding in a practical paper.

FIG. 6.



The side of the tooth may be formed either of a single arc or of two. As the arc is only an approximation, and is, strictly speaking, only exact at one point of the action, it will be better to adopt a figure composed of two arcs of circles, as we obtain two exact points; but in that case one arc should be concave and the other convex, in order to facilitate their junction and produce a wider base; and thus a figure is formed, as we have seen, very near to that usually adopted, the convex arc being of course given to that part which lies outside the pitch circle, and the concave to that which extends within it.

The angle ATP is arbitrary, and its value may therefore be determined from other conditions than those already stated. If, however, it be made a right angle, it is clear that the points P and Q vanish by coinciding with T; and if it be made a little less than a right angle, the points P and Q are thrown so near to T that the radii by which the arcs are struck become too short, and the points of the teeth too much rounded off.

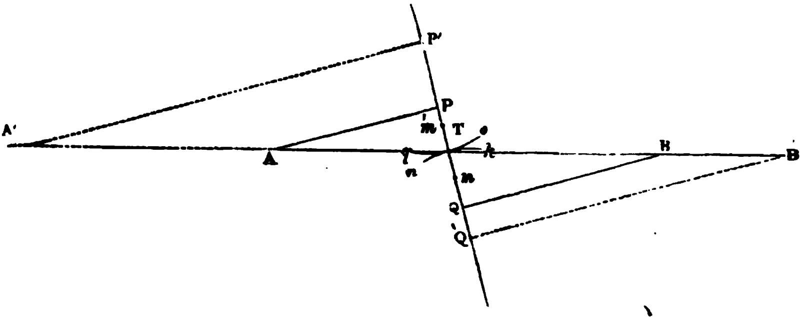
On the other hand, if the angle ATP is made too acute, the action of the teeth upon each other at the moment of passing the line of centres and elsewhere becomes very oblique, and an injurious pressure is thereby thrown upon their axes. By various trials I have fixed upon 75° as the value of the angle which appears to avoid these two extremes, and have accordingly employed it in the construction of the Odontagraph.

Again, the position of the point *m*, through which the arcs are to be struck, is also arbitrary, and must be determined by considering which point of the action we wish to make the correct point. If the teeth consist of a single arc each, the correct point may be fixed at the moment of passing the line of centres, and therefore the arcs must be struck through the point T; but if the side of the tooth be formed of two arcs joined, one lying within, the other beyond the pitch line, then the action of one of them will be confined to the approach of the point of contact of the teeth to the line of centres, and the action of the other to its recess from that line, and *m* must be assumed upon such a principle that the correct point of each arc shall fall nearly in the middle of its action, the mode of doing which will appear presently.

TO DESCRIBE TEETH CONSISTING OF A SINGLE ARC.

If the side of the tooth consist of a single arc, the system may be made exceedingly simple, for as the distance of the point K from T is arbitrary, when the points P or Q are not given, suppose it to be taken at an infinite distance, then (Fig. 6) APK and QBK will become parallel to TK, and perpendicular to PTQ, which shews that if lines be drawn from A and B (Fig. 7) perpendicular to PTQ, the points P and Q will be centres, whence if arcs be drawn through some common point *m*, or rather in this case T, these arcs will drive each other correctly.

FIG. 7.



If the angle PTA remain constant for a set of wheels of this kind, any two of them will work truly together, provided the arcs be struck through the point T; for let the wheel, whose radius is AT, be removed, and another whose radius is A'T be substituted, A'P' drawn perpendicular to TPP' will give the point P' as the centre belonging to the arc *oq*, and it is clear that this new arc *oq* will work as well with *kn* as the former one, and also that if the radius A'T had been substituted for BT instead of for AT, by placing it and its corresponding line A'P' in the situation indicated by the dotted lines B'Q', that still the conditions of the construction would have been satisfied, and these two wheels worked truly together, and the same may be shewn of any other pair of radii. But if the arcs were struck through a point *m*, not coinciding with T, then the wheels would fall into two groups, in one of which, as AT, A'T, the arcs are struck through a point *m* on the opposite side of the line of centres to the centre points P, P', and in the other, as BT, B'T, they are struck through a point *m* on the same side of the line of centres as the centre points Q Q′. Any wheel out of one of these groups will work correctly with any wheel taken from the other.

But suppose that a pair of wheels out of one of these groups be put together, for example, out of that in which the point *m* and the centre point of the arc, lie on opposite sides of the line of centres, and let A T and B T be the radii of the wheels in question. Now the relative positions of the points P and Q will still be true, but the arcs will no longer be struck through a common point, one of them being through *m*, the other through *m′* at the same distance on the opposite side of T, and therefore they will not work truly together. The arcs of the entire set must therefore be struck through T, and then any two wheels of the set will work.

The distance T P is equal to A T × cos A T P, and if A T P be fixed at 75° 30′, which is a convenient value, then $\mathrm{TP} = \frac{\mathrm{AP}}{4}$, whence the value is very easily found for any given radius, for in this case the value depends upon the radius alone and not on the pitch or number of teeth, as in the next example. The practical mode of setting out the teeth has been already explained.

On this system, however, the tooth has but one true point, that is to say, it is only strictly exact at the moment of passing the line of centres, and I therefore greatly prefer the construction about to be described, in which the side of the tooth is made up of two arcs united, and consequently has two points of accuracy. The tooth just described has considerable analogy to the involute, and like it has the fault of acting with too great a degree of obliquity. The teeth next to be described are of nearly the same form as that which has been so long in use, and have, as well as those of Fig. 8, the property of allowing any pair of wheels in a set to work together.

TO DESCRIBE TEETH CONSISTING OF TWO ARCS OF CIRCLES.

Figures 9 and 10, Plate 20, represent a pair of so constituted teeth in contact, Fig. 9 shewing their action before they reach the line of centres, and Fig. 10 after they have passed that line; each tooth is formed of two arcs of circles, *ab*, *bc*, *de*, *ef*, of which the concave ones, *ab*, *ef*, are situated within the pitch circles, and the convex ones, *bc*, *de*, extend beyond these circles; therefore, from well known principles, the concave arc *ab* will drive the convex arc *de*, until the point of contact reaches the line of centres, and then the convex arc *bc* will begin to drive the concave arc *ef*.

There are two points in the action of these teeth at which perfect accuracy is attained; one of them is when the teeth are in the position of Fig. 9, during the mutual action of *ab* and *de*, and the other when they are in the position of Fig. 10, during the action of *bc* and *ef*; and the arcs are so set out that these points of the action shall take place, the one nearly in the middle of the arc of motion before the line of centres is reached, and the other somewhere about the middle of the arc of motion that is traversed from the line of centres until the teeth quit contact.

The construction of these teeth in a set is as follows. AB, Figures 9 and 10, is the general direction of the line of centres; QPT, as before, is a line making a constant angle of 75° with the line of centres; KTK perpendicular to QPT and having its two points K set off at equal distances on each side of T, these points and the lines being invariable for the entire set.

The centres for the convex arcs are found by joining the centre of each wheel (A, Fig. 10; B, Fig. 9) with that point K which lies on the *opposite* side of the line QPT. Thus in Fig. 9, Q is the centre of the convex arc *de*, found by joining BK, and in Fig. 10, P is the centre of the convex arc *bc*, found by joining AK.

The centres for the concave arcs are found by joining the centre of each wheel with the K which lies *between it* and the line QPT; thus in Fig. 9, P is the centre of the concave arc *ab*, found by joining AK, and producing it to meet PQT, and in Fig. 10, Q is the centre of the concave arc *ef*, found by joining BK, and producing it to meet TPQ. Moreover, the whole of these concave and convex arcs are struck through a point lying beyond T at a constant distance, T*n*, or T*m*, which for simplicity's sake I have assumed equal to half the pitch; finding that this will place the correct points of the action at a sufficient distance o each side of the line of centres.

The consequences of this arrangement will be, that any pair of teeth so described will, when put together, answer the conditions of the construction already demonstrated. (Fig. 6.)

1st. (Fig. 9.) Before reaching the line of centres we have a concave arc *ab* driving a convex one *de*, of which the first has been struck from a centre P, derived from its nearest K, and the second from a centre Q, derived from its farthest K, consequently both derived from the same K; also the arcs have both been struck through a point *m*, at the same distance beyond T, and therefore will work truly together.

2d. (Fig. 10.) After passing the line of centres, a convex arc *bc* drives a concave arc *ef*, which in like manner are seen to have been derived from the same K, and to have been struck through a common point *n*, so that although the position of all these points is reversed, the arcs will, in this case, work truly together.

The same will manifestly be true for every pair of wheels in the set, for the distances TK and T*m*, or T*n*, are constant for the whole.

CONSTRUCTION OF THE ODONTAGRAPH.

To enable a workman to find these points P and Q at once in every case, I have contrived the instrument already described, (vide page 150,) which I have termed an Odontagraph, and have represented in Fig. 11, Plate 20, with the arrangements for describing the tooth *fed* of Figures 9 and 10. These three drawings being all made to the same scale will explain each other by comparison.

The instrument, as already mentioned, consists of a kind of bevil formed of a sheet of card paper, four times the lineal size of the drawing EF*t*D, the angle D*tk* is 75°, and the side *kt*F is occupied by a scale of equal parts numbered from *t* both ways. An example will show how this instrument is connected with the previous demonstration.

Let the example be a wheel of 26 teeth 3 inch pitch. Describe an arc *te*T of the required pitch circle, and set off upon it *t*T equal to the pitch and bisected in *e*, draw radial lines B*t*, BT. To describe the arc *ef* within the pitch circle, apply the slant edge D*t* of the scale to the upper radial line B*t*, placing its extremity *t* on the pitch circle, as in the figure. In the table headed "Centres for teeth within the pitch circle," look down the column of 3 inch pitch, and opposite to 26 teeth will be found the number 55. The point indicated on the drawing board by the position of this number at *q* on the scale of equal parts *t*F, which is marked *Scale of centres for teeth within pitch circle*, in the actual instrument, is the centre required, from which the arc *fe* must be drawn with a radius *qe*.

Now a comparison of this figure with Fig. 10, will show that thus far the relative positions, inclinations and distances have been indicated by the instrument for the point *q* (the Q of Fig. 10.) The line B*t*, Fig. 11, is the same as BT in Fig. 10.

The centre for the arc *ed*, which lies outside the pitch circle, is found in a manner precisely similar, by applying the slant edge of the scale to the lower radial line BT, placing the instrument in the position indicated by the dotted lines. The table of centres for teeth outside the pitch circle does not contain 26 in its column of Number of Teeth, therefore the nearest number must be taken, which in this case is 30, and the number 21 opposite to it in the column of 3 inch pitch, will indicate the position of the centre Q upon the scale T*k* of *centres for teeth outside the pitch circle*, this scale being so titled in the actual instrument. Here, again, a comparison of Fig. 11 with Fig. 9, will show that this new operation has given the true relative position of the point Q to the radial line BT and arc *de*.

I will now explain in a few words the mode of calculating the numbers in the table, by way of enabling other persons to alter any of the conditions. A formula for these numbers may be obtained as follows. (Vide Fig. 6, page 162.) From A draw AM perpendicular to TPP′, then from the similar triangles AMP, PTK we obtain $\mathrm{KT} = \frac{\mathrm{PT} \times \mathrm{AM}}{\mathrm{PM}} = \frac{\mathrm{PT} \times \mathrm{AM}}{\mathrm{TM} - \mathrm{PT}}$. Let $\mathrm{KT} = \mathrm{C}$, $\mathrm{AT} = \mathrm{R}$, $\mathrm{PT} = \mathrm{D}$, $\mathrm{ATP} = \theta$;

$$\therefore \quad \mathrm{C} = \frac{\mathrm{D.R.} \sin \theta}{\mathrm{R.} \cos \theta - \mathrm{D}} \; . \; . \; . \; . \; . \; (1)$$

Now the point P being in this case obtained from the K on the opposite side of T to A, this formula belongs to that part of the tooth which lies beyond the pitch circle, according to the principles already laid down. If TK″ be taken equal to TK on the line KT produced, and a point P″ obtained by joining AK″, and producing the line to meet P″PT, then P″ will belong to the part of the tooth within the pitch circle, and the similar triangles AMP″, P″TK″, will give us for this case the formula

$$\mathrm{C} = \frac{\mathrm{D}'\mathrm{R} \sin \theta}{\mathrm{R} \cos \theta + \mathrm{D}'} \; . \; . \; . \; . \; . \; (2); \text{ where } \mathrm{D}' = \mathrm{TP}''.$$

Now the value of C, which represents the equal lines

KT, or K″T, may be determined for the whole set, by considerations similar to those already employed in settling the diameter of the constant describing circle in the first section of this paper. If the radius AT of a wheel be assumed of such a length that AK″ fall perpendicularly upon K″T, then will the line AK″P″ become parallel to PTP″, and consequently the point P″ will go off to infinity, and the arc which should be struck from it to form the flank of the tooth will become a right line perpendicular to PTP″.

If the radius AT be taken still smaller with respect to K″T, it will be seen (by taking K′T larger than AT) that in such a case the point P, will make its appearance on the opposite side of T*, but this makes the flank of the tooth convex, and drawing inwards so as to be less at the base than at the pitch line, which is an impracticable form. To avoid this, and at the same time to make K″T as large as possible consistently with this limitation, assume K″T equal to R′ sin θ, where R′ is the least radius of the set. This value corresponds to the case in which AK″ is perpendicular to K″T, and necessarily excludes the impracticable forms; for since the least radius of the set now corresponds to that peculiar example in which AK″P″ is parallel to PTP″, every other value of AT being larger, will throw the points P″ on the opposite side of T to M, which is the thing required to produce the concave flank. These observations apply only to that value of KT which lies nearest the centre A, and therefore to the flank or portion of tooth within the pitch circle. As to the opposite value of TK, which corresponds to the portion of tooth beyond the pitch circle, and which it must be remembered is equal to TK″, it is clear from the figure that whatever value be given to it, its point P will always lie between T and M, and the arc of tooth be convex, supposing it to be struck, as it must be, through a point near to T.

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* Our formula then becomes $c = \frac{D' R \sin \theta}{D' - R \cos \theta}$.

The value selected for $K''T$ (namely $R' \sin \theta$) will therefore suit KT. Substitute now this value for C in the formulæ (1) and (2), and after arranging the terms we obtain the following values of D and D'.

$$D = \frac{R'R \cos \theta}{R + R'} \ldots\ldots (3) \qquad \text{and } D' = \frac{R'R \cos \theta}{R - R'} \ldots\ldots (4)$$

Now D and D' (that is TP and TP'') are the distances of the centre points of the arcs measured from T, and it will be seen by comparing the diagrams with the description of the Odontagraph, that the numbers in the columns of each pitch are the values of D and D', corresponding to the number of teeth in each wheel given in the first column, or, which is the same thing, to the values of the radii R and R'. To find these numbers for a given pitch, substitute in (3) and (4) the particular values of R' and θ, and by help of a table of logarithms, the values of D and D' belonging to as many values of R as may be thought necessary, may be computed, and thus the column of numbers obtained for that pitch. Those of the other pitches may be derived from the first by common proportion. In this way I formed the table, assuming 12 for the least number of teeth, and 75° for the value of θ, and employing a scale of half inches and tenths in which to express the values of D in the nearest whole numbers, because I found that a unit of the twentieth of an inch was sufficiently small to avoid practical error.

It is unnecessary to have numbers corresponding to every wheel, for the error produced by taking those which belong to the nearest as directed, is so small as to be unappreciable in practice. I have calculated the amount and nature of these errors by way of obtaining a principle for the number and arrangement of the wheels selected. It is unnecessary to go at length into these calculations, which result from very simple considerations, but I will briefly state the results.

The difference of form between the tooth of one wheel and of another is due to two causes, (1) the difference of curvature, which is provided for in the Odontagraph by placing the compasses at the different points of the scale of equal parts, (2) the variation of the angle *t*B T, (Fig. 11,) which is met by placing the instrument upon the two radii in succession.

The first cause is the only one with which these calculations are concerned. Now in three inch pitch the greatest difference of form produced by mere curvature in the portion of tooth which lies beyond the pitch circle, is only ·04 inch between the extreme cases of a pinion of twelve and a rack, and in the acting part of the arc within the pitch circle is ·1 inch, so that as all the other forms lie between these, it is clear that if we select only four or five examples for the outer side of the tooth and ten or twelve for the inner side, that we can never incur an error of more than the $\frac{1}{200}$th of an inch in three inch pitch by always taking the nearest number in the manner directed, and a proportionably smaller error in smaller pitches. But to ensure this, the selected numbers should be so taken, that their respective forms shall lie between the extremes at equal distances. Now it appears that the variation of form is much greater among the teeth of small numbers than among the larger ones, and that in fact the numbers in the two following series are so arranged that the curves corresponding to them possess this required property.

For the outer side of the tooth, 12, 14, 17, 21, 26, 34, 47, 73, 148, Rack.

For the inner side, 12, 13, 14, 15, 16, 17, 19, 22, 26, 33, 46, 87, Rack.

Now these numbers, although strictly correct, would be very inconvenient and uncouth in practice if employed for a table like that in question, where convenience manifestly requires that the numbers, if not consecutive, should always proceed either by twos or fives, or by whole tens, and so on. They are only given as guides in the selection, and by comparing them with the actual table, their use in the formation of the first column will be evident.

ESSAY II.

ON THE SHAFTS OF MILLS.

CHAPTER I.

181. To make these Essays useful to operative mechanics; to save engineers and managers of manufactories the trouble of much explanation in giving directions to foremen and others, who are to carry their ideas into effect; to give workmen some notion of the principles on which their work should be conducted; and to construct machinery upon true principles, which is ultimately the most economical plan of proceeding; we introduce the following extract from a respectable periodical publication*, as it appears applicable to the subjects of these papers, and may induce the reader's taste for entering upon a new and untrodden path.

"A country in which manufactures are extensively established, and conducted with spirit, as in Britain, becomes by degrees a country of machinery. For inventions to diminish the quantity of human labour employed, will be more ingenious in construction, more powerful in operation, and of more general use, in proportion to the necessity of furnishing a greater quantity of commodities at moderate and equable prices. The bodily exertions of workmen, in whatever branch of labour, have their limits, and excessive efforts, if unduly prolonged, irremediably destroy the health and vigour of those who pursue them. But machines may be continued in activity day and night, week after week, and month after month; having in themselves no life which suffers a sensible consumption, no principle of activity whose energy requires a pause to effect its recovery or renovation.

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* Eclectic Review, Dec. 1806, Art. XII.

"We have seen the manufactures of our own country solicit the aid of every hand that could be spared from its agriculture, and seek in distant lands for labourers of every age to supply the mill or to throw the shuttle. We have seen ingenuity exerted to its utmost, to contrive and to construct those machines which these labourers were to superintend and assist. We remember the time when these constructions were the dread and the hatred of the manufacturers, but we believe the most ignorant workman of the present day acknowledges their utility, and would with difficulty be induced to relinquish that very implement which his father or grandfather would have gladly committed to the flames.

"Considering then the importance of machines to shorten labour, and the number of persons who are interested in them, as proprietors, as inventors, or as constructors, it is wonderful that so little has hitherto been communicated on this subject by the medium of the press.

"The process towards perfection in complicated machinery is perhaps too generally the reverse of what might be expected. When practice has shewn the importance of a machine, science takes it up, investigates its principles, analyses its movements, and connects them by the assistance of mathematical precision.

"Mathematicians are seldom inventors, and workmen are rarely men of science, yet the mutual assistance of study and practice is necessary, to perfect the subject which each is intent on improving." It was Buchanan's aim, then, to come between these two classes, and to make them better acquainted, and more useful to each other. How far he has succeeded in the attempt, must be left to the determination of time.

In common with all writers on similar subjects, he experienced considerable difficulty in finding precise technical words to express the different parts of mill-work. Those which are used by millwrights in different districts being very different from each other. It is hoped, however, that the explanations which we have given of the terms, will make them sufficiently clear.

With regard to this particular Essay on the Shafts of Mills, the subject is treated in a manner similar to that followed in the Inquiry into the Strength and Durability of the Teeth of Wheels. For the reasons there given, Buchanan did not here enter into the demonstrations of the elementary propositions which serve to guide the inquiry. He was at pains, however, to collect and arrange facts respecting gudgeons and journals in actual use, upon which to ground calculations. This method he considered as being much more certain than rearing calculations on insulated experiments made on the cohesive strength of materials. These, however, are of great value, and in some cases he has endeavoured to apply them. The various Tables given in the course of the Essay, will be of great use to the millwright in finding without trouble the sizes of gudgeons and journals for any case that may occur in practice, and the principles on which they are formed are laid down in so plain a manner that he will easily understand and apply them. These Tables may be considered as certain great lines drawn to guide the millwright in his work, and even allowing they may not be absolutely true, he may find from experience how near they may be approached with safety.

182. To proportion the diameters of axles to the stress they have to bear, is in mill-work of great practical importance. On the one hand, if the shafts be made too weak, it is evident they must soon give way; and on the other hand, if made too strong, they occasion not only unnecessary expense in the construction of the machinery, but, what is in most cases still worse, a waste of power from unnecessary friction. It is therefore desirable, that the millwright should have some rules to guide him in this very important part of his business; a part which has hitherto in most cases been conducted entirely at random. This Essay gives such a practical view of the subject as shall enable the millwright to proceed with greater certainty.

183. Until of late years, most of the shafts used in mill-work were constructed of timber. The use of cast iron in this and other parts of mill-work, however, has now become almost universal. For this improvement we are perhaps indebted to those who are engaged in the cotton manufacture. After Arkwright's invention, it became a great object with them to save time in the erection of machinery, and to render it as durable as possible; for every stoppage was attended with great loss, by throwing idle the numbers of people necessary in cotton mills.

Besides the expense attending the repair, what had perhaps still more weight with them was, that the profits at that period on cotton spinning, were almost unparalleled in any other branch of manufacture.

Another circumstance which tended very much to the advancement of mill-work, arose from James Watt's improvement of the steam engine, which enabled cotton-spinners and other manufacturers who required power to work their machinery to carry on their business in towns. Hence power and people might, without trouble, be concentrated on the most eligible spot, and the great expense and disadvantages avoided which are attendant on colonizing the remote situations in which powerful falls of water are commonly found. The questions of health and morals belong to the legislator, not to the civil engineer.

The introduction of cast iron then may be considered as a kind of new era in the history of mills, without the use of which it would not have been possible, with the same number of operative mechanics, to have constructed one tenth part of the machinery which has of late years been erected in Great Britain.

CHAPTER II.

SECTION I.

GENERAL DESCRIPTION OF SHAFTS.

184. THE axles used in mill-work are commonly denominated, when of a large size, *shafts;* those which are smaller, are usually called *spindles.* Thus, for example, we say the shaft of a water-wheel; the spindle which carries the millstone of a corn-mill.

185. When shafts lie in a horizontal direction, they are called *lying* or *horizontal* shafts; when vertical, they are termed *upright* or *vertical* shafts.

186. Shafts are usually made of wood or of iron. Large wooden shafts are generally made either of solid oak, or are built of fir-logs. The scarcity of large oak occasioned the built shafts of fir to come into more general use. The latest improvement made on wooden shafts, was that of having what are called cross-tailed *gudgeons**. Before that improvement, it was attended with very great trouble and expense to keep the gudgeons from becoming loose in the shafts. Indeed, it was found impracticable to keep them fast for any considerable time.

187. Fig. 1, Plate II. represents a wooden shaft, with the gudgeons in use previously to the last improvement. They are called *laid-in gudgeons.* A B C is the gudgeon somewhat in the form of the letter T. One of the tails, C, was let into a mortice, and the rest of the gudgeon sunk into

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* The *gudgeon* is the arbour or spindle on which the shaft turns.

its place in the centre of the shaft. In order to accomplish this, it was necessary to cut out the part, from B to D, Fig. 1, No. 2. After the gudgeon was laid in its place, the vacant part was filled up by the piece of wood D E, Fig. 1, No. 1. The shaft was then hooped, and the end of it driven full of wedges, in order to fasten the gudgeon. This gudgeon is shown in perspective, Fig. 1, No. 3.

188. Fig. 2. represents a wooden shaft, with cross-tailed gudgeons. This kind of gudgeon is made of cast iron, and being thin in the cross-tails, let in from the end of the shaft, it leaves the wood much more entire than the *laid-in gudgeon*, while its cross-arms take a much firmer hold. After it is let in, the hoops are driven on the end of the shaft, when warm, and lay firm hold of the ends of the cross-tails. The wood is then wedged up, which makes the gudgeons perfectly fast. Fig. 3. is a perspective view of a cross-tailed gudgeon, and Fig. 4. its profile. It is cast with the round part undermost; for which reason the pattern must have a taper, to make it rise out of the sand. This taper has, in the cross-tails, another very important use, that of giving the gudgeon the advantage of *dove-tailed joints* with the timber of the shaft when it is wedged up. Instead of wrought iron hoops, cross-tailed gudgeons sometimes have a hoop of cast iron cast along with the tails, as represented by Fig. 5.

189. When it is considered that the direction of the stress which tends to loosen the gudgeon in a wooden shaft is continually changing, and that such action is exerted upon wood, a material that is so very easily permanently compressed, it will not be wonderful that it should have been found difficult to render them firm and lasting. The last method, viz. that where the hoop is cast along with the cross-tails, seems to be far preferable to the other; but perhaps it seldom happens that the hoop part is of sufficient length. It may be proved, that when the diameter of the

shaft is sufficient for the strain upon it, the length of the hoop should be equal to the square of the diameter of the shaft divided by the length of the shaft; otherwise there will be no certainty of the gudgeon remaining permanently fixed.

190. An improved method of fixing gudgeons is described in the Transactions of the Society of Arts, &c., Vol. xxxi. p. 223; it consists in casting the gudgeon with cross-arms, which fit into proper notches in an octagonal box of cast iron that has been previously fixed upon the end of the shaft. The arms of the gudgeon are retained in their places by screw-bolts.

In Plate [IV. A,] Fig. 1, 2, and 3, A A represents the end of the wooden shaft, which is supposed to be made of an octagonal form. B B is the cast iron box accurately fitted on the end of the shaft, and wedged tight. The end of the box has a projecting flanch *a a*, with four notches to receive the cross-arms *b b*, *d d* of the gudgeon C. These cross-arms are firmly fixed to the box by four screw-bolts, which pass through the flanch, and the ends of the cross-arms. The section, Fig. 3, shews the box B B on the end of the shaft, with the gudgeon C, and its cross-arms separated; to explain a further precaution which is necessary for strength. This precaution consists in the cross-arms having projections, *e e*, which enter the end of the box, and keep the gudgeon true to its centre, and prevent any lateral strain on the bolts. When the gudgeon of a wheel is fitted according to this method, it can be easily removed when it is so far worn that a new one is necessary; and the new one may be inserted without injury to the end of the shaft.

This improved method was invented by Robert Hughes.

It is obvious that the length of the box should be regulated by the rule stated in Art. 189*. The real advantage gained by this mode of fixing seems to be, that of retaining the end of the wooden shaft more perfect, with the means of renewing the gudgeon, without injury to the shaft.

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* The rule supposes the shaft to be proportioned to the stress upon it.

191. Cast iron shafts are sometimes made hollow cylinders, and sometimes they are made solid, and of various figures. It is demonstrable, that a hollow cylinder is much stronger, with the same quantity of matter, than it would be if made into a solid of the same length. This law is very observable in the beautiful economy of nature; for instance, the stalks of plants, the quills of birds, the bones of animals. But, in the works of art, numberless obstacles to perfection continually occur. In this particular case the expense of making small shafts hollow, would be very great; and another objection is, the difficulty of making such castings perfect. Shafts of a small diameter are, therefore, commonly made solid.

192. Fig. 6, Plate II. represents a cast iron cylindrical shaft. It consists of three parts, the body, A B C D, and the two gudgeons, A E C, and B P D G. The gudgeons are turned and carefully fitted into the ends of the body, which is bored and turned to receive them. They are then fixed with screw-bolts, which pass through the flanches, as may be seen by the figure [*].

This kind of shaft will obviously be variously constructed, according to circumstances. That represented in the figure was made for a cast iron water-wheel. The use of the small projections H, H, &c., is to prevent the eye of the arms from shifting round on the shaft. When cylindrical shafts are not used, what are called feathered shafts are often adopted.

193. Fig 7, Plate III. represents this construction of a shaft. It probably took its name from its resemblance to the feathered part of an arrow. It may be here remarked,

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[*] In this construction, the resistance to twisting depends entirely on the bolts, and some difficulties appear to be encountered in casting without corresponding advantages, either in strength or beauty.

that shafts of this species, as often constructed, are by no means calculated to withstand the twist brought upon them by the strain of the machinery. From the breadth of the feathers, their strength to withstand lateral pressure, is, no doubt, considerable; but wanting substance between the feathers, they are liable to continual tremor. Where feathers are applied to shafts it is preferable to keep the body of the shaft fully as strong as the gudgeon, or *journal**, and apply the feathers merely to prevent bending in the middle, as Fig. 7, No. 4. But the simple square, Fig. 8, is more easily made, and has been found in practice, at least as advantageous as any other form that has been tried for solid shafts†.

Having given this general account of shafts, we come next to consider the causes from which the stress on them arises.

SECTION II.

OF THE KINDS OF STRESS TO WHICH SHAFTS ARE SUBJECT.

194. There are two kinds of stress to which shafts are liable: first, *lateral stress*, by which they may be broken across: secondly, stress arising from *torsion*, by which they may be wrenched or twisted.

All horizontal shafts are liable to the first kind of stress, viz. lateral stress; and some have no other strain whatever; as, for instance, a water-wheel shaft, where the motion is communicated from teeth, on the shrouding. See Plate III. Fig. 9.

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* *Journals*, or *journeys*, are gudgeons subject to torsion.

† It is easily proved, that the best form for a revolving shaft is a cylinder, and that in any other form the flexure will be irregular, and consequently produce irregular wear on the gudgeons and brasses; but when a shaft is to be adapted for placing wheels on any part of its length, a square section is convenient; in all other cases, the section ought to be circular with projections, as at H, H, Fig. 6, Plate II. By making four of these projections continue throughout the length upon a cylindrical shaft, all the advantage and convenience of a square one would be obtained, with very little irregular flexure. The projections should not be greater than is necessary to fix the wheels firmly on the shaft.

In Fig. 10, the stress on the upright shaft arises from torsion only; excepting what may proceed from the inaccuracy of the teeth of the wheels, which, if great, will occasion a considerable lateral thrust.

A vertical shaft, which gives or receives motion by means of a pulley, has thereby a lateral pressure brought upon it.

In Fig. 11 and 12 the stress is compounded of *lateral pressure,* arising from the weight of the wheels A and B, and that of the shaft itself, and of the torsion or twist produced between the wheels A and B.

The following remarks of JOHN ROBERTON, engineer, relate to the subject of this Essay, and contain much useful matter which will be acceptable to the reader; to whom it must be demonstrable, "that by a judicious arrangement of wheels and pinions, in many cases much of the stress and friction may be avoided. This is a doctrine in practical mechanics of very great importance, when we consider, that in many cases almost the whole of the impelling power is expended in overcoming the friction of the machinery.

195. "Let A, Fig. 1, Plate IV., be an overshot water-wheel. Let the line *a b* represent the line of direction of the centre of gravity of the water in the buckets. On the extremity of this wheel let there be a toothed wheel acting into the pinion B. It is obvious that, independently of the weight of the wheel, the whole weight of the water will be supported by the axis *c*, and the teeth of the wheel at *d*; and the weight which each will sustain, will be in the ratio of *c e* to *e d*. That is, by the principles of the lever, the weight on the gudgeon will be as the distance *e d*, while that on the teeth will be as *c e*; or if the radius of the

toothed wheel were *c e*, the teeth would sustain the whole weight of the water, leaving no weight on the gudgeon but that of the wheel. Again, let the wheel B be removed to C, it is evident that the gudgeon *c*, would have to sustain the weight of the water, and a great deal more. That is, the weight on the gudgeon would be increased as *e d* to *e c*. The true relation of the stress in the two cases, is $ed : \frac{2(dc)^2}{dc+ed}$. So that it is evident, the stress and friction of the gudgeon must depend, in a great measure, on the size of the toothed wheel attached to the water-wheel, and to the situation of the pinion B.

196. " Again, let there be a wheel at A, Fig. 2, fixed on the end or middle of a shaft working into the wheel or pinion B, of any size.—Let the teeth move in the direction *a b*. It is evident, that the gudgeon or shaft will tend to move in the contrary direction, that is, in the direction *c d*, and with the very same force that the teeth act upon each other, as action and reaction are equal and in contrary directions, and for the same reason, the gudgeon of the wheel B, will tend to move in the direction *e b*, with the very same force, that is, the same force as the action of the teeth on each other. Indeed, in any single pair of wheels, of whatever form or construction, the tendency to break or bend the shaft, or cause friction, is the same as the action of the teeth on each other.

" Now, if the above wheels were made of a double size, it is evident that the acting power on them would be only one half, and consequently, one half of the strain to break the shaft or cause friction *.

197. " In the case of an intervening wheel, the force or

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* The object of this remark is, apparently, to show the superiority of large wheels; but it is clear that the acting power would be the same with the same first mover; and if the resistance be diminished on one shaft, another must be added to give the proposed velocity to the working point.

tendency to break the shaft depends on the situation of such intervening wheel. Thus, if it be placed in a direct line betwixt the centres of the conducted and conducting wheels, as at A, Fig. 3, the shaft or gudgeon will tend to move in the line *a b* or *b a*, (according to the direction of the conductor,) with double the force of the action of the teeth.

" If the axis of the intervening wheel form a right angle with the axis of the other two wheels, the force to break the shaft will be to that of a pair of single wheels, or the action of the teeth on them, as the diagonal of a square is to one of its sides. That is, the direction of the force, and its intensity, will be represented by the diagonal, it being evident, from the well-known laws of mechanics, that by the action of the wheel B, Fig. 4, No. 1, the centre of the intervening wheel A, would tend to move in the line *a c*, and by its action in the wheel *e*, it would tend to move in the line *a d*. Let *a d* and *a c* represent the forces in these directions, complete the square or parallelogram, the diagonal of which will both represent the force and its direction. See also Fig. 4, No. 2, and No. 3.

198. " On the other hand, if a wheel be placed betwixt two others, as A, Fig. 3, where A is the conductor, the teeth of which act with equal force on each of the wheels B and C, it is evident that the strain is wholly taken off the shaft, the forces being equal and opposite to each other*; and in Fig. 5, where A is supposed to be the conductor, and B and C the conducted wheels on which the teeth on each bear equally, —it is plain from what has been already stated, that the direction of the forces will be *d a* and *a c*, and letting *a d* and *a c* represent the direction and intensity of these forces, and

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* The shaft ought not, however, in any case, to be entirely freed from pressure in this manner; because its motion will not be so steady and regular as when there is some considerable pressure on the gudgeons. And since there is nothing more injurious in machinery than a hobbling, unsteady motion, this point requires the engineer's most careful attention.

completing the parallelogram, we have the diagonal *a b* to represent the compound direction and intensity of the force.

199. "There is another point that is worthy of attention: that is, the place on the shaft where the wheels are fixed. If a wheel is put on at the end of a shaft to drive any other or others, it is clear, that the whole or nearly the whole of the stress will be at the end of the shaft or journal. If the wheel be placed in the middle of the shaft, the strain to break it will be greatest at that part, but the force will be resisted equally by each journal. Indeed, on whatever part of the shaft a wheel is placed, at that very place is the greatest (cross) strain on the shaft, and the force on each journal will be in the inverse ratio of the distance of the wheel from the ends of the shaft.

"In Fig. 6, let A represent a shaft, either upright or lying. If a single wheel, as B, fixed upon it, drive two pinions, as C D, directly opposite to each other, the shaft or journal will not be affected thereby *, but if two wheels are placed on the shaft, driving each a pinion, as in Fig. 7, both journals will be affected, and the greatest strain to break the shaft will be at the arms of the wheel, or close to them, and betwixt the arms and journal. In this case, the middle point of the shaft, as at A, being in a state of contrary pressure, and therefore no strain on that part, it may be considered as a lever on each side of A; this being the fixed point, and the greatest strain on the shaft being at the wheels. The strain on the journal will be inversely as the distance of the wheels from the middle point A, it being understood that the wheels, pinions, and resistances are all the same.

200. "Again, if two wheels are placed on a shaft, and two pinions, both on the same side of the shaft, the journals must support the strain or force of the action of the teeth of both wheels, or indeed whatever number of wheels is on a shaft; and working into pinions on one side, the journal

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* See the note to Art. 198.

of that shaft must support a pressure equal to the whole of their action into the teeth of the wheels or pinions into which they are connected; and the strain on the shaft to break it, depends on the situation of the wheels, as they may be placed on the shaft: it being understood, as formerly, that directly at the place where the wheels or pinion is fixed on the shaft, that it must support a pressure equal to the pressure of the teeth of the wheel in its corresponding wheel or pinion. And according as these pressures combine, or act in a contrary direction to each other, so must the strain on the journals, or tendency to break or bend the shaft be.

"Hence the importance of placing wheels and pinions, so that their action on each other may be in contrary directions, or so as to avoid, as much as possible, the strain on the shaft or journals.

201. "In lying shafts, where circumstances may require, it may be advisable to have the main or heaviest shaft on the lift of the wheels, as this will take off a considerable deal of their weight or friction on the journals. The other shaft will naturally be on the fall of the wheel, (which is supposed in this case much lighter than on the main shaft,) and will be prevented from jolting upwards, from both its own weight, and a pressure equal to that on the teeth of the wheels, being supported by the journals. It being evident that whatever the wheels are, that the same pressure that is on the teeth of the wheels will be equally the same on both shafts, the one tending to increase the weight of the shaft on the journals, and the other to diminish it."

202. Roberton, in the course of his business, made several observations on the foregoing subjects. Late in his career, a case of this nature occurred, of considerable importance, at the flour-mill erected near the Slitt Mills of Patrick*. "The water-wheel, 16 feet diameter, makes about

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* On the banks of the Clyde.

10 or 11 revolutions per minute, driving in general two pair of stones; the pit wheel about 6½ feet diameter. The consequence was, that the machinery, framing, &c., were not of sufficient strength to bear the force applied by the pit wheels, (though they were very strong, and well executed for ordinary cases,) the shaft, framing, &c., were in a high state of tremor, the machinery working in a rough and straining manner, and the wooden teeth (5 inches broad, pitch about 4½) could not stand for any length of time. The mill was altered, by enlarging the pinion, to let the wheel run at 14 or 15 turns per minute; and the pit wheel enlarged about one foot diameter; afterwards the mill wrought very well.

" In common corn-mills, and many others, there is a great deal of the impelling power lost by the smallness of the pinions, &c., which causes a great friction on the shaft or spindle, as well as by the friction of the teeth. Were both wheel and pinion increased in diameter, much advantage would arise, not only in saving of power, but in the tear and wear of machinery. It is on this account that a corn-mill, constructed in the double way, other circumstances being the same, performs much more work than in the single way.

203. " In short, more things of this nature take place in machinery, than the most of operative mechanics, or even philosophers, are aware of; and there can be no doubt, that, with a knowledge of the affecting causes, a vast deal might be done in saving power. For example, in a cotton-mill, the main shaft generally makes from 40 to 50 revolutions per minute. Let the weight of the shafts and machinery, the size of the journals, and the effect of the wheels on these shafts, and friction of journals, be taken into the account, in the one case, and let the main shaft be supposed to be reduced to half of the former velocity, that is, from 20 to 25 turns per minute, and let the strength of the shaft,

together with a due proportion of wheels, be augmented, so as to preserve the same firmness in every part of the machinery as in the former, and that the ultimate part of the machinery may be brought to the same speed as formerly, it will be found on investigation, that in the last case there will be a considerable saving in the first, or impelling power.

"It must not be lost sight of, that, by reducing the velocity of the main shaft, a judicious increase of the diameters of the wheels thereon is absolutely necessary. Indeed, without strict attention to matters of that kind, the effect of any alteration that may be proposed or made is very precarious. A due regard to the proper diameter of wheels, according to the work they have to perform, is a matter of very great importance; and it will be found, on general inquiry, that in most cases of machinery, it would be prudent to have the wheels and pinions of large diameter; and, as we said before, by increasing their size, the force, strain, and friction on the shafts and journals are diminished in the same ratio.

"Suppose, in a mill, that a range of lying shafts, of 80 or 100 feet long, together with wheels, &c., fixed on them, weighed 7000 lbs; the journals 4 inches diameter, making 46 turns per minute; the surface of the journal would at that rate move at about 50 feet per minute: and supposing that the friction was equal to one third of the weight, we should have $7000 \div 3 = 2333 \times 50 = 116650 \div 44000 = 2{\cdot}66$; that is, nearly $2\frac{2}{3}$ horses' power expended in overcoming the friction of these shafts*.

"Again, were these shafts and journals extended to 5

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* That is, valuing the horses' power at 44000 lbs. 1 foot per minute. See page 88, "*Horses' Power*," Art. 108.

The quantity of friction is much overrated, but the loss of power would be very considerable on the lowest estimate; and, therefore, the proper situation for the first mover in a system of machinery is of some importance, inches diameter, and their number of turns reduced to one half of the former; that is, to 23 turns per minute, the surface of the journal would move at the rate of 31 feet per minute, and the weight of the wheel and shafts increased one half, or say, to 10000 lbs., we should have $10000 \div 3 = 3333 \times 31 = 103323 \div 44000 = 2{\cdot}348$, that is, nearly 2½ horses' power; so that in this last case there would be a saving of something more than three tenths of a horse's power, and the machinery would be, in every respect, improved.

204. " In a horse-gin, where a pinion is driven by a toothed wheel on the gin, the friction, or strain on the journals, depends on the situation of the horse-beam; at any moment of time when the lever or horse-beam is above or below the pinion, the friction on the shaft will be the least, and when in the opposite direction, the friction and force on the shaft will be the greatest, and the general friction will be as the size of the main wheel; that is, the smaller the main wheel is, the friction and force on the shaft will be the greater, and the larger this wheel is, the friction, &c., will be the less; it being understood that the friction or strain on the gudgeon, or axle, on account of the reaction of the teeth of the wheel is meant, and not the strain on the shaft, to twist it, nor any other friction or strain whatever.

205. " In a water-wheel turning machinery, in many cases, it is most advisable to have the toothed wheel of the same diameter. In this case, whatever power or force is applied to the wheel, the same must be resisted by the teeth of the wheel; and it also follows, of course, that whatever is the size of the wheel, or pinion which is driven by the main wheel, that the very same strain is on the shaft; that is, the same as on the teeth; and the shaft must be sufficiently strong to withstand the pressure. But another circumstance occurs, that by increasing the size of

the pinion, there must necessarily be increase of the diameter of the shaft to withstand the twist. The shaft by no means, however, keeps pace with the augmentation of the wheels, but is only as the cube-root; for instance, a water-wheel of 16 feet diameter would work very ill into a pinion of 12 inches, supposing its shaft or journal to be 3 inches diameter. Again, let the pinion be increased to 2 feet diameter, a shaft of 3¾ inches will be sufficient to withstand the twist, the friction on the journals will be much less, and the strength to resist the strain will also be much increased. General rules cannot be laid down with accuracy in these things. It is the particular circumstances of the case that will guide a machinist in the construction of any piece of machinery; and if he be not well acquainted with these, it cannot be expected that his schemes will be well arranged.

"The above observations are no doubt at variance with the opinions of those who are continually holding up the simplification of machinery. For instance, Fenwick's 'Essay on the Simplification of Machinery.'* In many cases it is prudent to make machinery of a more complex nature than it is sometimes constructed; and in order that it may be easier driven, that the tear and wear may be lessened, as well as the ultimate expense and trouble of attending it." †

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* The maxims of Fenwick consider friction only, and are so far correct; but of the wear and tear, and friction of teeth, he has taken no account. His 6th maxim, that "Small wheels are equally as generative as large wheels, if the same ratio of size be preserved," is true only within certain limits, (see Art. 33,) but these limits being assigned, his other maxims hold till the stress on the moving parts is less from a greater number of small wheels, than from fewer large ones.

† John Robbrton, to whom the world is largely indebted, not merely for the foregoing clever and scientific remarks, but for much of modern improvement, was, in his day, (the beginning of the present century,) one of the most distinguished millwrights or engineers in Glasgow.

206. In the essay to which Roberton in the foregoing observations alludes, Mr. Fenwick infers, (p. 64,) that "*the most perfect machine is that which operates with the fewest moving parts.*" But Fenwick seems to have been misled here by a desire to generalize. Simplicity is, no doubt, a most desirable quality in a machine, provided it can be obtained without making a sacrifice of power or of durability. A sledge has fewer moving parts, and in that sense is more simple than a steam carriage, yet no one with truth could say that a sledge is more perfect*.

Nor, perhaps, does the *simplicity* of a machine consist strictly in having *few moving parts*. If the parts of a machine be few, they are perhaps more easily taken in by the eye at one view, which may make them more easily comprehended by the mind, and in that sense be more simple. But in machinery, the kind of simplicity at which we ought to aim, has more regard to the manner *of action* than to the number of the moving parts. Thus, for example, when a weight is to be raised, if one wheel worked by a screw be employed, the machine *consists of fewer parts* than two wheels and two pinions, applied to the same purpose. But in this last case, the *manner of action* is really more simple; for the action and resistance are directly opposed in the same line; whereas, in the case of the screw, the action is oblique, and experience shows that it has much more friction, and much less durability.

What is here said of *manner of action* is applicable in comparing machines, consisting each of trains of wheels and pinions; for the most durable, by longest maintaining the true figure of the teeth, will ultimately be the most simple in the manner of its action. It is evident, that when the teeth become much worn, that the *manner of*

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* The Essay by Fenwick here alluded to, is wholly confined to the *application* of wheel-work; consequently the remarks in this and the following paragraph are not applicable to his Essay.

action becomes proportionably more oblique and less simple. Respecting the durability of the wheel-work of mills, we may refer the reader to Art. 100.

It can be of very little use to give rules for the diameters of gudgeons, or for the strength of shafts, unless they be accompanied with some method of estimating the straining force. Our author has not touched upon this part of his subject; and therefore we have inserted Roberton's remarks, with the view that the information which they convey, respecting the stress on shafts and gudgeons, may be studied in their proper order; we shall add some additional inquiries to these articles.

207. Let A B, Plate [IV. A.] Fig. 4, represent a shaft, with one wheel at D, and another at C; these wheels being of any size whatever. If a power act upon the wheel D at the point P, and the resistance be at W, the stress arising from these forces will cause a pressure on both the gudgeons; and the line W P being drawn, cutting the axis at some point E; the stress upon the shaft and gudgeons will be the same as if a force equal to the power and resistance together, were applied at the point E. Hence it is clear, that when the wheels differ considerably in size, the gudgeon next the smaller wheel will have to sustain the greater part of the stress.

208. But if the resistance were at *w*, the power and resistance in this case being at the same side of the shaft, the pressure will be downward on one gudgeon and upward upon the other. For the descending power P is resisted by the support of the gudgeon at A, and by the resistance at *w*; but the power not falling between these points, it will tend to raise the gudgeon B.

209. Again, if the resistance be at the point C on the upper or under side of the wheel C, the pressure at the gudgeon B will be wholly in a lateral direction; conse-

quently, when the impelling power moves with the wheel, the stress on the gudgeons will vary considerably both in intensity and direction.

If the plan of the shaft and wheels be drawn to a scale, it will be easy to compute the pressures in these different cases, and to compare them; and perhaps a young machinist will feel some pleasure in such comparisons, where he would have been fearful of engaging with a mass of algebra.

Case 1. The power and resistance being at opposite sides of the shaft. Draw the line AB in the middle of the shaft, and also draw the line WP. Then to find the stress upon the gudgeon B, we shall have AB : AE :: power added to the resistance : stress on the gudgeon B; or $\frac{AE \times (P + W)}{AB} =$ the stress upon the gudgeon B. Also AB : BE :: power added to the resistance : stress on the gudgeon A = $\frac{BE \times (P + W)}{AB}$.

Case 2. The power and resistance being at the same side of the shaft. Draw the lines A*w* and PB which cut one another at F. Then, the line PB may be considered a lever with its fulcrum at F; and to find the stress necessary to keep the gudgeon B down, we have BF : PF :: power at P : stress at B = $\frac{PF \times P}{BF}$ = the stress on the gudgeon B. This stress will obviously be opposed to the weight of the shaft and wheels. Also, AF : *w*F :: the resistance at *w* : stress on the gudgeon at A, = $\frac{W \times wF}{AF}$. This stress will be to add to the stress from the weight of the shaft and wheels.

Here it may be remarked, that it is desirable that the greater pressure on any gudgeon should always, when convenient, be in the direction of gravity, to prevent the unpleasant jolts which take place when the machine is put into motion, when the pressure is upwards upon any of the gudgeons.

210. But the preceding cases suppose that the power at P and the weight or resistance at W, are parallel, they are, however, often oblique in respect to one another. Let E, Figs. 5, 6, and 7, on a plan of such wheels, be the axis; let P be that point in the circumference of one of them where the power acts in the direction DP; and let W be the point where the resistance acts in the direction DW. Then DE will be the direction of the stress upon the axis, and if D*c* be made proportional to the resistance; or D*b* proportional to the power; and if the parallelogram D*cab* be completed, *a*D will be proportional to the whole stress upon the axis, which is obviously greatest in Fig. 5, and least in Fig. 6. Now, make *cd* perpendicular to DE, then D*d* is the pressure on the axis caused by the resistance at the circumference of the lesser wheel; and *ad* will be the pressure on the axis from the power at the circumference of the large wheel. But it must be remarked, that when the direction of the stress on the axis falls between the direction of the power and that of the resistance, as in Figs. 5 and 7; the whole stress will be either in the direction DE, as in Fig. 5, or ED as in Fig. 7, and proportional to D*a*. Whereas, if the directions of the power and resistance be both on the same side of the axis, as in Fig. 6, D*d* will be the pressure on the axis at the place of the small wheel W, in the direction ED, tending to raise the axis; and *da* the pressure on the axis at the large wheel P, in the direction DE; or opposite to the pressure at the wheel W; while *a*D is the difference between these pressures, showing the whole tendency of the axis to rise.

The figures are all drawn to the same size, and the power and resistance being represented by equal lines in each figure, the advantage or disadvantage of any particular construction will be seen by inspection; and the reader may easily multiply examples, by drawing more figures.

The relation and intensity of the pressures might have been shown in a more elegant manner by the arithmetic of sines, but perhaps not so satisfactorily to the minds of most of my readers. And when the rule and compasses are in the hand, the relations are more easily, and with less risk of mistake, ascertained by drawing the wheels and axis as shown in the figures; as it merely requires a little knowledge of the composition and resolution of forces.

Thus, make D*c*, from a scale of equal parts, equal to the number of cwts. in the resistance at W, and make *ac* parallel to the power PD, and *cd* perpendicular to DE. Then D*d*, measured from the same scale, is the stress at the centre of the wheel W in cwts. and *ad* the stress in cwts. at the centre of the wheel P.

The stress from the resistance and power being determined, that which is caused by the weight of the shafts and wheels themselves will be easily calculated. See Art. 245 and the following articles.

The case of lateral stress being the most simple, we shall, in the first place, examine it, confining, for the present, our attention to the gudgeons only. The bodies of shafts will be afterwards considered.

CHAPTER III.

SECTION I.

OF THE STRENGTH OF GUDGEONS WHERE THE STRESS IS PRODUCED BY LATERAL PRESSURE ONLY.

211. THE gudgeons having all the weight on the shaft to support, ought to be made sufficiently strong for that purpose; while, to avoid unnecessary friction, they should be made as small in diameter as possible, consistently with sufficient strength and durability.

When we are able to determine the diameters of the gudgeons, or journals, this serves as a foundation for the proportions of the other parts of the shafts.

Although wrought iron will bear a greater weight than cast iron, yet cast iron being not only cheaper, but much more easily formed into convenient shapes, gudgeons are now most commonly made of that material. We shall, therefore, in the first place, confine our attention to the diameters of gudgeons made of cast iron. Here it may be proper to state the following proposition:

212. Prop. I.—*Solid cylinders of the same length have their lateral strength as the cube of their diameters* [*], *for in general, the lateral strength of any pieces of iron or timber, whose sections are similar figures, are as the cubes of the similar sides of the sections.*

That is, if a gudgeon of two inches be sufficient to support a certain weight, a gudgeon of four inches will support eight times as much.

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[*] See Emerson's 4to edition, prop. 67, cor. 2. Gregory's Mechanics, vol. i. art. 170. cor. 3.

From this law, it is evident, that were all gudgeons made of iron of the very same quality, knowing the strength sufficient in any one case, it would be easy to calculate what it should be in any other case.

But as there is a great variety, in point of strength, in different kinds of iron; Welch cast iron, for instance, being stronger, as some think, by one fourth, than that made in Scotland; it is prudent to calculate upon the weakest. Mr. Banks observes *, that, "Iron is much more uniform in its strength, than wood; yet it appears that there is some difference in different kinds of ore, or iron-stone; there is also a difference from the same furnace, perhaps owing to the degree of heat which it has when poured into the mould."

213. The strength of a gudgeon is limited by the strain it will bear without permanent derangement of its structure, for the length is always so small in regard to the diameter that the flexure will be, in all practical cases, insensible.

The calculated stress should include every kind of force acting on the axis or shaft; and the diameter should be determined, so that the gudgeon would be capable of resisting the whole stress if it were thrown upon the extreme point of its bearing, (see Art. 217.)

When w is the utmost amount of the stress in cwts., and l the length of the gudgeon in inches, from the shoulder to the extreme point of bearing, it is shewn (Essay on Cast Iron, Art. 138.) that $\frac{(112\mathrm{w} \times \frac{1}{12} l)^{\frac{1}{3}}}{5} = d$, the diameter of the gudgeon in inches. Or, $0{\cdot}42\,(\mathrm{w}\,l)^{\frac{1}{3}} = d$.

214. But an allowance should be made for wear, which will be nearly directly as the stress, and inversely as the

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* Banks's Power of Machines, p. 94.

length of the gudgeon's bearing; consequently the length of the gudgeon should be greater in the same ratio as the stress is greater. And till some more certain principles of proportioning the gudgeons of a machine so as to be of equal duration shall be found, we may allow one fifth of the diameter. as a provision against wear where no gritty substance is likely to affect it, and one third in all cases where the gudgeons are exposed to gritty matters.

Rule.—In the former case, the rule will become $0{\cdot}5\,(w\,l)^{\frac{1}{3}} = d$. That is, multiply the stress in cwts. by the length of the gudgeon in inches, and the cube root of the product being multiplied by 0·5, will give the diameter of the gudgeon in inches.

When a gudgeon is likely to wear much from the nature of the machine or its particular situation, multiply the cube root of the product by 0·6, instead of 0·5. Gudgeons of water-wheels may be included in the class which are exposed to considerable wear.

If the stress on one gudgeon be equal to the weight of the wheel, and the wheel be at the middle point, that part of the stress which is produced by the action of the moving power and resistance, must be considered equal to half the weight of the wheel; for only half the weight will bear on one of the gudgeons in this case. But it often happens that the wheel is considerably nearer to one bearing than the other, and in such cases, the rule of the author would be likely to mislead.

Since our author has given it as a general principle, that the diameter of a gudgeon should be equal to the cube root of the weight supported in cwts., it will be desirable to compare our rule with that principle; first assuming that the weight is actually equal to the stress upon the gudgeon. Now, in that case, the rules will be the same when $0{\cdot}5\,l^{\frac{1}{3}} = 1$, or $l = 2^{\frac{1}{3}} = 1{\cdot}26$. And in the second rule, when $l = \left(\frac{1}{\cdot 6}\right)^{\frac{1}{3}} = 1\cdot185$ inches. Therefore whenever the length of the gudgeon exceeds about 1 inch and ¼, the rule gives the diameter too small. Again, if we take the actual stress to be only half the weight of the wheel in cwts., which is clearly in all cases less than the real stress, the above numbers should be multiplied by the cube root of 2, which will show that the author's rule becomes in defect again when the length exceeds 1·587 inches.

We shall proceed in this inquiry, on principles similar to those which have been followed in the *Inquiry respecting the Strength and Durability of the Teeth of Wheels*, namely, by taking a number of cases from mill-work in actual use, and drawing inferences from them. This method of making inferences, by collecting and arranging facts respecting mill-work, being much safer and more useful, than founding calculations upon experiments, often made on a small scale, and under circumstances very different from those which occur in practice with machinery. We begin with considering the gudgeons of water-wheels.

SECTION II.

OF GUDGEONS OF WATER-WHEELS.

215. In the following table, water-wheels of various weights are collected, and the diameters of the gudgeons in actual use, stated. The weights of the cast iron wheels were found, from the weight of the castings, &c., of which they are composed. The wooden wheels are estimated, making allowance for the wood becoming heavier by being soaked in water. We are aware, however, that besides the mere weight of the wheel, other circumstances should be

taken into account*, such as the weight of water in the buckets, and the pressure brought on the gudgeon by the resistance of the work, &c. But we shall follow the general result of those cases only, in which the gudgeons have been found sufficiently strong, and we apprehend that those extraneous causes on the one hand, will not affect our rules more than the different qualities of cast iron will the strength of gudgeons on the other hand†.

The water-wheels in the table were all in the middle of their respective shafts. Both gudgeons of each wheel had, therefore, equal stress.

216. *Description of the first Table of Gudgeons of Water-wheels.*

Column 1. contains letters to distinguish the wheels.
2. shews the material of which the wheel is made.
3. the diameter in feet.
4. the width in feet.
5. the kind of wheel.
6. the diameter of the gudgeon in inches.

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* See Encyclopædia Britannica, article Rotation.

† These remarks of Robertson Buchanan render it necessary to say a few words on rules founded on empirical principles, and particularly when the circumstances are not minutely detailed for those cases on which the rules are founded.

The rule for the strength of gudgeons in the text, supposes the stress to be always proportional to the weight of the wheel, but this supposition scarcely ever corresponds with the truth; consequently, in the cited practical cases, if the stress was not the greatest possible, in regard to the weight of the wheel, and the metal of an inferior quality, this rule may lead us into serious errors. But the practical cases are not described, and therefore we cannot judge, from any thing in the text, of the safety of the rule.

When a rule is to be formed by any process, every cause of stress should be considered, and where simplicity is desirable, the stress should be represented by a quantity which is certain to equal it, even in an extreme case. We cannot err greatly, if the error be always on the side of strength.

Column 7. the weight of some of the wheels, in tons and cwts.
8. the weight of some of the wheels, in cwts. and qrs.
9. contains the cube root of the weight.

The use of this column is to compare the several gudgeons with the law contained in Prop. I. For, were all the gudgeons duly proportioned to the weight they have to sustain, they would be to one another as the cube roots of their weights.

TABLE I.—GUDGEONS OF WATER-WHEELS.

1	2	3	4	5	6	7		8		9	
	Wheel made of	Diameter of wheel in feet.	Width in feet.	Kind.	Diameter of cast iron gudgeon in inches.	Weight of wheels in tons and cwts.		Weight of wheels in cwts. and qrs.		Cube roots of weight in cwts.	Remarks.
A	Wood	24	12	Overshot	7	23	14	474	..	7·796974	
B	Cast iron	16	6	ditto	7	12	..	240	3	6·214464	
C*	Cast iron	16	8	ditto	6½	16	10	330	..	6·910423	This Gudgeon broke from being a bad casting.
D	Cast iron wheel and buckets	32	4½	ditto	..	24	..	480	..	7·829735	
E	Wood	16	9	ditto	6	10	11	211	..	5·953341	
F	Wood	12½	7	ditto	6	5	14	114	..	4·848808	
G	Wood	32	11	ditto	10	..	..	..	..	....	
H	....	21	10	ditto	7	..	..	..	..	....	Broke, owing to their too great length; they were 11 inches long.
I	Wood	28	6	ditto	8	..	..	..	..	....	
K	Fir	18	10	ditto	10	..	..	..	..	....	

* The hollow shaft of C being 16 inches diameter, and 2 inches thick, broke in consequence of the gudgeon getting loose about March 7, 1808.

OBSERVATIONS ON THE FIRST TABLE OF GUDGEONS.

217. Particular care should be taken that the axis of the gudgeon be exactly in a line with the axis of the shaft which it supports, otherwise the motion will be unequal, and at one part of the revolution the stress will be thrown

to the point of the gudgeon; this would endanger its breaking, more particularly if very long, though otherwise sufficiently strong*. In the case H, we have an instance of a gudgeon breaking, from being made too long. In practice it is a good method to turn the gudgeons of a wooden shaft, after they are fixed in their places, a second time, in order to render them quite true.

218. From comparing the diameters of the gudgeons (column 6) with the cube root of the weight in cwts. of the wheel, (col. 9,) it will be found that they approach one another. In other words, the cube root of the weight in cwts. is nearly equal to the diameter in inches. In the case C, the gudgeon broke only from being a bad casting, although it is smaller in proportion to its weight, than most of the other cases. Since it was renewed of the same size, it has continued to support its work. When, therefore, we can ascertain the weight of a water-wheel, we have a very simple rule for finding the diameter which the gudgeon ought to have.

RULE I.

219. *The cube root of the weight of a water-wheel, in hundredweights, is nearly equal to the diameter in inches of a cast iron gudgeon sufficiently strong to support such wheel*†.

We say *nearly*, it being evidently most prudent to make the gudgeon a little more rather than less in diameter, and to make allowance for wearing.

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* The possibility of such a cause of failure should be guarded against in proportioning the strength of a gudgeon. See Art. 213.

† In water-wheels, the stress is not proportional to the weight of the wheel. See Art. 212—215 inclusive, where more correct principles are investigated. Also see the cautions in note to Art. 215.

EXAMPLE.

Suppose a water-wheel to weigh 12 tons, 0 cwt. 3 qrs. what ought to be the diameter of a cast-iron gudgeon, sufficiently strong to support the wheel?

12 tons = 240 *cwt.* 3 *qrs.*

The cube root of 240·75 = 6·221 *Answ.* That is, the diameter of the gudgeon should not be less than 6·$\frac{2}{10}$ diameter. It ought to be rather more, to allow for wearing, &c. See B, in the first table of gudgeons.

220. As the weights of wooden water-wheels cannot be accurately known, without a good deal of calculation, it is desirable to have some more ready method for practical purposes.

The weights of overshot, or bucket water-wheels, will be to one another nearly as their circumferences, or diameters and breadth.—We say *nearly*, because the arms will make large wheels heavy in rather a greater proportion. Hence the following rule is formed on the direct proportion of the sole and buckets, adding one-half of the diameter increased in the duplicate ratio or square of the diameter.

RULE II.

For wooden water-wheels, multiply the diameter in feet by the width also in feet, to which add the square of half of the diameter. The cube root of the sum will be nearly equal to the diameter of the gudgeon in inches.

EXAMPLE.

Suppose a wooden water-wheel 12 feet diameter and 7 feet wide, (see E in Table II. of Gudgeons.)

$$12 \times 7 = 84$$

The square of $6 = 36$

$$120 \text{ the cube root of which} = 4{\cdot}932424;$$

that is the gudgeon should not be less than about 5 inches diameter.

EXPLANATION OF TABLE II. OF WATER-WHEELS.

221. All the columns, except No. 10, are the same as in Table I.* The column 10 shews the result by Rule II.

TABLE II.—GUDGEONS OF WATER-WHEELS.

	Wheel made of	Diameter of wheel in feet.	Width in feet.	Kind.	Diameter of cast iron gudgeon in inches.	Weight of wheel in tons.		Weight of wheel in cwts.	Diameter of gudgeon by calculating the weight.	Diameter of gudgeon of wooden water-wheels by Rule II.	Remarks.
A	Wood	24	12	Overshot	7	23	14	474	7·796974	7·559525	
B	Cast iron	16	16	ditto	7	12	2	242	6·231678		
C	Cast iron	16	8	ditto	6¾	16	10	330	6·910423		
D	Cast iron wheel and buckets	32	4½	ditto	9	20	..	400	7·368063		
E	Wood	12	7	ditto	6	5	14	114	4·848808	4·932424	
F	Wood	32	11	ditto	10	..	..	..	....	8·471647	
G	Wood	21	10	ditto	7	..	..	..	....	6·839903	
H	Wood	28	6	ditto	8	..	..	..	....	7·140037	
I	Wood	16	9	ditto	6	10	11	211	5·958341	5·924991	
K	Wood	18	10	ditto	10	..	..	..	....	6·390676	

SECTION III.

OF CAST IRON GUDGEONS FOR VARIOUS PURPOSES.

222. Taking it for granted that the cube root of the weight of a water-wheel, in hundredweights, is nearly equal to the diameter in inches of a cast iron gudgeon, sufficiently strong to support such wheel, the following

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* In these tables the weight of the wheel B is different. We suppose the first table to be the correct one, but the difference is inconsiderable.

table of the diameters of gudgeons, and the weights which they may be supposed to sustain, is formed. It may be of use for finding the diameters of journals in all cases of stress arising from lateral pressure, such as grindstones, intermediate spindles, &c. where the pressure can be ascertained, as well as water-wheels*.

EXPLANATION OF THE TABLE OF CAST IRON GUDGEONS.

223. Column 1 contains the diameter in inches, from 1 to 11 inches.

Column 2 contains the cube of that diameter, or the hundredweights which the gudgeon may sustain.

N.B. We have already remarked, (Art. 219,) that it would be most prudent to make the gudgeon a little more in diameter than the cube root of the hundredweights.

TABLE OF CAST IRON GUDGEONS.

Diameter in inches.	Cube of diameter, or cwts. which the gudgeon may sustain.	Diameter in inches.	Cube of diameter, or cwts. which the gudgeon may sustain.
1·	1·	6·25	244·140625
1·25	1·953125	6·5	274·625
1·5	3·375	6·75	307·546875
1·75	5·359375	7·	343·
2·	8·	7·25	381·078125
2·25	11·400625	7·5	421·875
2·5	15·625	7·75	465·483375
2·75	20·796875	8·	512·
3·	27·	8·25	561·515625
3·25	34·328125	8·5	614·125
3·5	42·875	8·75	669·921875
3·75	52·734375	9·	729·
4·	64·	9·25	791·453125
4·25	76·765625	9·5	875·375
4·5	91·125	9·75	926·859375
4·75	107·171875	10·	1000·
5·	125·	10·25	1076·890625
5·25	144·703125	10·5	1157·625
5·5	166·375	10·75	1242·296875
5·75	190·109375	11·	1452·
6·	216·		

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* See Art. 212.

USE OF THE TABLE.

EXAMPLE I.

224. Suppose a cast iron gudgeon of 5¾ inches diameter, what weight of a water-wheel would it be capable of sustaining?

In the first column find 5·75. Opposite to which will be found 190·109375 hundredweights, which is rather more than 9½ tons, which is the answer. But in practice, it would perhaps be prudent not to load this gudgeon with more than nine tons.

EXAMPLE II.

Suppose a grindstone weighing 15½ hundredweights, required the size of a gudgeon sufficient to sustain this weight. Look in the second column for the nearest weight to that given, which will be found to be 15·625, opposite to which, in the first column, is 2½ inches, the diameter of the gudgeon required.

But in practice the spindles of grindstones are commonly of wrought iron, which metal we shall presently consider as applied to gudgeons.

SECTION IV.

OF MALLEABLE OR WROUGHT IRON GUDGEONS.

225. Professor Robison states*, that the cohesive force of a square inch of cast iron is from 40,000 to 60,000 lbs., wrought iron, from 60,000 to 90,000 lbs.

In the year 1795, Buchanan had occasion to substitute cast iron gudgeons for those of wrought iron, and made some experiments on those metals, from which he drew the following inference: *that gudgeons of the same size, of cast and of wrought iron, in practice, are capable, at a medium, of sustaining weights without flexure, in the proportion of* 9 *to* 14*.

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* Encyclopædia Britannica, article Strength of Materials, 40.

Taking it for granted that this proportion is near the truth, we may find the diameter which any wrought iron gudgeon ought to have when its lateral pressure is given, in the following manner:

226. 1. Find the diameter which a cast iron gudgeon should have to sustain the given pressure, then say, as 14 is to the cube of the diameter of the cast iron gudgeon, so is 9 to the cube of the diameter of the wrought iron gudgeon.

2. The root of this last number gives the diameter required of the wrought iron gudgeon.

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* According to Tredgold's experiments, the stiffness of good cast iron is to that of good English malleable iron as 1 is to 1·3 nearly. (Essay on Cast Iron, Art. Iron.) But the stiffness of malleable iron is much increased by hammering, &c.; and in his trials some pains were taken to obtain the resistance unhammered, which most probably causes the difference.

It is further necessary to observe, that in calculating the strength of gudgeons, the resistance to permanent alteration is the proper measure, because they are too short to admit of sensible flexure; and the strain which produces permanent alteration in malleable iron is only 1·12 times that producing a like alteration in cast iron.

Hence the diameter of a malleable iron gudgeon should be 0·963 times that of a cast iron one to bear the same stress. For if a be the diameter of the cast iron gudgeon, and b the diameter of the wrought iron one; then $a^3 \times 1 = b^3 \times 1{\cdot}12$ when their strengths are equal; consequently $\frac{a^3}{1{\cdot}12} = b^3$; or $\frac{a}{(1{\cdot}12)^{\frac{1}{3}}} = b$; but $\frac{a}{(1{\cdot}12)^{\frac{1}{3}}} = 0{\cdot}963\,a = b$.

The proportion given by our author is about $0{\cdot}863\,a = b$, but his proportion is made from the relative stiffness, while the rule applies to the strength, and therefore it is not correct.

EXAMPLE.

Suppose the lateral pressure to be 125 hundredweights, the cube root of which is 5, the diameter in inches of the cast iron gudgeon: then say,

As 14 : 125 :: 9 : 80·357.

The cube root of which is 4·30887*.

Upon this principle the following table is calculated to show the proportionate diameters of cast iron and wrought iron gudgeons.

EXPLANATION OF THE TABLE OF CAST AND WROUGHT IRON GUDGEONS.

Columns 1 and 2 are the same as those in the table of cast iron gudgeons.

Column 3 contains numbers in the proportion of 9 to 14 less than those of column 2.

Column 4 contains the cube root of column 3, or the diameters of wrought iron gudgeons, having the same strength as those of cast iron in column 1.

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* To find the proportion by the preceding note, multiply the diameter of the cast iron gudgeon by 0·963. Thus 5 inches × 0·963 = 4·815 inches, the diameter for a wrought iron one.

TABLE OF CAST AND WROUGHT IRON GUDGEONS.

1	2	3	4
Diameter of gudgeons in inches.	Cube of diameter of cast iron gudgeons, or the cwts. which the gudgeons may sustain.	Cube of diameter of wrought iron gudgeons.	Diameter of wrought iron gudgeons, in inches and parts.
1·	1·	·6428571	·863
1·25	1·953125	1·2555803	1·063340
1·5	3·375	2·1696427	1·259921
1·75	5·359375	3·4453125	1·514825
2·	8·	5·1428571	1·709976
2·25	11·400625	7·3289732	1·912933*
2·5	15·625	10·0446428	2·154435
2·75	20·796875	13·3694196	2·351335
3·	27·	17·3571428	2·571282
3·25	34·328125	22·0670803	2·802039
3·5	42·875	27·5625	3·018294†
3·75	52·734375	33·9006696	3·239612
4·	64·	41·1428571	3·448217
4·25	76·765625	49·3493303	3·659306
4·5	91·125	58·5803571	3·881936
4·75	107·171875	68·896	4·101566
5·	125·	80·357	4·308870
5·25	144·763125	93·023	4·530655
5·5	166·375	106·955	4·747459
5·75	190·109375	122·213	4·959675
6·	216·	138·857	5·180101
6·25	244·140625	156·948	5·394690
6·5	274·625	176·545	5·609376
6·75	307·546875	197·709	5·828476
7·	343·	220·500	6·041877
7·25	381·078125	244·979	6·257324
7·5	421·875	271·205	6·471274
7·75	465·484375	299·240	6·686882
8·	512·	329·143	6·903436
8·25	561·515625	360·975	7·120367
8·5	614·125	394·795	7·337234
8·75	669·921875	430·664	7·553688
9·	729·	468·643	7·769462
9·25	791·453125	508·791	7·984344
9·5	875·375	562·741	8·257263
9·75	926·859375	595·837	8·415541
10·	1000·	642·857	8·631103
10·25	1076·890625	692·287	8·845085
10·5	1157·625	744·187	9·061309
10·75	1242·296875	798·619	9·279308
11·	1452·	933·428	9·771484

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* Brewster says, "A gudgeon 2 inches diameter, will sustain 9239 lbs. avoirdupois." Brewster's Edition of Ferguson's Lectures, Vol. I. p. 157.

† The wrought iron grindstone spindle used by Southern, in his experiments on Friction, was 3½ inches diameter, and carried a stone weighing 3700 lbs. See Phil. Mag. Vol. XVII. 123.

USE OF THE TABLE.

EXAMPLE.

227. To find the diameter of a wrought iron gudgeon of the same strength with one of cast iron of 3 inches diameter. Look in the 1st column for 3, and on the same line in the 4th column will be found 2·571282, that is, a little more than 2½ inches, the diameter required of the wrought iron gudgeon.

The numbers in the 3rd column, being the cube of those in the 4th, another use may be made of this part of the table. For, supposing the 4th column to represent cast iron gudgeons, then the 3rd column will represent the hundredweights which cast iron gudgeons of those diameters should sustain.

Before proceeding to consider the bodies of shafts subject to *lateral stress*, we shall inquire into the strength of *journals* of shafts subject to torsion.

CHAPTER IV.

SECTION I.

OF THE STRENGTH OF JOURNALS, WHEN THE STRESS ARISES FROM TORSION OR TWISTING, IN ADDITION TO LATERAL STRESS*.

228. In our Inquiry into the Strength and Durability of the Teeth of Wheels, we have used what is called the *horses' power*, as a measure for the strain. We refer the reader to what we have there said (Art. 108—114) in explanation of that term which we shall use here, in measuring the strain brought on shafts by torsion or twisting.

In this case of torsion, as well as that of lateral pressure, the law of proportionate strength *is as the cubes of the diameters*†.

It may be proper here to remark, that what we had to say respecting journals, relates to those of cast iron, for although wrought iron will bear more lateral stress, as we have seen, (Art. 225,) yet it is a fact, perhaps not generally known, that wrought iron will not resist torsion equal to cast iron‡.

In some cases a journal has not only torsion to resist, but also to carry a very heavy fly wheel, and it is prudent in such cases not merely to make an allowance for the weight properly balanced, but also for any inaccuracy which may occasion swagging, which greatly adds to the stress: but others have hardly any other resistance but what arises from torsion.

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* When a shaft has a support between the points where the power and resistance are applied, the part of the shaft which revolves on this support is called a *Journal*.

† See Gregory's Mechanics, Vol. I. Art. 191.

This reference is to a statement that the strength is in this proportion, but it is not demonstrated.

‡ The author seems to be under a mistake in regard to this fact.

It is further observable that the value for 10 horses in the smaller engines is much less than in the larger. For this difference what we have said respecting the weight of the fly in a great measure accounts: and not only is the heavier fly to be considered, but also the greater danger of accidents from a large fly than from one that is of a smaller diameter.

SECTION II.

OF PROPORTIONING JOURNALS TO THE STRESS WHICH THEY HAVE TO SUSTAIN.

229. The stress any journal has to sustain being as the horses' power to which the resistance is equal *directly*, and the number of revolutions which the shaft makes *inversely*, it follows: That a resistance for example of 32 horses' power on a journal making 50 revolutions per minute, has the very same stress with a resistance of 16 horses' power on another journal making 25 revolutions per minute.

32 divided by 50 is equal to 16 divided by 25, each of which gives a quotient of 0·64. Therefore in all cases when the horses' power divided by the revolutions per minute produces the same quotient the stress is the same.

Thus a resistance equal to 50 horses' power making 50 revolutions per minute, produces the very same stress as 10 horses' power making 10 revolutions per minute.

Having therefore fixed on any journal which has been found sufficiently strong, we may make any other to have the same strength in proportion to the resistance which it has to overcome in the following manner.

230. RULE.—If it so happen that the horses' power, and the revolutions per minute be the *same number*. For instance, 50 horses' power making 50 revolutions, 50 ÷ 50 = 1; then the cube of the diameter of the journal will be a multiplier, by which to find the cube of the diameter of the required journal.

But in case the horses' power and the revolutions per minute are *different numbers*, then you must suppose them both the same, and calculate (as in Ex. II.) what, in that case, would be the proportionate diameter of the journal.—The cube of this diameter will be a multiplier, the same as mentioned above.

Having found the multiplier, to find the diameter of the required journal.

Divide the horses' power by the revolutions per minute.

Multiply the quotient by the *multiplier*, the cube root of the product will give the diameter of the journal required*.

EXAMPLE I.

To find the multiplier from a journal 7½ inches diameter, where there is an engine of 50 horses' power turning a shaft, at the rate of 50 revolutions per minute.

Divide the power by the revolutions, that is 50 divided by 50 is equal to 1; the diameter of the journal is 7½ inches; the cube of this is 420, which multiplied by 1 produces 420.

In this case it happens that the horses' power, and the revolutions per minute, are *the same number*, therefore we with little trouble find the multiplier.

EXAMPLE II.

From a journal of 4 inches diameter, where the horses' power is 12, and the revolutions per minute 48; to find the multiplier.

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* See Art. 233.

Now let us suppose both numbers the same, that is, 12 horses' power, and 12 revolutions.

Here it is evident that there will be four times the stress brought on the journal. Its actual diameter was 4 inches.

Then the cube of 4 is 64,
64 multiplied by 4 is 256 inches,
The cube root of which is 6.35,

which is the diameter which the journal ought to have, to be in proportion to the velocity.

The cube of 6·35 is 26,

which is the multiplier required.

It is to be observed, that in the latter case a much smaller steam engine is employed than in the former, and, therefore, for reasons already given, (Art. 228,) has less stress brought upon the journal. This accounts for the multiplier being less.

We shall now give an example of the application of a *multiplier;* let us take that found in the case Example I., viz. 420, and see what size of the journal it would give in the case Example II., which is an engine of 12 horses' power and journal making 48 revolutions per minute.

12 divided by 48, equal to ·25, then multiplied by 420, gives a quotient of 105, that is, the strength of the journals must be as 420 to 105; but the cube root of 420 is 7½, and the cube root of 105 is 4¾, which points out that the journals 7½ and 4¾ are proportioned to their respective engines. In like manner the following table is calculated; the multiplier being 420.

231. *Description of the Table of Journals, proportionate to* D, *having* 420 *as a multiplier.*

Column 1 contains letters to distinguish the cases in which D is the same as Example I. (Art. 230,) and E, Example II. of the same Art.

Column 2 the horses' power.

Column 3 the revolutions of the journal per minute.

Column 4 the product of the horses' power divided by the revolutions of the shaft.

Column 5 contains the proportionate strain on each journal, represented in whole numbers, which are found by multiplying the product in column 4, by 420, as a multiplier.

Column 6 diameters of journals, as really executed in several steam engines.

TABLE OF JOURNALS

Proportionate to D, *having* 420 *as a multiplier.*

1	2	3	4	5	6
	Numbers of horses' power.	Revolutions of journal per minute.	Product of power divided by the rev. of the journal.	Proportionate strain on journal.	Diameters of journals from observation.
A	32	58	0·55	231	7½
B	32	19	1·67	701	9½
D	50	50	1·0	420	7½
E	12	48	0·25	105	4
F	9	55	0·16	67	4

OBSERVATIONS.

232. We have already observed, (Art. 228,) that, besides torsion, the journals of fly-wheel shafts have considerable lateral and other stress, arising from the weight and swagging of their fly wheels, and therefore they ought to be made stronger than shafts, in other situations. The multiplier 420, therefore, which we have used in the table,

would give diameters too great, for some other parts of machinery. A journal, for instance, subject to torsion, immediately connected with a water-wheel, has, from the weight of the wheel and other causes, considerable lateral stress; but not so much as that of a steam engine. The journal may, therefore, be considerably smaller than would be required for a steam engine fly-wheel shaft subject to the same degree of torsion*.

Again, a secondary shaft driven from a steam engine, a water-wheel, or horse-gin, by means of wheels, has in general very little lateral stress, compared with the two cases just stated; and may therefore have a journal smaller than either, when the degree of torsion is the same.

233. For these reasons the three following multipliers will probably approach near the truth; that is, for journals of steam engine fly-wheel shafts (where the power is moderate) 400

Journals in immediate connexion with water-wheels†, or other heavy work 200

Journals for the ordinary kind of internal mill-work 100

EXAMPLE.

Suppose B, in the table, (Art. 231,) 1·67 multiplied by 400, is 668, the cube root of which is 8·7416 inches, diameter of journal.

234. When the resistance of a journal is equal to the twisting stress, the strain not being sufficient to produce permanent derangement in the material, the cube of the diameter of the journal will be equal to 3·78 times the

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* It seems a better method to use a mode of calculation which includes the effect of lateral stress; see Art. 233.

† The reader will please to observe, that, when Buchanan uses the word journal here, he supposes it subject to torsion. Where there is lateral pressure only, and no torsion, he invariably uses the word gudgeon.

number of horses' power divided by the number of revolutions in a second*.

But some allowance must be made for wear, and if this be made so that the journal shall have sufficient strength when it is worn down one-sixth of its diameter, the number 3·78 should be made 6·01; or with sufficient accuracy 6.

If the number of revolutions in a minute be employed instead of those in a second, the constant multiplier, 6, must be multiplied by 60; and therefore the constant multiplier will become 360; and a less number ought not in any case to be employed, because there will always be some lateral stress in addition to the twisting stress. The resistance of a journal, or its diameter as regards the twisting strain, may be always calculated by the following rules.

RULE.—If N be the number of revolutions in a minute, and d the diameter of the journal in inches, then $\frac{N.d^3}{360} =$ the number of horses' power the journal is sufficient to resist.

RULE.—If N be the number of revolutions per minute, and H the number of horses' power moving the train of machinery, then $7{\cdot}12 \times \left(\frac{H}{N}\right) = d$, the diameter of the journal in inches.

EXAMPLE.

Let it be required to find the diameter of a journal for case B in the table of journals, Art. 231.; then we have H equal 32 horses, and the number of revolutions N equal 19; therefore $\frac{32}{19} = 1{\cdot}68421$. The cube root of 1·68421 is found to be 1·19; and $7{\cdot}12 \times 1{\cdot}19 = 8{\cdot}4728$ inches, the diameter of the journal, or nearly 8½ inches.

* The reason of this rule will be given in treating of the resistance of shafts.

235. But the effect of lateral stress ought always to be considered, and we have found the strength of a gudgeon to be $0{\cdot}6\,(\mathrm{w}\,l)^{\frac{1}{3}} = d$; where the stress is wholly lateral, (Art. 212.) Or, $0{\cdot}216\ \mathrm{w}\,l = d^3$. And the strength of a journal, where the stress is altogether twisting, to be $\frac{360\,\mathrm{H}}{\mathrm{N}} = d^3$; therefore, since journals bear both kinds of stress, we have as a general

Rule. — $\left(0{\cdot}216\ \mathrm{w}\,l + \frac{360\,\mathrm{H}}{\mathrm{N}}\right)^{\frac{1}{3}} = d$, the diameter in inches. Where w is the lateral stress upon the journal in cwts., l the length of the journal in inches, H the number of horses' power moving the train of machinery, and N the number of revolutions of the journal per minute.

SECTION III.

236. When the diameter of a journal and its revolutions per minute, are given, in order *to find the horses' power* to which it is equal; we must invert the preceding operation, and convert the *multiplier* into a *divisor.*

Rule.—Cube the diameter of the journal, divide the cube by the *divisor.* The quotient multiplied by the revolutions per minute, gives the horses' power, to which the journal is equal.

EXAMPLE I.—F IN THE TABLE.

Suppose the journal of a steam engine to be 4 inches diameter, making *55* revolutions per minute, then we shall use 400 as a divisor.

The cube of 4 is 64, divided by 400 equal to 0·16, then this quotient (0·16) multiplied by *55*, gives 8·8 the horses' power, to which the journal is equal.

EXAMPLE II.

Suppose again, the same size of a journal, connected with heavy machinery, then we must use 200 as a divisor.

The cube of 4 = 64 ÷ 200 = ·32 × 55 = 17·6 horses' power.

EXAMPLE III.

We shall take the same journal for internal work, of the ordinary kind, and use 100 as a divisor.

The cube of 4 = 64 ÷ 100 = ·64 × 55 = 35·2 horses' power.

CHAPTER V.

SECTION I.

ON THE BODIES OF SHAFTS.

237. From what is stated in the preceding part of this Essay, the millwright will be enabled to approach sufficiently near to the truth, for all practical purposes, in proportioning gudgeons and journals to the stress which they have to sustain. Taking this for granted, it may be proper for us next to consider what relates to the *bodies* of shafts, or those parts which lie between the gudgeons or journals. In this part of our inquiry, we may derive assistance from the principles which have been applied by writers on mechanics, to the stress of timber and other materials*. The generality of writers on mechanics, however, as Dr. Young justly observes, (vol. i. page 136,) have confined their attention to *strength* (resistance to fracture) alone, although there be other very important properties, which required their consideration. The most usual as well as the most important effect, produced by the application of force is *flexure:* (p. 138, ibid.) *stiffness* therefore, as well as *strength,* ought to be considered in determining the form, as well as the quantity of materials, for any mechanical purpose, more particularly that of a shaft in mill-work, which in theory may be considered as an *inflexible* straight line. The practical reader should attend to the distinction between *stiffness* and *strength.*

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* Few operative mechanics have a distinct notion of the difference between *strength* and *stiffness.*

Stiffness is that property which resists *flexure* or *bending*.

Strength that which resists fracture or breaking.

The consideration of their limits, may make this plainer.

The limit of *stiffness* is *flexure ;* the limit of *strength* is *fracture*.

The *stiffness* of a beam follows laws very different from those which determine its *strength ;* these laws we shall presently consider, and endeavour to shew their application to practice, with regard to some cases of shafts. But although those laws may throw considerable light on the subject, yet it must be confessed, that there are many cases in practice, in which it is very difficult, if not impossible, to apply them; for it is very often difficult to estimate what may be the amount of the lateral pressure on shafts, arising not only from their own weight, and that of the wheels, upon them, but also from the thrust, proceeding from the action of the toothed wheels, and other extraneous causes. In cases of this nature, where calculation fails, much must be done, by what Smeaton calls *feeling**, which will direct the experienced millwright to make a due allowance for whatever accidental strain may be likely to occur.

We shall now proceed to state and apply some of the laws, respecting stiffness and strength, with regard to force, applied transversely.

OF LATERAL STIFFNESS AND LATERAL STRENGTH.

238. The "*stiffness* of any substance, is measured by the force required to cause it to recede, through a given small space, in the direction of the force," (Young's Nat. Phil. vol. i. p. 139.) Its transverse *strength* is measured by the pressure required to produce its fracture, or, in other words, to break it.

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* Smeaton's Account of Eddystone Lighthouse, p. 136.

PROPOSITION II.

239. *Any beams of equal length have their lateral stiffness,* [*to bear a load at any point in the length,*] *as the breadth and cube of the depth,* (Young's Nat. Phil. vol. i. p. 139, or ii. art. 333,) *and have their lateral strength, as the breadth and square of the depth**, (Gregory, vol. i. art. 169, cor. 1. Emerson, prop. 67.)

Thus, if a square beam measure twice as much, on the side, as another of equal length, it would be sixteen times as stiff. In other words, it will sustain sixteen times the weight, without bending.

But, if a square beam be twice as much on the side as another, both being the same length, it will be only eight times stronger.

Hence we see, that when beams or shafts are of equal lengths, their *stiffness*, by any increase of thickness, increases in a higher proportion than their *strength.*

EXAMPLE I.

If a beam or shaft be four inches square throughout, and another five inches, both of equal lengths; what is their comparative *stiffness?*

The cube of 4, is 64,
$64 \times 4 = 256$,
The cube of 5, is 125,
$125 \times 5 = 625$,

That is, the shaft of five inches is nearly two and a half times stiffer than that of four; in other words, it would require nearly two and a half times the weight to bend it.

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* That is, as the cube of the side of a square beam, and in general the lateral strength of any beams whose sections are similar, as the cube of the similar sides or diameters of the sections.

EXAMPLE II.

If a beam or shaft be four inches square throughout, and another five inches, both of equal lengths; what is their comparative strength?

The cube of 4, is 64,
The cube of 5, is 125.

That is, the five inches shaft is nearly twice as strong as that of four inches; in other words, it would require nearly double the weight to break it.

PROPOSITION III.

240. *Any beams of different lengths have their stiffness [to bear a load at any point in the length] directly as the breadth and the cube of the depth, and inversely as the cube of the length,* (Young's Nat. Phil. ii. art. 333,) *and have their strength directly as the breadth, and as the square of the depth, and inversely as the length**, (Young, vol. ii. art. 335.)

Thus, if a beam be twice as long as another, of the same breadth and depth, it will have only one eighth of the stiffness, while it will have one half of the strength. Hence the *stiffness* of shafts or beams by any increase of their length, decrease in a much higher proportion than that of their *strength.*

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* This is not strictly true in practice, for "some experiments appear to shew, that the strength is diminished, in a proportion somewhat greater than that in which the length is increased." (Young's Nat. Phil. vol. ii. p. 147.)

The variation is caused by the increase of strain which takes place when the flexure is considerable, (see Elementary Principles of Carpentry, Art. 18,) and some decrease of cohesive power when the natural arrangement of the particles of a body is disturbed more than in a certain degree; but these causes are insensible in a practical point of view, because we can never allow the stress to produce so much flexure, nor the strain to be so near to fracture, as to make it necessary to allow for such circumstances.

EXAMPLE I.

Suppose a beam or shaft, four feet long and four inches square throughout, and another eight feet long and seven inches square; what is their comparative stiffness?

The cube of 4 feet, is 64,

The cube of 8 feet, is 512,

512 divided by 64, is equal to 8, that is, when we double the length, we decrease the stiffness eight times.

The cube of 4 inches is 64, which multiplied by 4 is equal to 256, a number representing the stiffness of the four inch shaft.

The cube of 7 inches is 343, multiplied by 7 is equal to 2401, divided by 8 is equal to 300·1, then as 256 is to 300·1, so is the stiffness of the shaft of four inches to that of a shaft of seven inches.

EXAMPLE II.

Suppose a beam or shaft, four feet long and three inches square, and another eight feet long and seven inches square, what is their comparative strength?

The cube of 4 is 64, which represents the strength of the four inch shaft.

The cube of 7 is 343; but the shaft being of double length, we must halve this sum, to find the number representing its strength, viz. $343 \div 2 = 171{\cdot}5$ divided by 100, that is, as sixty four is to a hundred and seventy-one and a half, so is the strength of the short shaft to that of the long one. Thus the shaft of seven inches, eight feet long, has nearly two and six tenths times the strength, of the four inch four feet long.

PROPOSITION IV.

241. *Supposing a tube, indefinitely thin, to be expanded into a similar tube of a greater diameter, but of equal lengths, the quantity of matter remaining the same, the* STIFFNESS *will be increased, in the ratio* of the square of the diameter, and the* STRENGTH *in the ratio of the diameter*†.

Thus, if the one tube be double the diameter of the other, it will have four times its *stiffness*, but only double the strength.

Hence, hollow cylinders of equal lengths, by any increase of diameter, increase in *stiffness*, in a much higher proportion than in *strength.*

EXAMPLE I.

Suppose two thin narrow cylindrical cast iron shafts, of equal lengths and weights, the one of one foot diameter, and the other three feet diameter, required their comparative *stiffness?*

The square of 1 is 1,
The square of 3 is 9,

that is, the shaft of three feet diameter, is nine times stiffer than that of one foot.

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* Ratio, that is, proportion.

† This proposition is taken from Dr. Young's Nat. Phil. vol. ii. art. 339, where it is followed by this essential limitation. "When a beam of finite thickness is made hollow, retaining the same quantity of matter, the strength is increased in a ratio somewhat greater than that of the diameter, because the tension of the internal fibres at the instant of breaking is increased." Dr. Young has given the correct rule for estimating the strength and stiffness of a hollow cylinder, at p. 84, (Nat. Phil. vol. ii.) "The strength of a tube may be found by deducting from the strength of the whole cylinder that of the part removed, reduced in the ratio of the diameters." And observes, that "the strength is in this case in the same ratio as the stiffness."

EXAMPLE II.

Suppose the same shafts, as in example first, required their comparative strength?

Diameter one foot,
Diameter three feet,

that is, the three feet shaft is just three times stronger than that of one foot diameter.

242. In these examples, we have supposed the weight of the shafts equal, that is, the area of their ends to be equal, but the strength of any of them would be increased in proportion to their weight, or the areas of their ends and diameters, conjointly. (Gregory, vol. i. art. 172, cor. 3.)

Thus, suppose two shafts of equal length and diameter, the one double the weight of the other, it will be double the strength*.

243. Professor Robison justly observes, "that this property of hollow tubes is accompanied also with greater stiffness, and the superiority in strength and stiffness is so much the greater, as the surrounding shell is thinner in proportion to its diameter. Here we see the admirable wisdom of the Author of nature in forming the bones of animal limbs hollow. The bones of the arms and legs have to perform the office of levers, and are thus opposed to very great transverse strains. By this form they become incomparably stronger and stiffer, and give more room for the insertion of muscles, while they are lighter and therefore more agile; and the same wisdom has made use of this hollow for other valuable purposes of the animal economy. In like manner, the quills in the wings of birds acquire by their thinness the very great strength which is necessary, while they are so light as to give sufficient buoyancy to the animal, in the rare medium in which

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* See note to Art. 236.

it must live and fly about. The stalks of many plants, such as all the grasses, and many reeds, are in like manner hollow, and thus possess an extraordinary strength." (Ency. Brit. article Strength.)

Long before this eminent philosopher, the celebrated Galileo made similar observations, and goes on to say that "if a wheat straw, which supports an ear that is heavier than the whole stalk, were made of the same quantity of matter but solid, it would bend or break with far greater ease than it now does. And with the same reason art has observed and experience confirmed, that a hollow cane or tube of wood or metal, is much stronger and more firm than if, while it continued of the same weight and length, it were solid, as it would then of consequence be not so thick.

It may be proper now to consider the effects called *stress*, which are produced on beams or shafts lying horizontally by weights or pressures brought on various parts of them.

SECTION II.

OF LATERAL STRESS.

244. The *stress* or *strain*[*] are terms used to express the force which is excited in any body tending to break it. The meaning of the term *stress* may perhaps be more clearly understood by contrasting it with the term *strength.*

Strength, as we have already observed, is the property which *resists fracture.*

Stress is that which has the tendency to produce *fracture ;* and *lateral stress* is that particular application of it, which has the tendency to break a body *across.*

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[*] Strain is the effect of stress: it is the derangement from the natural state which is caused by stress.

PROPOSITION V.

245. *The stress on a beam arising from one weight hung upon it, is proportional to the rectangle of the parts of the beam, and is greatest when the load is laid on the middle of the beam.* (Ency. Brit. art. Roof, § 19.)

What is meant by the expression *rectangle of the parts*, is the product of parts multiplied into each other. Thus, for example: if a beam be ten feet long, and the weight hung two feet from one end, the parts are 2 and 8, which multiplied together, would be equal to 16; but supposing the weight were hung in the middle, the parts are 5 and 5, which multiplied together would produce 25.

246. The ends of beams having the whole weight to support, the end which is nearest the weight has to support the greatest proportion of it, in the inverse proportion of the distance of the weight from the end. This will be easily understood from the properties of the lever. For, suppose the beam instead of being supported by two props or walls, as in Fig. 9, No. 1, to be hung from each end by a rope, as in Fig. 9, No. 2, it is plain that the beam would receive the same support, and suffer the same stress, as if lying on props or walls; now suppose the weights A and B, to balance the weight W, then A and B, taken together, must be equal to W, but A must be greater than B in proportion as W is near to it.

247. Hence when any beams or shafts are loaded exactly in the middle, each of the ends of the beams or gudgeons of the shafts has half the weight to support, and when the weight is nearer one end, the end or gudgeon to which it is nearest, has the stress in the inverse proportion of the distance. In this last case, therefore, the one gudgeon might be smaller than the other.

248. "We may always consider the weight which is

uniformly diffused over any part of a beam as united in the middle of that part, and if the load is not uniformly diffused, we may suppose it united at its centre of gravity." (Ency. Brit. article Roof, § 20.)*

249. It is evidently of importance that a beam or shaft should in every part be able to resist the strain excited in that part. "It should therefore be equally strong, because the piece will nevertheless break where it is not stronger throughout, and it is useless to make it stronger (relatively to its strain) in any part, or it will nevertheless equally fail in the part that is too weak." (Ibid.)

250. From what we have said respecting lateral stress, it is evident than when a beam lying between two props is loaded at some intermediate part, that part has to sustain more stress than the rest. In order to resist this strain, therefore, and to render the beam equally strong throughout, it should have its section enlarged at the place of greatest stress, and hence shafts subject to lateral stress should swell in the middle, and it will be found that when each section is made proportional to the stress it has to sustain, that the sides of the shaft will form curves.

251. When the transverse sections of a beam are all similar, such as circles, squares, or polygons, and the weight is laid on one place, in order to make it equally strong throughout its length, the curve of the sides of the beam becomes what mathematicians call a cubical parabola. (Ency. Brit. Strength of Materials, 87.) But when the weight is uniformly diffused all over the beam, the sides of

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* When the weight is uniformly diffused, the stress is the greatest at the middle of the length, and is equal to half the weight collected in the middle. But the flexure in the middle produced by a weight which is uniformly diffused, is the same as when five eighths of the load is collected in the middle of the length. (See art. Carpentry, Supplement to Encyclopædia Brit. 1817, Prop. F., or Barlow's Essay on the Strength of Timber, p. 117.)

the beam must be a different parabolic curve called a semicubical parabola*.

We come now to examine some of the laws respecting twisting or torsion.

SECTION III.

OF TORSION.

PROPOSITION VI.

252. *In general the strength of a cylinder or solid axle by which it resists being wrenched asunder by twisting is as the cube of its diameter.* (Ency. Brit. art. Strength of Materials, 123.)

Thus, if a solid cylinder be double the diameter of another, it would require eight times the force to wrench it asunder.

OF HOLLOW AXLES.

253. Hollow axles are stronger to resist twisting than solid ones containing the same quantity of matter. For if a hole be bored out of an axle of half its diameter, this reduces its *weight one-fourth*, (because circles are to one another as the squares of their diameters,) but the strength of solid cylinders being as the cubes of their diameters, the part taken out by boring had only the eighth part of the strength of the whole cylinder, and therefore when taken out would reduce the *strength* of the whole *one eighth*.

Thus, let the external diameter of the hollow axle be five inches, and that of the hollow part of it four inches, then the diameter of another cylinder made solid, having the same quantity of metal with the tube, is three inches.

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* "The parabola is a conic section, arising from a cone being cut by a plane parallel to one of its sides, or parallel to a plane that touches one side of the cone."

For 5 multiplied by 5 is equal to 25
4 multiplied by 4 is equal to 16

Difference 9

The square root of 9 is 3. The strength of the solid cylinder of five inches diameter may be expressed by the cube of 5, or 125. Of this the internal part, four inches diameter, exerts 64, that is, the cube of 4; therefore the strength of the tube is 64, subtracted from 125, is equal to 61, but the strength of the solid axle of the same quantity of matter, and three inches diameter is expressed by the cube of 3 or 27, which is not half of that of the tube. (Ency. Brit. art. Strength of Materials, 124.)*

254. The superiority of strength of hollow tubes over solid cylinders is much greater in resisting torsion than transverse or lateral stress. We have seen above, that the strength to resist torsion of the tube was to that of the cylinder as sixty-one is to twenty-seven; but Professor Robison estimates, that their strength to resist transverse strain is only as sixty-one is to thirty-two and a half nearly —and if we calculate according to Dr. Gregory's corollary, Vol. I. page 109, (see Art. 237 of this Essay,) the result will be still more in favour of strength to resist torsion; for by the last mode of calculation the tube would be to the cylinder only as forty-five is to thirty-six; but the Professor's mode of calculation, though less simple, is probably more accurate than that above alluded to†.

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* These calculations are founded on the erroneous supposition that the tension is equal in every part of the section; and consequently they are widely distant from the truth. (See note to Art. 254.)

† It has been shewn that the resistance of a cylinder to torsion is $124{\cdot}8d^3 = \text{R W}$; where W is the stress in lbs. and R the leverage in feet it acts with; d being the diameter of the cylinder in inches. (Essay on Cast Iron, Art. 227.) And when the straining force is considered to act at the surface of the shaft $\frac{d}{2} = 12\,\text{R}$; and therefore in this case $20{\cdot}8\,d^2 = \text{W}$.

Professor Robison mentions (in Ency. Brit. art. Strength of Materials, § 128) that "when the matter of the axle is of the most simple texture, such as that of metals, we do not conceive that the length of the axle has any influence on the fracture. It is otherwise if it be a fibrous texture, like timber; the fibres are bent before breaking, being twisted into spirals like a corkscrew. The length of the axle has somewhat of the influence of a lever in this case, and it is easier wrenched asunder if long.—Accordingly we have found it so; but we have not been able to reduce this influence to calculation*.

255. All shafts are exposed to lateral stress and twisting, but it will commonly happen that one of these forces will vastly exceed the other; and consequently, we need only adapt the shaft to the resistance of the greater power;

By the same reasoning it may be proved that, in a hollow cylinder, where D is the exterior diameter, and n D the diameter of the hollow part, $124{\cdot}8\,D^3 (1 - n^4) = R\,W$; when the force W acts with the leverage R in feet; but when the force is applied at the surface of the shaft, $20{\cdot}8\,D^2 (1 - n^4) = W$.

Hence when the strain is at the surface of the shaft, the resistance of a solid cylinder is to that of a hollow one as $d^2 : D^2 (1 - n^4)$; and taking the same example, which our author has quoted from Professor Robison, we have $d = 3$, $D = 5$, and $n D = 4$, or $n = {\cdot}8$, the ratio is $9 : 25 (1{\cdot} - 4096,)$ or as $9 : 14{\cdot}76$, when the stress is reduced to the surface of the shaft or cylinder.

But the ratio is $d^3 : D^3 (1 - n^4)$ when the leverage is constant; that is, $27 : 73{\cdot}8$, instead of 27 to 61. It must however be observed, that these ratios obtain only in the particular case for which they are here calculated, in the example where the leverage is constant the general ratio is $(1 - n^2)^{\frac{3}{2}} : (1 - n^4) ::$ strength of a solid cylinder : that of a tube containing the same quantity of matter. And it may be very easily provéd that the lateral strengths are in the same ratio.

* In wood the lateral adhesion of the fibres being much inferior to their direct cohesion, it is much weaker to resist torsion; the fibres sliding one upon another very considerably before the rupture takes place. But this gives the length no sensible influence, when the strain is kept within proper limits.

but in the first place we must be able to ascertain when the one or the other must be calculated for.

The resistance of a shaft to a lateral stress must obviously be measured by its stiffness to resist flexure, because it would otherwise play too much on its brasses, couplings, &c., and occasion irregular action in the machine.

From a comparison of shafts in use, it appears, that about the $\frac{1}{100}$th part of an inch, for each foot in length, is the quantity of flexure that may be allowed without sensibly affecting the regularity of its motion.

In a cast iron shaft, supposing it to be a solid cylinder, if W be the stress when referred to the middle of the length of the shaft; $l =$ the length in feet, and $d =$ the diameter in inches; the deflexion in the middle being $\frac{l}{100}$ of an inch, then $\frac{\mathrm{W} l^2}{250} = d^4$. (Essay on Cast Iron, Art. 218.) or $\mathrm{W} = \frac{250 d^4}{l^2}$.

256. Now if $\frac{740\mathrm{H}}{v}$ be the twisting power collected at the surface of the shaft, where v is the velocity of that surface in feet per second, and H the greatest number of horses' power that is necessary to work the train of machinery to which the shaft belongs, (see note to Art. 106,) we have $124{\cdot}8 d^3 = \frac{740\mathrm{H}}{6v}$, (Essay on Cast Iron, Art. 227,) but the velocity of the surface of the shaft is equal to its circumference in feet multiplied by the number of revolutions in a second. And, if this number of revolutions be N, then $v = \frac{11\mathrm{N}d}{42}$; and the equation between the stress and strain, reduces to $d^4 = \frac{3{\cdot}78\mathrm{H}}{\mathrm{N}}$.

It therefore appears, that when $\left(\frac{3{\cdot}78\,\mathrm{H}}{\mathrm{N}}\right)^{\frac{1}{3}} = \left(\frac{\mathrm{w}\,l^2}{250}\right)^{\frac{1}{4}}$ the strain from torsion will be equal to that from lateral pressure; whence if $\frac{1472}{l^2} \times \left(\frac{\mathrm{H}}{\mathrm{N}}\right)^{\frac{4}{3}}$ be less than w, the diameter of the shaft must be determined by the rule for lateral stress; if it be greater than w, calculate the diameter by the rule for torsion.

For the advantage of those who are not much versed in calculation, it may be of use to remark, that in common cases when the stress in lbs. multiplied by the square of the length in feet, is less than 3,000; the twisting strain will be the greater; and the reverse.

In a series of lying shafts, as in Fig. 1, Plate V. or Fig. 21, Plate VIII. it is necessary to make the journals equal to the twisting strain with the addition of the necessary allowance for wear; hence it will often be necessary to adjust the shafts to both strains: but in all instances where the lateral stress is less than $\frac{1472}{l^2} \times \left(\frac{\mathrm{H}}{\mathrm{N}}\right)^{\frac{4}{3}}$ the bodies of the shafts need not be greater than the journals.

In other cases, as in Fig. 12, Plate III. the principal twisting strain will be in the part of the shaft between the wheels A and B, while the twisting strain on the gudgeons will be equal only to the friction.

Having thus stated these laws, as far as seems to be necessary for the purpose of our present inquiry, it may be proper next to endeavour to apply them more particularly to practice, with regard to the proportion of shafts.

SECTION IV.

OF THE PROPORTION OF SHAFTS.

257. It was already observed, (Art. 211.) that the gudgeons or journals having to support the whole stress of the

shafts to which they belong, their diameters being determined, serve to guide in determining the proportions of the whole shaft. They are subject to wear, which the body seldom is. They ought, therefore, to be sufficiently large to allow for that wear. It frequently happens, that a shaft has no lateral pressure excepting that which arises from its own weight, for instance in a line of coupled horizontal shafts conveying motion to a distance. In the case of vertical shafts also there is often little or no lateral stress. In such cases, when solid cast iron shafts are of moderate lengths, it is found from experience, that making them square of the same size throughout*, between the journals, and the measuring a little more on the side than their diameters, gives them sufficient stiffness. Even where there is considerable lateral stress when the shafts are but short, making them in this manner is found to give sufficient stiffness. This square form, in many cases, affords great convenience for hanging or fixing wheels, pulleys, &c., upon them.

258. The gudgeons of water-wheels are often so near the wheel, that the stress is, in a great measure, taken off the shaft. Hence some water-wheels are made without shafts, the gudgeons being fixed to the arms at each side of the wheel. The sole of the wheel, in these cases, may be *considered as a large hollow axle*†.

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* See note to Art. 193.

† For a wheel of considerable breadth it will in many instances be an advantage to employ a comparatively small axis, and to render the wheel firm by arms and braces. But if the axis be dispensed with, the same degree of firmness will be gained only by a greater quantity of matter.

The author seems to have had in view the ingenious method of constructing a water-wheel executed by Mr. Burns, at Cartside; (Dr. Brewster's edition of Ferguson's Lectures, Vol. II. p. 55;) but the addition of an axis would be of great use if it were only to prevent the racking strain of the gudgeons upon the cross arms.

CAST IRON SHAFTS.

259. 1st. Let us now suppose a square cast iron shaft required to be made eight feet long, with gudgeons four inches diameter, having considerable lateral stress in the middle—were the shaft of equal size throughout, it would evidently be weakest in the middle. Its section, in that place, ought therefore to be considered. We have already seen (Art. 257) that square shafts, when short, are sufficiently stiff, if made as large as the gudgeons; this shaft may therefore be four inches square at the end. Supposing the section in the middle enlarged to five inches, swelling with a regular curve (Art. 187) from each end (by Prop. II.) then

The cube of 4 is 64;
The cube of 5 is 125;

that is, nearly double.—Therefore, the *strength* of shafts being inversely as their length, (Prop. III.,) the section, as thus increased, would make the eight feet shaft nearly as strong as a shaft subject to the same stress, only four feet long and four inches in the middle.

Let us next see what would be the *stiffness* of this shaft, as thus swelled in the middle to five inches. Then (by Proposition II.) the cube of 4 is 64, which, multiplied by 4, is equal to 256. The cube of 5 is 125, multiplied by 5, is equal to 625; therefore, the stiffness being inversely as the cube of the lengths—

The cube root of 256 is 6·3.
The cube root of 625 is 8·5.

Therefore, the section as thus increased, would make the eight feet shaft about as stiff as one of six feet, subject to the same stress, and four inches throughout.

260. 2nd. Let us next suppose the point of greatest lateral pressure to be two feet from one end.

From the properties of the lever, the gudgeon next the point of greatest pressure has three-fourths of the whole to sustain.

Now the cube of the diameter of the gudgeon is 64; multiplied by 2, is equal to 128, which represents the strength of both journals taken together.

3-4ths of 128 is equal to 96.

The cube root of 96 is 4·578, the diameter which the largest gudgeon ought to have.

1-4th of 128 is 32.

The cube root of 32 is equal to 3·174, the diameter which the smallest gudgeon ought to have, or that furthest from the point of pressure.

Having thus obtained the diameters of the two gudgeons, each end of the shaft may be made square, equal to its respective gudgeon. (Art. 257.)

The stress on the shaft being as the rectangle of the parts, (Prop. V.); in this case it is less than the former, as twelve is to sixteen—for four feet multiplied by four feet is equal to 16, which expresses the stress when in the middle—6 feet multiplied by two feet is equal to 12, which expresses the stress when two feet from one end.

This shaft would therefore be as *strong* as the former, if its greatest section were reduced to 4·54.

For the cube of 5 is 125, as 16 : 12 :: 125 : 93·75, the cube of which is 4·54.

It would be as *stiff* as the former, if its greatest section were reduced to about 4·65:

For the cube of 5 is equal to 125; multiplied by 5 is equal to 625, which expresses the stiffness of the former shaft, as 16 : 12 :: 625 : 468·75, which expresses the required stiffness; and the cube of 4·65 is equal to 100·5444625, which, multiplied by 4·654, is equal to 467·934, which is nearly equal to the required stiffness.

By examining shafts in this way, the millwright, according to the nature of the case, will be enabled to judge how much they should swell at the place of *greatest stress.*

261. As far as regards the strength these examinations will apply; but not to stiffness; because the rule for the stiffness supposes the shaft to be everywhere of the same section. And it may be here remarked that a cylinder is stiffer than any figure which can be inscribed within it; hence there is not that advantage in diminishing a shaft towards the points of support which many people have imagined.

Supposing the reader to have considered the principles of estimating the pressure on a shaft (Art. 207) with sufficient attention to be able to ascertain the greatest stress upon one with as much precision as is necessary in these calculations; we shall here give rules for computing the diameters of different forms of cast iron shafts to resist lateral stress.

CYLINDRICAL SHAFTS OF CAST IRON.

If the stress be in the middle, and equal to w cwts. the flexure in the middle not to exceed as many hundredths of an inch as the shaft is feet in length; then by (Art. 255) $(\frac{1}{2}w)^{\frac{1}{4}} \times l^{\frac{1}{2}} = d$ the diameter in inches. That is, the fourth root of half the stress in cwts. multiplied by the square root of the length in feet is equal to the diameter in inches.

262. If a cylindrical shaft has no other lateral stress to sustain than its own weight, then by making the proper substitutions, the rule becomes $\sqrt{\cdot 007\, l^3} = d$, the diameter in inches.

That is, multiply the cube of the length in feet by ·007; and the square root of this product is the diameter in inches.

The rule now stated may be easily applied to any case with the advantage of both accuracy and simplicity, because it enables us in every instance to include the effect of the weight of the shaft itself; when it is computed in a tabular form. Let the stress, supposed to be at the middle, be n times the weight of the shaft, then $\sqrt{\cdot 012 l^3 n} = d$ in inches.

263. *Table of Shafts of Cast Iron to resist lateral Pressure.*

Length in feet.	Diameter in inches.	Diameter in inches.	Diameter in inches.	Diameter in inches.	Diameter in inches.
2	·237	·31	·44	·54	·62
4	·67	·88	1·24	1·5	1·76
6	1·23	1·61	2·28	2·79	3·22
8	1·9	2·48	3·51	4·30	4·96
10	2·65	3·47	4·9	6·0	6·93
12	3·48	4·55	6·44	7·89	9·10
14	4·38	5·74	8·12	9·94	11·48
16	5·36	7·01	9·92	12·15	14·02
	Own weight only.	Stress equal to its own weight, or $n=1$.	Stress double its own weight, or $n=2$.	Stress three times its own weight, or $n=3$.	Stress four times its own weight, or $n=4$.

HOLLOW CYLINDRICAL SHAFTS OF CAST IRON.

264. To compute the diameter of a shaft when it is to be a hollow cylinder, it is necessary to assume the ratio between the diameter of the hollow part and that of the exterior of the shaft, in order to avoid a complicated rule. Let D be the exterior diameter, and N D the interior one, then W being in cwts. the rule will be $\left(\frac{W l^2}{2(1-N^4)}\right)^{\frac{1}{4}} = D$ the diameter in inches. But if the shaft supports n times its own weight, then $\sqrt{\frac{\cdot 012\, l^3 n}{1+N^2}} = D$, when the necessary sub-

stitutions are made. If however some convenient number be fixed upon for N, the rule may be still further simplified. And I think one that will be well adapted for practice is to make the hollow part six-tenths of the exterior diameter. According to this proportion, the rule is $\sqrt{\cdot 009\, l^3 n} = \text{D}$. That is, the cube of the length in feet, multiplied by ·009, and also by the number of times the weight of the shaft is contained in the stress, then the square root of this product is the diameter in inches.

The weight of the shaft in lbs. will be very nearly equal to the square of the exterior diameter in inches, multiplied by 1·6 times the length in feet.

265. *Table of Hollow Shafts of Cast Iron to resist lateral Stress.*

Length.	Exterior diameter in inches.	Interior diameter in inches.	Exterior diameter in inches.	Interior diameter in inches.	Exterior diameter in inches.	Interior diameter in inches.	Exterior diameter in inches.	Interior diameter in inches.
4	1·5	0·9	1·9	1·1	2·2	1·3	2·4	1·4
6	2·8	1·6	3·5	2·1	4·0	2·4	4·5	2·7
8	4·3	2·5	5·3	3·1	6·1	3·6	6·9	4·1
10	6·0	3·6	7·4	4·4	8·5	5·	9·5	5·7
12	7·9	4·7	9·8	5·8	11·2	6·7	12·6	7·5
14	10·0	6·0	12·3	7·3	14·2	8·5	15·9	9·5
16	12·2	7·3	15·0	9·0	17·3	10·3	19·4	11·6
	Stress four times the weight of the shaft.		Stress six times the weight of the shaft.		Stress eight times the weight of the shaft.		Stress ten times the weight of the shaft.	

It will be necessary to refer to the tables of resistance to torsion, previous to fixing on the diameter of a shaft.

WROUGHT IRON SHAFTS TO RESIST LATERAL STRESS.

266. If the diameter of the shaft be calculated for cast iron, and this diameter be multiplied by ·935, the product will be the diameter of a wrought iron shaft of equal stiffness.

For in order that shafts of these metals may be equally stiff, the stiffness of wrought being 1·3, when that of cast iron is unity, (see Art. 162, note,) $c^4 \times 1 = w^4 \times 1{\cdot}3$ or $c \times {\cdot}935 = w$; where c is the diameter of the cast iron shaft, and w that of the wrought iron one.

WOODEN SHAFTS.

267. Suppose a cast iron shaft five inches square at the point of greatest lateral stress—Required the size it should be when made of oak to have the same strength?

The cube of 5 is 125.

Cast iron is four times stronger than oak, (see Banks on Powers of Machines, &c., p. 94,) therefore $125 \times 4 = 500$.

The cube root of 500 is 7·93 inches.

268. Shafts should be compared by their stiffness, or their resistance to torsion; but here the question is the comparative stiffness. Now the stiffness of good oak is to that of cast iron as $\frac{1}{11{\cdot}2} : 1$ (Essay on Cast Iron, Art. Oak.) Therefore $c^4 \times 1 = o^4 \times \frac{1}{11{\cdot}2}$ when the shafts are of equal stiffness; hence $1{\cdot}83\,c = o$; where c is the diameter of the cast iron shaft, and o that of the oak one. And consequently an oak shaft should be 1·83 times the diameter of a cast iron one to be equally stiff.

And the stiffness of square shafts being as the fourth powers of the sides of the shafts, the side of an oak shaft should be 1·83 times that of an iron one to be of equal stiffness.

269. Required the size of a fir shaft to have the same strength as one of cast iron?

The cube of 5 is 125.

Cast iron—5·5 times stronger than fir, (Banks,) therefore $125 \times 5{\cdot}5 = 687{\cdot}5$; the cube root of $687{\cdot}5$ is equal to $8{\cdot}82$.—*Ans.*

270. The stiffness of red or yellow fir is to that of cast iron as $\frac{3}{26} : 1$ (Essay on Cast Iron, Art. Fir.) Therefore c being the diameter of a cast iron shaft, and f that of a fir one to resist the same stress, $c^4 \times 1 = f^4 \times \frac{3}{26}$, or $c \times \left(\frac{26}{3}\right)^{\frac{1}{4}} = f$, that is $1{\cdot}716\, c = f$. Whence it appears that a shaft of yellow fir should be 1·716 times the diameter of a cast iron one, to resist the same stress.

Shafts that are square should be in the same ratio.

271. In the same manner we might examine various kinds of cylindrical shafts, but what has been already said will suffice. In order to shew, however, that considerable allowance should be made for accidental stress, we may mention the following fact which lately occurred.

272. The hollow shaft of a water-wheel, in consequence of one of the gudgeons getting loose, broke quite through near one end, although it was 16 inches diameter, and two inches thick in the shell. (See Art. 216, 1st table of gudgeons.) The gudgeons of this shaft were not secured by flanches.

273. Hollow cylindrical shafts are often made of equal size throughout, in order to get large flanches, the better to secure the gudgeon.

SHAFTS SUBJECT TO TORSION.

274. We saw (Art. 254) that the length of a cast iron shaft has no influence on its resistance to torsion, and we have already said all that seems to be necessary respecting them. The case is, however, different with regard to wooden shafts; but we are yet in want of facts to reduce

the influence which their length may have, to calculation. As it may give some idea of this influence, we may state one fact which came under Buchanan's observation.

A shaft about 15 feet long, made of fir, had cast iron journals $2\frac{9}{10}$ diameter; one of the journals broke from fair stress, after working about 16 years; for a considerable time before it broke, the resistance was equal to 7 horses' power, making $11\frac{1}{2}$ revolutions per minute. It gradually wore until it broke; when that happened, the shaft seemed strained to the utmost, so that it might be reckoned as just equal in strength to the gudgeon; the shaft was $9\frac{3}{4}$ inches square.

The cube of $9\frac{3}{4}$ is 926,

926 divided by 5·5 (Art. 269,) is 16·8;

Cube root of 16·8 is 2·6. That is, the fir shaft would be equal to resist the same *lateral stress* as a square cast iron shaft 2·6 on the side.

The cross-tailed gudgeons of wooden shafts commonly require them to be made sufficiently large to withstand the stress which is brought upon them; often indeed, formerly, they were much weakened by mortices cut through them for inserting the arms of wheels; this practice is now, however, almost entirely abandoned.

275. The power of a shaft to resist torsion, has been calculated in comparing the resistance to torsion with that to lateral stress, in Art. 255, therefore it only remains to apply the equation.

OF CYLINDRICAL SHAFTS OF CAST IRON TO RESIST TORSION.

We have found the equation expressing the relation between the stress and strain to be $\frac{3{\cdot}78\,\text{H}}{\text{N}} = d^3$, the diameter in inches. But it will be accurate enough for our purpose to increase the constant multiplier to 4, in order to render the computation easier; with this change, when N is the revolutions per minute, we have as a practical rule $\frac{60 \times 4\,\text{H}}{\text{N}} = d^3$, or $\left(\frac{240\,\text{H}}{\text{N}}\right)^{\frac{1}{3}} = d$. That is, divide 240 times the number of horses' power by the number of revolutions per minute, and the cube root of the quotient will be the diameter of the shaft in inches.

But the reader must remember that when, in this or any other case, he represents the power of the first mover by a certain number of horses, he must be certain to make an ample allowance for any temporary increase of the action of the moving power.

The following table is calculated by the above rule, and it will be found useful to compare with the table, Art. 260.

276. *Table of Cylindrical Shafts of Cast Iron to resist Torsion.*

Diameter of shafts in inches.	REVOLUTIONS OF THE SHAFTS IN A MINUTE.					
	5 rev.	10 rev.	20 rev.	30 rev.	40 rev.	50 rev.
	Horses' power.	Horses' power.	Horses' power.	Horses' power.	Horses' power.	Horses' power.
2	0·17	0·33	0·66	0·99	1·33	1·66
3	0·56	1·13	2·25	3·37	4·5	5·62
4	1·33	2·66	5·33	7·99	10·66	13·33
5	2·6	5·2	10·4	15·6	20·8	26·0
6	4·5	9·0	18·00	27·0	36·0	45·0
7	7·15	14·3	28·6	42·9	57·2	71·5
8	10·66	21·33	42·66	64·0	85·0	106·6
10	20·83	41·66	83·33	125·0	166·0	208·3
12	36·00	72·00	144·0	216·0	288·0	360·0
14	63·83	127·66	255·33	383·0	510·0	638·3
16	85·33	170·66	341·33	512·0	682·0	853·3

277. The same table will serve for hollow cylindrical shafts to resist torsion, if the diameter be multiplied by 1·05, and the diameter of the hollow part be six-tenths of

the exterior diameter; for in that case $\left(\frac{1}{1-n^4}\right)^{\frac{1}{3}} = 1{\cdot}05$. (See Art. 254, note.)

		Inches.	Inches.	Inches.	Inches.	Inches.
Shafts of solid cylinders;	diameters	8·	10·	12·	14·	16·
Hollow shafts of equal strength;	exterior diameter	8·4	10·5	12·6	14·7	16·8
	interior diameter	5·	6·3	7·5	8·8	10·

278. This table applies to vertical shafts, but in horizontal ones there is an additional stress if it be only from their own weight, and much more should there be wheels on the shaft. Where the lateral stress is small, it may be allowed for by adding something to the diameter shown by the table, or it may be calculated by the rule at the end of this article.

Example.—Suppose a vertical shaft is to make 20 revolutions per minute, the power of the first mover being equal to 18 horses. Look in the column of horses' power under 20 revolutions; and opposite 18, the diameter will be found in the first column to be 6 inches, for cast iron.

279. If the shaft is to be of wrought iron, then multiply the diameter found by the rule or the table, by 0·963. (Art. 225, note.) Thus, in the above example, 6 × 0·963 = 5·778 inches, the diameter of a wrought iron shaft to make 20 revolutions per minute, when the power of the first mover is equal to 18 horses.

280. If the shaft be of oak, then the power of oak being $\frac{1}{11{\cdot}2}$ when that of cast iron is 1[*], (Art. 268,) and the resistance to torsion being as the cube of the diameter, we have $(11{\cdot}2)^{\frac{1}{3}} = 2{\cdot}238$; and multiply the diameter found by the rule or table for cast iron shafts by 2·238, and it will be the diameter for an oak shaft. Thus in the preceding example, 6 × 2·238 = 13·428 inches for the dia-

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[*] The relative stiffness is used instead of the relative strength, to reduce the quantity of torsion in wooden shafts.

meter of an oak shaft to make 20 revolutions per minute, the first mover being equal to 18 horses.

281. When fir is to be used for a shaft, its diameter should be $\left(\frac{26}{3}\right)^{\frac{1}{3}}$ times that of a cast iron one for the same purpose; but $\left(\frac{26}{3}\right)^{\frac{1}{3}} = 2\cdot06$ nearly; therefore it should be 2·06 times the diameter of the cast iron one.

Example.—Let the moving force be equal to 7 horses, and the number of turns per minute 11½, (see the case cited in Art. 274,) then by the rule $\frac{7 \times 240}{11\cdot5} = 146\cdot09$, and the cube root of 146·09 is 5·267 nearly; or practically 5·3 inches should be the diameter of the shaft were it of cast iron. And $2\cdot06 \times 5\cdot3 = 10\cdot918$ inches, or nearly 11 inches for the diameter of a fir shaft. It seems that a shaft of 9¾ inches square, of fir, was found equal to the strain; and one 11 inches diameter is at least $\frac{1}{18}$ stronger. We have made these calculations directly from the theory of equal cohesion, but it is so well known a fact that the lateral cohesion of fir is vastly inferior to the direct cohesion, that in the rule for fir shafts to resist torsion, an increase of diameter should be allowed by considering the number of horses' power about ⅓ more than it is intended to be; at least, till experiment shall have given the precise effect of lateral cohesion in decreasing the force of shafts to resist torsion.

282. If a shaft have to sustain both lateral stress and torsion, then the sum of the straining forces must be taken; and hence by Art. 261, and 274, we have $\frac{240\text{N}}{\text{N}} + \frac{\text{W}l^2}{2d} = d^3$. But in this equation it is difficult to calculate the value of the diameter, as it is what algebraists call an equation of the fourth degree. This difficulty may however be easily avoided by considering $2d$ to be 2 only, for then the error

will always be in excess, except when d is less than unity; and it is much better to be in excess than defect. Consequently we have as a practical rule $\left(\frac{240\,\text{H}}{\text{N}} + \frac{\text{W}\,l^2}{2}\right)^{\frac{1}{3}} = d$, the diameter of the shaft in inches, when of cast iron. Where l is the length in feet between the bearings, H the number of horses which are equal the power of the first mover, N the number of revolutions to be made by the shaft in a minute, and W the lateral stress in cwts.

EXAMPLE.—Suppose that a cylindrical shaft of cast iron is to make 34 revolutions per minute, the power of the first mover being equal to 3 horses, the length of the shaft 8 feet, and the lateral stress 3 cwts., when reduced to the middle point, then $\left(\frac{240 \times 3}{34} + \frac{3 \times 8^2}{2}\right)^{\frac{1}{3}} = (21{\cdot}18 + 96)^{\frac{1}{3}} = (117{\cdot}18)^{\frac{1}{3}} = 4{\cdot}893$ inches.

283. With regard to the making of patterns of cast iron shafts, the reader is referred to what has been said in the first Essay, relative to the making of patterns for cast iron wheels, which is, in a great measure, applicable to those of shafts. Nor is there any thing on this subject to be added here; except to remind the millwright that he make the allowance for contraction of metal, of one-eighth of an inch to the foot in the pattern.

284. The following table contains the dimensions of shafts subject to torsion, and to considerable lateral pressure, as they were executed by a respectable millwright. It will serve to shew the sizes of the parts as found in practice sufficiently strong, and may be found useful to compare with those which would be produced, calculating on the principles laid down in this Essay.

Column 6th, therefore, shews the diameters which these journals ought to have, were 400 used as the multiplier. (See Art. 233.)

TABLE OF SHAFTS.

Name.	1 Horses' power.	2 Revolutions per minute.	3 Diameter of journals in inches.	4 Length of shaft.	5 Section in the middle.	6 Diameters of journals by calculating from a multiplier of 400.	Remarks.
Cast iron lying shaft.	20	20	6	11·	7¾	7,368	Feathered shafts.
	18	22	5¾	11·	7	6,889	
	16	22	5¼	10·6	7	6,621	
	14	24	5	10·	6¾	6,153	
	12	25	5	9·6	6	5,768	
	10	25	4¾	9·	6¼	5,428	
	8	27	4	9·	6	4,904	
	6	28	4¼	8·6	5¾	4,414	
	5	30	3⅞	8·6	5¼	4,061	
	4	32	3¾	8·	4⅝	3,484	
	3	34	3¼	8·	4	1,203	Square shafts.
	2	46	2¾	8·	3	2,802	
	1	40	2	8·	3¼	2,154	
Malleable iron lying shaft.	6	28	3	8·6			
	5	30	3¼	8·			
	4	32	2	8·			
	3	34	2	8·			
	2	36	1¾	8·			
	1	40	1	7·6			

APPENDIX.

COHESIVE STRENGTH OF DIFFERENT METALS.

285. “We shall take for the measure of cohesion the number of pounds avoirdupois which are just sufficient to tear asunder a rod or bundle of one inch square. From this it will be easy to compute the strength corresponding to any other dimension.

“ 1st, Metals.

		lbs.
“ Gold cast		20,000 24,000
Silver cast		40,000 43,000
Copper cast	Japan	19,500
	Barbary	22,000
	Hungary	31,000
	Anglesea	34,000
	Sweden	37,000
Iron cast		42,000 59,000

		lbs.
Iron bar	Ordinary	68,000
	Stirian	75,000
	Best Swedish and Russian	84,000
	Horse nails	71,000
Steel bar	Soft	120,000
	Razor temper	150,000
Tin cast	Malacca	3,100
	Banca	3,600
	Block	3,800
	English block	5,200
	——— grain	6,500
Lead cast		860
Regulus of Antimony		1,000
Zinc		2,600
Bismuth		2,900

" The only author who has put it in our power to judge of the propriety of his experiments is Muschenbroëk. He has described his method of trial minutely, and it seems unexceptionable. The woods were all formed into slips fit for his apparatus, and part of the slip was cut away to a parallelopiped of $\frac{1}{5}$th of an inch square, and therefore $\frac{1}{25}$th of a square inch in section. The absolute strengths of a square inch were as follow.

	lbs.		lbs.
Locust tree	20,100	Pomegranate	9,750
Jujeb	18,500	Lemon	9,250
Beech, Oak	17,300	Tamarind	8,750
Orange	15,500	Fir	8,330
Alder	13,900	Walnut	8,130
Elm	13,200	Pitch Pine	7,640
Mulberry	12,500	Quince	6,750
Willow	12,500	Cypress	6,000
Ash	12,000	Poplar	5,500
Plum	11,800	Cedar	4,880
Elder	10,000		

"Muschenbroëk has given a very minute detail of experiments on the ash and the walnut, stating the weights which were required to tear asunder slips taken from the four sides of the tree, and on each side in a regular progression from the centre to the circumference. The numbers of this table corresponding to these two timbers may therefore be considered as the average of more than 50 trials made of each, and he says that all the others were made with the same care. We cannot therefore see any reason for not confiding in the results; yet they are considerably higher than those given by some other writers. Pitot says, on the authority of his own experiments, and of those of Parent, that 60 pounds will just tear asunder a square line of sound oak, and that it will bear 50 with safety. This gives 8640 for the utmost strength of a square inch, which is much inferior to Muschenbroëk's valuation.

"We may add to these—

	cwt.
Ivory	16,280
Bone	5,250
Horn	8,750
Whalebone	7,500
Tooth of sea calf	4,075

"The reader will surely observe that these numbers express something more than the utmost cohesion, for the weights are such as will very quickly, that is, in a minute or two, tear the rods asunder. It may be said in general that two thirds of these weights will sensibly impair the strength after a considerable while; and that one half is the utmost that can remain suspended at them, without risk, for ever; and it is this last allotment that the engineer should reckon upon in his constructions. There is however a considerable difference in this respect. Woods of a very straight fibre, such as fir, will be less impaired by any load which is not sufficient to break them immediately.

"According to Emerson, the load which may be safely suspended to an inch square is as follows:

	lbs.
Iron	76,400
Brass	35,600
Hempen Rope	19,600
Ivory	15,700
Oak, box, yew, plum-tree	7,850
Elm, ash, beech	6,070
Walnut, plum	5,360
Red fir, holly, elder, plane, crab	5,000
Cherry, hazle	4,760
Alder, asp, birch, willows	4,290
Lead	430
Freestone	91

"He gives us a practical rule, that a cylinder whose diameter is *d* inches, loaded to one fourth of its absolute strength, will carry as follows:

	cwt.
Iron	135
Good rope	22
Oak	14
Fir	9

"The rank which the different woods hold in this list of Emerson's is very different from what we find in Muschenbroëk's. But precise measures must not be expected in this matter. It is wonderful that in a matter of such unquestionable importance the public has not enabled some persons of judgment to make proper trials."

286. Mr. Banks (Powers of Machines, &c., p. 94) takes iron at an average to be four times as strong as oak, and 5½ times as strong as deal or fir.

287. "According to the experiments of various authors, the cohesive strength of a square inch of razor steel is about 150 thousand pounds, of soft steel 120, of wrought iron 80, of cast iron 50, of good rope 20, of oak, beech, and willow wood, in the direction of their fibres 12, of fir 8, and of lead about three thousand pounds; the cohesive strength of a square inch of brick 300, and of freestone 200; teak wood, the tectona grandis, is said to be still stronger than oak.

"The strength of different materials in resisting compression, is liable to great variation. In steel and in willow wood, the cohesive and repulsive strength appear to be nearly equal. Oak will suspend much more than fir, but fir will support twice as much as oak, probably on account of the curvature of the fibres of oak. Freestone has been found to support about 2000 pounds for each square inch; oak, in some practical cases, more than 4000.

"The strongest wood of each tree is neither at the centre nor at the circumference, but in the middle between both; and in Europe it is generally thicker and firmer on the south-east side of the tree. Although iron is much stronger than wood, yet it is more liable to accidental imperfections; and when it fails, it gives no warning of its approaching fracture. The equable equality of steel may be ascertained by corrosion in an acid, but there is no easy mode of detecting internal flaws in a bar of iron, and we can only rely on the honesty of the workmen for its soundness. Wood, when it is crippled, complains, or emits a sound, and after this, although it is much weakened, it may still retain strength enough to be of service."*

288. The cohesive force of metals has been examined by several experimental inquirers, besides those noticed in the extracts made by our author, and our knowledge of

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* Young's Nat. Phil. vol. i. p. 151.

this subject has been recently extended very considerably by the experiments of Telford, Brown, and others. Tredgold collected all the most important experiments on the cohesive force of metals, (Phil. Mag. vol. l. p. 421,) and omitting those which are given in the preceding articles, the table is here re-arranged by him.

TABLE OF EXPERIMENTS ON THE DIRECT COHESION OF METALS.

Description of metal.	Force (in lbs.) that would tear asunder a bar of one inch square.	Experimentalist.	Quoted from
I. STEEL.			
Cast steel, previously tilted.	134,256	Rennie.	Phil. Mag. vol. liii. p. 167.
Cast steel	63,065	Brown.	Barlow's Essay, p. 234.
Blister steel, reduced by the hammer.	133,152	Rennie.	Phil. Mag. vol. liii. p. 167.
Blister steel	32,973	Brown.	Barlow's Essay, p. 234.
Shear steel, reduced by the hammer.	127,632	Rennie.	Phil. Mag. vol. liii. p. 167.
II. MALLEABLE IRON.			
Iron wire	113,077	Sickingen.	Ann. de Chimie, xxv. 9.
Iron wire	93,964	Telford.	Barlow's Essay, p. 222.
Iron wire	85,797	Buffon.	Œuvres de Gauthey, ii. 153.
German bar, mark BR, highest result.	93,069	Muschenbroëk.	Intro. ad Phil. Nat. i. 426.
Swedish bar, highest result.	88,972	Idem.	
German bar, mark L, highest result.	85,900	Idem.	
Liege bar, highest result.	82,839	Idem.	
Spanish bar	81,901	Idem.	
Bar................	80,833	Soufflot.	Rondelet's L'Art. de Bâtir, iv. 500.

TABLE CONTINUED.

Description of metal.	Force (in lbs.) that would tear asunder a bar of one inch square.	Experimentalist.	Quoted from
Oosement bar, highest result.	76,697	Muschenbroëk.	Intro. ad Phil. Nat. i. 426.
Swedish bar, reduced by the hammer.	72,064	Rennie.	Phil. Mag. vol. liii. p. 167.
Common round iron.	71,300	Telford.	Barlow's Essay, p. 230.
German bar, mark L.	69,538	Muschenbroëk.	Intro. ad Phil. Nat. i. 426.
Common Staffordshire bar.	69,440	Telford.	Barlow's Essay, p. 230.
Common German bar.	69,133	Muschenbroëk.	Intro. ad Phil. Nat. i. 426.
Swedish bar	68,728	Idem.	
Oosement bar, the same.			
Welsh bar.	66,752	Telford.	Barlow's Essay, p. 228.
Bar of the best quality.	66,000	Rumford.	Phil. Mag. x. 51.
A bar of Welsh, one of Swedish*, and one faggoted scrap Iron, each gave a result of	64,960	Telford.	Barlow's Essay, p. 229.
Liege bar..........	62,369	Muschenbroëk.	Intro. ad Phil. Nat. i. 426.
Staffordshire bar .	61,600	Telford.	Barlow's Essay, p. 229.
German bar, mark B R.	61,361	Muschenbroëk.	Intro. ad. Phil. Nat. i. 426.
Bar (mean of 33 experiments).	61,041	Perronet.	Œuvres de Gauthey, ii. 154.
Russian old sable, mark CCN.	59,472	Brown.	Barlow's Essay, p. 233.
English bar, reduced by the hammer.	55,872	Rennie.	Phil. Mag. vol. liii. p. 167.
Welsh bar (3 experiments).	55,776	Brown.	Barlow's Essay, p. 233.
Bar of good quality.	55,000	Rumford.	Phil. Mag. vol. x. p. 51.
Swedish bar, (3 experiments).	53,244	Brown.	Barlow's Essay, p. 232.

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* The Swedish bar broke at a flaw.

TABLE CONTINUED.

Description of metal.	Force (in lbs.) that would tear asunder a bar of one inch square.	Experimentalist.	Quoted from
Bar (fine grain)..	49,982	Rondelet.	L'Art de Bâtir, iv. 502.
—— (medium fineness.)	34,081	Idem.	
—— (coarse grained.)	20,460	Idem.	
III. CAST IRON.			
Bar, spec. grav. 7·807.	68,295	Muschenbroëk.	Intro. ad Phil. Nat. i. 417.
Bar, cast vertically.	19,488	Rennie.	Phil. Mag. vol. liii. p. 167.
Bar, cast horizontally.	18,656	Rennie.	Idem.
Bar, Welsh pig...	16,264	Brown.	Barlow's Essay. p. 235.
IV. COPPER.			
Wire..............	61,228	Sickingen.	Ann. de Chimie, xxv. 9.
Wrought copper, reduced by the hammer.	33,792	Rennie.	Phil. Mag. vol. liii. p. 167.
Cast, Barbary, spec. grav. 8·182.	22,570	Muschenbroëk.	Intro. ad Phil. Nat. i. 417.
Cast, Japan, spec. grav. 8·726.	20,272	Idem.	
Cast	19,072	Rennie.	Phil. Mag. vol. liii. p. 167.
V. PLATINUM.			
Platinum wire, specific gravity 20·847.	56,473	Morveau.	Ann. de Chimie, xxv. 8.
Platinum wire ...	52,987	Sickingen.	Idem, p. 9.
VI. SILVER.			
Silver wire.	38,257	Sickingen.	Ann. de Chimie, xxv. 9.
—— cast, spec. grav. 11·091.	40,902	Muschenbroëk.	Intro. ad Phil. Nat. i. 417.
VII. GOLD.			
Gold wire.........	30,888	Sickingen.	Ann. de Chimie, xxv. 9.

TABLE CONTINUED.

Description of metal.	Force (in lbs.) that would tear asunder a bar of one inch square.	Experimentalist.	Quoted from
Gold cast, spec. grav. 19·238.	20,450	Muschenbroëk.	Intro. ad Phil. Nat. i. 417.
VIII. ZINC.			
Zinc wire	22,551	Morveau.	Ann. de Chimie, lxxi. 194.
—— sheet.........	16,600	Tredgold.	Phil. Mag. vol. l. p. 422.
—— cast.	2,689	Muschenbroëk.	Intro. ad Phil. Nat. i. 407.
IX. TIN.			
Tin wire...........	7,129	Morveau.	Ann. de Chimie, lxxi. 194.
English block, cast.	6,650	Muschenbroëk.	Intro. ad Phil. Nat. i. 417.
——— spec. grav. 7·295.	5,322	Idem.	
Cast	4,736	Rennie.	Phil. Mag. vol. liii. p. 167.
Banca tin, cast, specific gravity 7·2165.	3,679	Muschenbroëk.	Intro. ad Phil. Nat. i. 417.
Malacca tin, cast, specific gravity 6·1256.	3,211	Idem.	
X. LEAD.			
Milled sheet, spec. grav. 11·407.	3,328	Tredgold.	Phil. Mag. vol. l. p. 422.
Wire.	3,146	Muschenbroëk.	Intro. ad Phil. Nat. i. 452.
Wire, spec. grav. 11·282.	2,581	Idem.	
Wire.	2,547	Morveau.	Ann. de Chimie, lxxi. 194.
Cast lead.	1,824	Rennie.	Phil. Mag. vol. liii. p. 167.
—— English, spec. grav. 11·479.	885	Muschenbroëk.	Intro. ad Phil. Nat. i. 452.
XI. BISMUTH.			
Bismuth, cast, spec. grav. 9·810.	3,250	Muschenbroëk.	Intro. ad Phil. Nat. i. 417.
——— spec. grav. 9·926.	3,008	Idem.	
XII. ANTIMONY.			
Antimony, cast, spec. grav. 4·500.	1,060	Muschenbroëk.	Intro. ad Phil. Nat. i. 417.

As this table, the most extensive of the kind, exhibits at one view the chief results of the experiments on the direct cohesion of the metals in English avoirdupois pounds when the area is a superficial inch, as well as references to the works wherein those experiments are described, it will be useful to direct the labours of future inquirers to such experiments as are best adapted to increase or correct our knowledge on this subject. It was collected at various times for Buchanan's information, and will be equally useful to others. When experiments are not reduced to a common standard, they cannot be compared without much labour: in the original descriptions of these experiments, this has not been done; they are described chiefly as they were made, and for further information, to these descriptions the reader must be referred. If he be interested in these researches, the works of Muschenbroëk, Rondelet, Barlow, Tredgold, and others, will afford him much information.

289. From the simple metals we naturally look to the alloys, some of which are of much importance. Here the curious but important fact that the union of two metals produces a compound of greater tenacity than either of the metals it is formed of, will be noticed.

TABLE OF EXPERIMENTS ON THE DIRECT COHESION OF ALLOYS.

Alloy of		Force (in lbs.) that would tear asunder a bar of one inch square.	Specific gravity of the alloy.	Experimentalist.
Parts.	Parts.			
Copper 10	Tin 1	32,093	8·351	Muschenbroëk, Intro. ad Phil. Nat.
Ditto 8	ditto 1	36,088	8·392	Idem.
Ditto 6	ditto 1	44,071	8·707	Idem.
Ditto 4	ditto 1	35,739	8·723	Idem.
Ditto 2	ditto 1	1,017	....	Idem.
Gun metal, hard	....	36,368	....	Rennie, Phil. Trans.
Brass, fine yellow	....	17,968	....	Idem.
Tin, English.. 10	Lead 1	6,904	....	Muschenbroëk.
Ditto 8	ditto 1	7,922	....	Idem.
Ditto 6	ditto 1	7,997	....	Idem.
Ditto 4	ditto 1	10.607	....	Idem.
Ditto 2	ditto 1	7,470	....	Idem.
Ditto 1	ditto 1	7,074	....	Idem.
Tin, Banca... 10	Antimony 1	11,181	7·359	Muschenbroëk.
Ditto 8	ditto 1	9,881	7·276	Idem.
Ditto 6	ditto 1	12,632	7·228	Idem.
Ditto 4	ditto 1	13,480	7·192	Idem.
Ditto 2	ditto 1	12,029	7·105	Idem.
Ditto 1	ditto 1	3,184	7·060	Idem.
Tin, Banca... 10	Bismuth 1	12,688	7·576	Muschenbroëk.
Ditto 4	ditto 1	16,692	7·613	Idem.
Ditto 2	ditto 1	14,017	8·076	Idem.
Ditto 1	ditto 1	12,020	8·146	Idem.
Ditto 1	ditto 2	10,013	8·580	Idem.
Ditto 1	ditto 4	7,875	9·009	Idem.
Tin, Banca... 10	Zinc, Indian 1	12,914	7·288	Muschenbroëk.
Ditto 2	ditto 1	15,025	7·000	Idem.
Ditto 1	ditto 1	15,844	7·321	Idem.
Ditto 1	ditto 2	16,023	7·100	Idem.
Ditto 1	ditto 10	5,671	7·130	Idem.
Tin, English.. 8	Zinc, Goslar 1	10,607	....	Muschenbroëk.
Ditto 4	ditto 1	10,258	....	Idem.
Ditto 2	ditto 1	10,964	....	Idem.
Ditto 1	ditto 1	9,024	....	Idem.
Tin, English.. 1	Antimony 1	1,450	7·000	Muschenbroëk.
Ditto 3	ditto 2	3,184	....	Idem.
Ditto 4	ditto 1	11,343	....	Idem.
Lead, Scotch 1	Bismuth 1	7,319	10·931	Muschenbroëk.
Ditto 2	ditto 1	5,840	11·090	Idem.
Ditto 10	ditto 1	2,826	10·827	Idem.

Brass is an alloy of copper and zinc, gun metal is an alloy of copper and tin, sometimes in the proportion of 96 parts of copper to 11 parts of tin, but perhaps more usually 108 parts of copper to 11 parts of tin*. It will be seen by this table that the proportion of six of copper to one of tin, is the most tenacious compound. A proportion very near to this is used for bearings, bushes, and some purposes in machinery; but it is too hard and brittle for many uses. It is worthy of remark, that copper and tin are soft and malleable metals, but when combined, they form a tenacious, brittle, and hard alloy. Both the hardness and brittleness is increased by augmenting the proportion of tin.

Tables of the cohesive force of woods of various kinds may be seen in Tredgold's Elementary Principles of Carpentry, Sect. II.; also in Muschenbroëk's work above quoted, or in Barlow's Essay on the Strength of Timber.

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* These numbers give the nearest chemical proportions to those in use among founders. For further information on this subject, see the Art. Brass, Supplement to Ency. Brit.

ESSAY III.

ON THE CONSTRUCTION AND DURABILITY OF THE LONGITUDINAL CONNEXIONS OF SHAFTS, DENOMINATED COUPLINGS.

PREFACE.

HAVING treated of Wheels and Shafts, both of which may be considered as essential parts of mill-work, the next subject in point of order is *the means of connecting shafts longitudinally*, denominated couplings, which accordingly forms the subject of the present essay.

In examining this subject, there have been collected and described a number of different methods which have been employed in the coupling of shafts. These methods are arranged under two classes; practical observations are made on each coupling. These observations are the result of Buchanan's great experience, and that of many others well acquainted with the subject, with whom he had taken many opportunities of conversing; nor will these observations be altogether without benefit, should they only lead practical men to make others more extensive, judicious, and useful.

A number of facts relative to the subject are also stated, which will not be without advantage. For however useful abstract reasoning may be, yet a theory excluding some apparently trivial or minute circumstances, is often rendered altogether uncertain in its application to practice. Of this we have remarkable instances in calculations made not many years ago by some of the most eminent philosophers then in Europe, relative to the motion of water in canals and pipes. The Academy of Sciences at Paris overestimated the quantity of water to be delivered by an aqueduct so far, that it was, when executed with the greatest care, found to be deficient in the proportion of five to nine. Desaguliers made an error of five parts out of six; and the celebrated M'Laurin of ten parts out of eleven, in estimating the water to be conveyed for supplying the city of Edinburgh*.

Smeaton, who was certainly well able to appreciate science, yet seemed to place more value on the writings of practical men than those merely of a theoretical nature, for he says, "I have myself always found that exact accounts of buildings [*and of course, of other such works*] which were in any degree remarkable, and actually executed, were much more instructive to my mind than *systematical* writing."† The professional learning of engineers is now better cultivated, and they are not required to gather the chief part of their instruction from the practical experiments of predecessors. No doubt different minds require different modes of instruction; nevertheless, a systematic course of study seems to be vastly preferable to a desultory course, and also to have been preferred by all teachers either of art or science. But men like Smeaton, advanced in years, and full of occupation, have seldom either inclination or leisure to follow a systematic course. They seek for information only when compelled by professional difficulties. They rely upon force of genius to supply the wants of the moment, as an Indian hunter on his exertion in the chase; and like him they have no idea of laying up a stock to provide for unforeseen exigencies. How different would be the powers of a man, of equal genius, with the advantage of a systematic course of study; where reasoning was joined with experiment! Might we not then look forward to a time when theory and the laws of nature would be merely different terms for the same thing? It is not however to be expected that this perfection will ever be attained while theory is confined to matter divested of its natural properties.

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* Professor Robison mentions these circumstances in the Ency. Brit. Art. River. This subject is now better understood. See Phil. Trans. Hydraulic Investigations, subservient to an intended Croonian Lecture on the Motion of the Blood, by Dr. Young. Read before the Royal Society, May 5, 1808.

† See Description of Eddystone Lighthouse, p. 7.

But while Buchanan spoke in favour of practical works, he did not depreciate those of science. They may be of great mutual benefit, and while we listen with reverence to the voice of experience*, by sound reasoning on her dictates, we may extend and apply them to purposes more various and useful than those to which they originally related.

"The man of science," says Dr. Robison, "who visits our great manufactures, is delighted with the ingenuity which he observes in every part, the innumerable inventions which come even from individual artisans, and the determined purpose of improvement and refinement, which he sees in every workshop. Every cotton-mill appears an academy of mechanical science; and mechanical invention is spreading from these fountains over the whole kingdom; but the philosopher is mortified to see this ardent spirit so cramped by ignorance of principle, and many of these original and brilliant thoughts obscured and clogged with needless and even hurtful additions, and a complication of machinery which checks improvement even by its appearance of ingenuity. There is nothing in which this want of scientific education, this ignorance of principle, is so frequently observed, as in the injudicious proportion of the parts of machines and other mechanical structures; proportions, and forms of parts, in which the strength and position are nowise regulated by the strains to which they are exposed, and where repeated failures have been the only lessons."

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* "Experience, slow preceptress, teaching oft
The way to glory by miscarriage foul."

COWPER.

CHAPTER I.

ON THE LONGITUDINAL CONNEXIONS OF SHAFTS, DENOMINATED COUPLINGS.

290. It is well known to those who are in any degree acquainted with mill-work, that it is very frequently necessary to convey motion much farther than would be practicable by any one shaft; it is therefore often requisite to connect two or more shafts together*. These connexions are denominated *couplings*, and may be divided into two classes; viz. 1st. Those having *two* bearings: 2nd. Those having *one* bearing. Couplings having two bearings are mentioned first, because they were long in use before those having *one* bearing, and because they are, generally speaking, more simple in their construction.

CLASS I.

OF COUPLINGS WITH TWO BEARINGS.

291. By the *bearings* of shafts are meant the parts which support their pivots, arbours, or journals. When a coupling has double bearings, each shaft is supported by two bridges, as represented A, B and C, D, Plate V. Fig. 1.

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* For, though it be most desirable that machinery should be concentrated as much as possible, long ranges of lying shafts are unavoidable in large mills, in cotton, linen, and woollen manufactories, breweries, &c., &c. Hence the efficient and durable connexion of these shafts is an object of considerable importance.

COUPLING I.—Fig. 1 and 2.

OF THE SQUARE COUPLING.

292. This kind of coupling is formed by making the ends of the shafts to be coupled, square. These squares project beyond the journals, and are in the spaces B C and D E, between the bridges. One of the squares is made as long as what is called the *coupling-box* F. The use of the coupling-box, which is made of iron, is to receive both the squares, so as that when the one shaft is moved, the box connects it with the other shaft in such a manner that they must move together. But when occasion requires, the box may be slipped back upon the longest square, and so give liberty to take out any one of the shafts, independently of the rest, however great the number may be. When the shafts are engaged, the box is kept in its place by the pin G.

The coupling at B C is represented as engaged, and at D E as disengaged.

Fig. 2. represents this kind of coupling upon a larger scale. The same letters refer to the same parts, as in Fig. 1.

Sometimes, instead of the section of the couplings forming a square, as Fig. 2, No. 2, it is made of an oblong form, as represented in Fig. 2, No. 3; but this form is more difficult of execution than the square. Also, instead of having the coupling-box solid, it is frequently made in two pieces, Fig. 2, No. 4, in which case it embraces the whole length of both squares, there being no occasion then for room to slide back the box in order to disengage the couplings.

OBSERVATIONS.

293. Were the axes* of these shafts truly in one straight

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* *Axis.*—The line, real or imaginary, that passes through any thing on which it may revolve.

line, and the squares made and fitted to the box with perfect accuracy, the motion would be perfectly smooth, but in large machinery this is almost impracticable, and even if practicable when new, would not long continue to be the case. The brasses wear unequally, or the framing sinks more in one place than another. In some part of each revolution, therefore, one or other, or both the shafts, will be lifted off their bearings. The two adjoining journals then come to act like one twisted piece of iron, and must obviously occasion an unsteady motion, and much friction. This imperfection is sometimes called a *lift*. This kind of coupling has, for these reasons, been in a great degree abandoned for mill-work. But in small machinery, such as in coupling the rollers of those machines for spinning cotton called *mules*, it is still used; because in that kind of machinery it can be executed with a very great degree of accuracy, and is not so liable to wear out of truth as in larger works.

294. Of this species of couplings with double bearings are many varieties, made according to the whim of different workmen; such, for instance, as is represented in Fig. 3, which has a small projection from each angle of the square, differing in no other respect from the common square coupling, and liable to nearly the same faults.

COUPLING II.—FIG. 4.

OF THE ROUND COUPLING.

295. The round coupling has the parts between the bridges cylindrical. The coupling-box C is made to fit those parts, and to slip backward when occasion requires, as was described of the square coupling. When the shafts are engaged, two bolts, D E, and F G, pass through the box at right angles to each other, and one of them through each

of the shafts; this being done, when the one shaft is moved, it will evidently carry round the other along with it.

OBSERVATIONS.

296. The effects of the round coupling are nearly the same as the square coupling, but as the parts may be all turned, it is more easily made true at first; and when the bolts which prevent it from twisting, wear, they may, with little trouble, be renewed; but, as the whole stress comes on a small surface at these bolts, both the bolts themselves and the holes very soon wear. For this reason, after this kind of coupling was some time tried for the rollers in cotton spinning, it was abandoned, and the square in which the strain is diffused over a greater surface, substituted.

COUPLING III.—Fig. 5.

OF CLUTCHES OR GLANDS.

297. Couplings which have no coupling-boxes, are denominated *clutches* or *glands*. They may, without impropriety, come under this class of couplings having *double bearings*.

Fig. 5. represents a coupling of this kind, it consists of two crosses, A A and B B, one fixed to each shaft; B B has its ends bent forward, and lays hold of A A, and thus turns round the other shaft.

OBSERVATIONS.

298. Glands are an excellent mode of coupling for *double bearings*, and have the advantage of throwing the stress further from the centre of motion, than in the square

coupling as commonly executed; but few workmen are able to execute glands with accuracy, and if this be not the case, they make a very disagreeable movement; it may therefore be not improper to describe how this may be accomplished.

299. When the axes of two journals are put as near to a straight line as possible, by observation from the workman's eye, it often happens, that they may be in a bad situation for working, either because the axes do not really correspond, or, even though they did correspond, yet the arms of the glands do not take hold both together, or perhaps from both causes. To adjust these arms, observe, in the *first* place, when the glands turn round, if one of the tails be in continual contact, and the other tail always preserve an equal distance from being in contact, in that case, the centres are perfectly opposite, and axes in one line, and consequently right; but if the distance of the other point of the glands vary in its distance, or becomes so irregular as to free the other point of the gland, then, in that case, the centres are wrong, and must be so adjusted, that one point of the gland bear equally all round, while the other preserves an equal distance. The *next* object is, to adjust the points of the glands so that they be both in contact. This may be done by chipping and filing, and in that case they will convey themselves all around in contact with each other.

COUPLING IV.—FIG. 6.

BORING MILL CLUTCH.

First Construction.

300. Fig. 6. represents a boring mill clutch; ABC is a round plate of cast iron firmly fixed on the shaft M, next the moving power; DE is a lever connected with the boring

shaft N, but which is moveable in one direction on a bolt at F, so that it may be moved to lay hold of the projections H, H, H, H, on the plate A B C, which carries round the lever D E along with it, and so moves the boring shaft N; by pulling the lever backward, the boring shaft N may be stopped at pleasure. It is represented in Fig. 6, No. 1, as disengaged.

OBSERVATIONS.

301. This kind of coupling is applicable to such cases only as have the motion very slow. Pressing on one side only of the centre, although the axes of the shafts should not be exactly on a line, there will be no lift. For the parts, in that case in contact, slide upon one another.

302. It is found, however, to have a great tendency to force the bridges O, P, on end. In order to lessen this tendency, the projections H, H, should be made as far from the centre as conveniency will admit.

COUPLING V.—FIG. 7.

BORING MILL CLUTCH.

Second Construction.

303. This coupling, like the first construction, has a lever D E, for disengaging and re-engaging; but instead of being hung immediately from the end of the shaft, turns on a bolt at F in a large cast iron plate I K L. The other parts having the same letters of reference as Fig. 6, resemble them, and are for the same use. There are three spare sets of ears, Q Q, &c. (which support the lever near the point of pressure) cast on the plate I K L, to be used in case of those in action breaking.

OBSERVATIONS.

304. The manner in which the lever D E is hung in the plate I K L, at a distance from the centre N, and is supported near the point of pressure by the ears Q Q, takes the stress entirely off the bolt F, and indeed off the lever D E, except near the ears Q Q.

305. This, therefore, is evidently a stronger and better clutch, and is accordingly used in boring the larger cylinders, whereas, that on the first construction is generally used for the smaller kinds of work.

COUPLING VI.—FIG. 8.

306. This coupling is constructed by having two round cast iron plates, A B and C D, of the same size and form. Each of these plates has a part F, cut out, *fghi*, No. 2, and a projection L, corresponding to the part cut out. The projection of the one plate is inserted into the opening in the other, and thus serves to engage the shafts *.

OBSERVATIONS.

307. This coupling is simple and durable. It is in fact a species of *glands*, (Art. 297.) It affords an excellent mode of adjustment. The pivot and hole for fixing the plates ought to be round, and the end of the shaft turned along with the journal. When the couplings are fitted into each other, care should be taken that those parts of the clutch which are opposite each other will take hold together.

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* A slight variation of this coupling has been found a very good one. It consists in making the circular heads toothed, so as to fit together; the teeth being wedge-shaped. This mode of coupling is not affected by any slight variation or settlement of the bearings, and it is very durable.

Shafts coupled upon this plan, when occasion requires, are easily moved out of their places, independently of each other.

COUPLING VII.

308. Fig. 9, Plate VI. represents the *coupling link* used by Messrs. Boulton and Watt in their portable steam-engines. From one of the arms A, of the fly-wheel, projects a strong iron pin P. On the end of the shaft R, which is to be coupled to that of the fly wheel A B, there is a crank C, which has the same length of an arm that the pin is distant from the centre of the fly wheel. The pin and the crank are connected by a link L, so that when the fly shaft moves, it carries round the other shaft R, along with it.

OBSERVATIONS.

309. This is a very simple and durable contrivance, and if the two shafts be only parallel to each other, they may work with great smoothness, although their axes should be in different lines, for the links, in that case, move without any twisting. But if the axes be not parallel, or if they be not in one line, a twist will take place at the link, which may be injurious.

The crank ought to be considerably longer than those in general use, to prevent, as much as may be, the continual drag on one side of each of the journals.

COUPLING VIII.

310. Fig. 10. represents a coupling, which is sometimes used to convey motion from the fly wheel shaft A, of a steam-engine; and is so contrived, that in case the fly should turn the wrong way, the mill-work remains at rest,

and so prevents accidents. This effect is produced by means of a joint C, on the arm B, resembling the joint of a table, or of a pocket foot-rule. When the fly wheel turns the proper way, the arm D, upon the end of the fly shaft, acts against the face of the arm B, on the mill shaft F; and as the joint does not yield in that direction, the mill shaft is carried round by the fly shaft. But, if from any accident, the fly turns the wrong way, the arm D strikes the back of the arm B, the joint yields, and the mill remains at rest.

OBSERVATIONS.

311. This coupling may be made sufficiently durable. Its principles are nearly the same as the boring mill clutch, Fig. 6. In the Repertory of Arts and Manufactures, Vol. II. p. 19, there is a description of an alteration on the cattle mill, to answer the purpose of these couplings, in making the mill go in its proper direction only. Buchanan was led to this contrivance from the accidents to which carding machines were liable when driven by horses, and in the year 1790 he erected several mills on that plan*. This has since been simplified and applied to thrashing mills.

GENERAL OBSERVATIONS.

312. All other couplings having two bearings, that he recollected having seen, from the reasons already given, are attended with much friction. From that fault, they have in a great measure been abandoned, and those with one bearing substituted.

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* The plate referred to in this paper will serve to give some idea of the manner in which mill-work was constructed about the year 1790, in this part of the island. Cast iron was then but little used in cattle mills. See *Introduction to Essay Second, on the Shafts of Mills.*

313. It is, however, proper here to remark, that although the friction, and expense of erection of couplings with one bearing, are considerably less than those having two bearings, where the chief object is to convey motion to a distance, or where there is no great weight or lateral stress on the shafts, yet all circumstances should be duly considered before adopting either plan: for if heavy drums or wheels are to be placed near both ends of the shafts, or any other thing that will occasion much lateral pressure, it will be advisable to use two bearings. In the next chapter, are examined couplings having *one* bearing.

CHAPTER II.

CLASS II.

OF COUPLINGS HAVING ONE BEARING.

SECTION I.

314. THIS class of couplings, when properly constructed, has, to a certain degree, the property of being flexible in all directions, like the well known contrivance invented by Dr. Hook, (Fig. 11,) called the universal joint. This joint is sometimes constructed by a cross, as represented in the figure, and sometimes with its four pivots fastened at right angles, upon the circumference of a hoop, or on the surface of a solid ball. The moving parts are evidently alike in all these cases.

315. It is sometimes applied to communicate motion, instead of bevelled gear, when the angle does not exceed 30 or 40 degrees, and where the number of revolutions is to be continued the same; also, where equality of motion is not required, for as it recedes from a right line, its motion becomes irregular. The property of the universal joint for conveying angular motion, is of great use when it can be attained in couplings, in order to allow for the inaccuracy which arises from the settling of the framing or the wearing of the brasses. It need not, however, yield further than what is really necessary for that purpose, which is so little, that no irregularity of motion that can be hurtful in practice can arise.

316. The disadvantage of the universal joint, as represented by Fig. 10, and as commonly made, is, that it has not sufficient strength to resist great strains. There is, however, afterwards described a modification of it, well adapted to bear very considerable stress. (See Fig. 18, Art. 329.)

317. Most part of the couplings described in Chap. I. may, with some small addition or alteration, be converted into couplings having one bearing. Of this description *is the square coupling*.

COUPLING IX.

THE SQUARE COUPLING.

318. It consists of a square coupling box C, Fig. 12, which fits a square on the end of each shaft. The square of the shaft A is close to its journal. The square of the shaft B is at the end furthest from its journal.

In order to support the square of B, and keep it on a line with A, there is a round hole D, bored out of the centre of the square of B; to fit this hole, there is a round projection D, from the square of A. Instead of having the projection D, sometimes there is a hole in the end of both shafts, into which is fitted an iron or steel dowel or pin.

The coupling box C, covers both squares. It is kept in its place by two iron pins or bolts, G, G, one of which passes through each shaft. These bolts also serve to keep the shafts from withdrawing from each other.

OBSERVATIONS.

319. This coupling is used with good effect in conveying motion through a great length of shafts, where there is but

little lateral pressure; but where there is much lateral pressure, for instance in *drum-shafts*, which give motion to carding engines by belts, it has been found, that the round projection and socket soon wear and get loose. This fault perhaps arose from the square being too small to admit the projection, and the socket to be sufficiently large. But though this be a better mode of coupling than many in use, yet the difficulty of having the squares accurately adjusted is great, as is also the fitting of the inside of the coupling box, from the ordinary mode of casting. For these reasons, in many cases it is very liable to lifting or straining.

320. In England, most of the rollers of those machines, denominated *mules**, for spinning cotton, have but one bearing; the end of one roller being squared and inserted into the end of that next to it, as represented in Fig. 13. But as there is a great lateral pressure, the end of the roller is liable to wear and become wide, and occasion a hobbling and inaccurate motion. From the accuracy required in *mule* and *throstle*† rollers, it is not only necessary that the axes be exactly in one line, and accurate in their diameters, but also that they run true without hobbling or jolting. Buchanan had never seen any method by which this was attainable in one bearing; even though the utmost attention of the most accurate workmen had been bestowed

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* The *mule* was invented about the year 1777, by Samuel Crompton, formerly of Hall-in-the-Wood, near Bolton, in Lancashire, a person of very great ingenuity, and to whom the country is indebted for many other useful improvements; this machine probably received its name for having rollers like Arkwright's machine, at the same time that it retained the carriage and spindles of hand-spinning machines called common jennies.

In 1769, Arkwright obtained his patent for spinning, and in 1775, for preparing cotton by machinery.

† *Throstle*, a machine for spinning cotton, compounded from the inventions of Arkwright and Crompton.

to accomplish this end. In Scotland, two bearings are generally used in the coupling of all rollers employed in spinning cotton.

COUPLING X.

OF THE ROUND COUPLING.

321. This coupling having only one bearing, is represented by Fig. 13, and has similar additions to those of the last described coupling, Fig. 12, and which will appear sufficiently from the figures, without further description.

OBSERVATIONS.

322. The observations which were already made on the round coupling with two bearings, (Art. 296, Fig. 4,) are in a great measure applicable here, as are those relative to lateral stress, in the observations (Art. 319,) on Coupling IX.

COUPLING XI.

323. Fig. 14 represents a coupling used in several of the mills at Manchester. It consists of a scarfed joint, (like that used in carpentry,) and when the shafts are engaged, it is firmly bolted, as shewn in the figure.

OBSERVATIONS.

324. It was probably owing to the defect relative to lateral stress mentioned in observations, (Art. 319,) Coupling IX., that this contrivance was adopted. But it seems to me to have all the defects attending the solid shaft with more than two bearings, namely, that it could not for any length

of time move properly on all its bearings, without a continual bending of the solid metal, which would occasion a waste of power, besides a great risk of breaking the shaft.

COUPLING XII.

325. The coupling represented by Fig. 15, Plate VII., is only a variety of Coupling XI., and, therefore, the same observations are applicable to this also.

COUPLING XIII.

326. That shewn in Fig. 16 may also be considered as only another modification of Coupling XI., the one shaft being firmly fixed to the other by flanches and bolts. The same observations are also applicable here.

COUPLING XIV.—Fig. 17.

327. Has the bearing and the joint of the coupling at the same parts of the shaft, so that the journal is not solid, but composed of the ends of each shaft, which are there formed into quadrants. Each shaft having two projections, corresponding to two recesses in the other, one of the projections is represented at A, No. 1, as engaged; No. 2 represents the coupling as disengaged.

OBSERVATIONS.

328. This is obviously so bad a contrivance, that little need be said with regard to it. The bad effects of it were lately brought under Buchanan's observation at an extensive set of calico printing works. It was there applied to drive dash-wheels for washing calicoes, and had been attended with great trouble and expense; for the

shafts very frequently broke at the couplings, and were thus rendered useless. Besides, while they did last, the couplings thus formed in the journals, acted as a kind of cutters, producing great friction, and tearing down the brass.

COUPLING XV.—FIG. 18.

329. This consists of three distinct parts, A A, B B, and C C; A A and B B are, each of them, firmly fixed to its respective shaft, and are so formed, that they join partly into one another, in some degree, like the parts of a common *papier maché* snuff-box.

C C consists of a solid ring of cast iron, into which is screwed four strong steel or wrought iron pins D, D, D, D. These pins serve to act as the four pivots of the universal joint already described in Fig. 11.

A A and B B are so contrived, that each of them lays hold of two opposite iron pins, the whole combined thus, forming an universal joint. There is a space of about a quarter of an inch left between A A and B B, in order to allow the joints to play; and A A and B B are commonly made about 14 inches diameter outside.

There is a small mortise in each pin, to receive a cotteril, to prevent the pin from coming out in the course of working.

In the plate, so many different views of this coupling are given, that it is hoped no further description will be required.

OBSERVATIONS.

330. This coupling is somewhat expensive in its first erection; but it being evidently, as far as is necessary in such a coupling, a complete universal joint, from its durability and saving of power which its pliancy must occasion,

it seems to be perhaps the best thing of the kind that has yet come under Buchanan's observation.

COUPLING XVI.—FIG. 19, PLATES VII. AND VIII.

331. This is a kind of coupling which was in Buchanan's time executed at Manchester. The coupling box C is made very long, and is square in the inside, excepting at D E, where there is a kind of partition, with a large round hole truly bored in it. Into this hole, each of the shafts A and B, are accurately fitted. The round part of the shaft B, however, is made so long, as to allow the liberty of slipping back the coupling box, in order to disengage the shafts. When engaged, the box is kept in its place by the pin B. H represents the journal.

OBSERVATIONS.

332. The principles of this coupling are nearly the same as those of the common square coupling, (X. Fig. 1,) (Art. 292,) but the greater length of the box, as well as the greater strength of the round parts intended to keep the axis true, give this last coupling very material advantage.

333. This kind of coupling has another advantage which the greatest part of single bearing couplings have not, viz.: when the coupling-box is shifted off, any shaft may be taken out, without affecting those adjoining.

COUPLING XVII.

334. Fig. 20, Plate VIII. represents a coupling used in one of the cotton mills last erected, and one of the most extensive at Glasgow. The external part, coupling box C, is cylindrical. On the inside, the parts A, A, A, A project,

and are fitted into the shafts, which have similar projections, E, E, E, E fitted into the coupling box. Through the centre of the coupling box there passes a bolt H H, to keep it in its place; and, at the further ends of the row of lying shafts, they are kept together by working against a kind of step formed of brass. Were this not the case, two bolts would be necessary in each coupling, to keep the shafts from separating.

OBSERVATIONS.

335. The advantages of this kind of coupling, seem to be these two: 1st. The projecting parts E, E, &c., can be accurately turned and fitted to the coupling box. 2d. These projecting parts tending to the centre, are strong, and little liable to wear.

SECTION II.

OF THE COUPLINGS OF UPRIGHT SHAFTS.

336. Hitherto we have considered couplings for lying shafts only. But of upright shafts, little need be said; having in general little lateral pressure, they are seldom made with two bearings; so that by placing any one of the couplings with one bearing, already mentioned, in a vertical position, an idea will be obtained of the mode of coupling upright shafts.

The square coupling, (IX., Fig. 12, Plate VI.,) for instance, may easily be applied to an upright shaft.

COUPLING XVIII.—FIG. 21.

337. A represents the journal of the lower shaft, (which is almost always that which has the bearing,) B is the lower end of the upper shaft, and C the coupling box.

COUPLING XIX.—FIG. 22.

338. A represents part of the under shaft, B the lower end of the upper shaft, C the journal. The termination of A is made square, to correspond with which there is a socket formed in B, which answers the purpose of a coupling box.

OBSERVATIONS.

339. This coupling is often used for light work, particularly in flour-mills, for connecting the feeder with the top of the stone-spindle.

COUPLING XX.—FIG. 23.

340. A represents the lower shaft, D the upper, which is above the journal. Projecting and receding quadrants, the same as in Fig. 17, serve to connect the shafts.

OBSERVATIONS.

341. This is a very good and simple mode of coupling upright shafts. By their own weight, together with that of wheels that may be on them, the projecting and receding quadrants D, D, are pressed home into their holds, and are not subject to get loose in their sockets or clutches, which would be the case were the shafts lying horizontally. A great many other schemes for couplings have been introduced; most of them ingenious; but the late Mr. Tredgold had not examined any, which are sufficiently simple and likely to be durable, to offer here as improvements.

CHAPTER III.

GENERAL OBSERVATIONS.

342. It may be proper to observe, that the larger the parts of the coupling can conveniently be made the better. In other words, the further the point of stress is from the axis, the couplings will be the more durable. This being a thing to which too little attention in practice is paid, it will not be improper here, in a popular way, to endeavour to explain the reasons of this greater durability.

343. The strain on the point of stress is inversely as the velocity of that point. (Essay II.) Now the revolutions being the same in a given time, the further the point of stress is from the axis, the greater will be the velocity of the point, and, consequently, the less the stress. Experience has taught this to those unacquainted with science; for every one knows that a handspike, capstan-bar, or other similar lever, requires to be largest near the fulcrum, (or point on which it turns in raising a weight,) and may be diminished in proportion to the distance from the centre of motion.

344. Thus, for example, at two feet from the centre, the stress is only one half of what it is at one foot from the centre. This is almost evident to the feeling, from the force the hand has to apply at those different distances. Now, the larger the parts are, the stress must be thrown the further from the centre of motion, and the acting parts will therefore be the more durable. Hence also the advantage in machinery of having large wheels and large

pulleys. (See Essay I., *General Observations on the Wheel-Work of Mills*, Art. 101.)

345. It is also proper to observe, that when there is a long line of shafts, the couplings, where there is only one bearing, should, if practicable, be so arranged as that the unsupported end of the shaft should be as far as may be from the part subject to lateral pressure. For instance, in Fig. 21, Plate VIII., the couplings and journals are better as there represented, than had they been at A A, in the middle, between the wheels.

346. The oiling of couplings is found to render them more durable. This fact has been fully ascertained in one of the most extensive cotton manufactories in Britain, in the machinery of which the couplings were formerly very liable to wear, but, since using oil, they have been found sufficiently durable. The squares of couplings nearest the journals, from accidentally getting oil, are also least worn.

347. A fly-wheel is often of use in a long line of coupled shafts. In the vicinity of Glasgow, motion was conveyed from a steam-engine by means of lying shafts, to the distance of ninety-three yards. When those shafts were first tried, from the elasticity or spring of so great a length of shafts, and the play of the couplings, the motion at the further end was so very irregular, that it could not be applied to work a "calender." A fly-wheel near the "calender," connected with the lying shaft by pulleys and a belt, was resorted to in order to cure this evil, and this simple contrivance had the desired effect; for the calender ever after gave satisfaction in its work.

A table respecting the dimensions, stress, and durability of couplings is annexed.

348. *Facts respecting Couplings.*

	1 Resistance, horses' power.	2 Revolutions per minute.	3 Years at work.	4 Side of square in inches.	5 Length of box.	6 Power divided by revolutions per minute or proportionate strain.
A, Cast iron. Square coupling, one bearing, (see Coupling IX. Fig. 12,) coupling next the steam-engine, worn off each angle of the square about three-fourths of an inch; the square was originally 5 inches; the box 10 inches long....	16	40	7	5	10	·40
B, Cast iron. Same line of shafts further on; all things but the resistance the same. Some of the couplings not perceptibly worn, others worn off about three-eighths of an inch at the angles	6	40	7	5	10	·20
C, Cast iron. Same kind of couplings as A and B, square originally 6 inches, box 12 inches long, not perceptibly worn........................	17	38	5	6	12	·44
D, Cast iron. Same kind of coupling, not perceptibly worn................	6	38	5	3	6	·15
E, Wrought iron. Same kind, much worn..................................	1	38	5	2	5	·02
F, Cast iron. Same kind, not worn .	12	50	8	4½	8	·24
G, Cast iron. Worn one half inch...	9	55	4	4	8	·16
H, Cast iron. Not perceptibly worn.	12	50	7	4	6	·24
I, Cast iron. Not perceptibly worn .	6	50	7	3¼	5	·12

DESCRIPTION OF THE TABLE.

Column 1 contains the resistance in horses' power.
Column 2 contains the revolutions per minute.
Column 3 contains the years at work.
Column 4 contains the side of the square in inches.
Column 5 contains the length of the box.

Column 6 is found by dividing the power by the revolutions per minute, which represents the comparative stress, (see Essay II. Chap. IV.)

OBSERVATIONS.

I.

349. These couplings do not seem to have been durable in proportion to their stress; but this may be in part at least accounted for, from difference of workmanship, and of degrees of hardness of metal, or perhaps accidentally getting oil.

II.

350. Two circumstances must materially affect the *durability* of couplings. 1. The extent of surfaces in contact at the place of pressure. 2. The distance of the surface of pressure from the centre of motion.

It is probable, therefore, that, all other circumstances being the same, the durability of couplings increases in a ratio compounded of those two circumstances, or nearly as the *squares* of the sides of such couplings as have square coupling boxes.

Thus, for example, B, in the table, is 5 inches on the side; admitting the above ratio to be near the truth, in order to have the case A made in proportion to B, it should be increased to rather more than 7 inches, for

The square of 5 is equal to 25, and as the stress on A is double, 25 multiplied by 2, is equal to 50, the square root of which is 7·07.

III.

351. But we may suppose it prudent to take a standard somewhat larger than the side of B, for all the additional

weight of the parts would never be felt hurtful in practice; let us suppose 6 inches therefore a proper standard for eight horses' power, at forty revolutions per minute.

Then the square of 6 is equal to 36, and 36 multiplied by 2 is equal to 72, the square root of which is 8·5 nearly, or the size which the coupling A ought to have had.

IV.

352. In Observations II. and III. cases have been considered in which the revolutions per minute were both the same; but we conceive that *velocity* must materially affect durability, because the grinding or wearing, where there is any play, must be increased by an increase of velocity.

V.

353. With respect to durability, couplings may be considered under two distinct classes.

1. Those having boxes.
2. Those without boxes, having legs, such as glands, &c.

The durability of the latter class will probably increase, all other circumstances being equal, nearly in the ratio of the distance of the parts of pressure from the centre of motion. For, in glands, the pressure is commonly confined to a small space compared with that of coupling-boxes.

SUPPLEMENTARY OBSERVATIONS.

I.

354. The durability of couplings depends upon so many circumstances, that it is difficult to form general rules with

regard to them. The two circumstances already mentioned (Art. 350) are important, but others merit at least equal consideration; the angle which the surface makes with the direction of the motion. Thus a square coupling will be more durable than an octagon of the same size, because the acting surface makes a greater angle with the tangent to the circle in which the acting part moves. A right angle to the tangent, or the radius, will be the maximum.

II.

355. The durability of couplings depends greatly upon the accuracy of the execution. For instance, a well-fitted square coupling will have one fourth, or perhaps one third of the surface of each side acting. But if fitted as is common in practice, they will have little more than the corners acting. One half of each side is the maximum.

III.

356. As square couplings of a large size are commonly fitted, there is so little of the surface acting, that we suppose their durability will be nearly as the length of the box, multiplied by the velocity of the corners. But if fitted as they ought to be, their durability will be as the rectangle of the acting parts multiplied by the velocity. Or, to simplify the case, (as the ratio will be the same,) as the rectangle of the side, multiplied by the velocity.

These observations may perhaps suggest matter of useful practical reflection to the considerate millwright.

ESSAY IV.

ON THE METHODS OF DISENGAGING AND RE-ENGAGING MACHINERY,

WHILE IN MOTION.

INTRODUCTION.

THE subject of this Essay is so intimately connected with that of Essay III. on Couplings, that in some cases they are really blended; and while a contrivance is employed for disengaging and re-engaging machinery, it also serves as a longitudinal connexion of shafts.

In viewing for the first time a cotton-mill, few objects attract more attention, or excite more pleasing surprise, than the facility with which even children stop or set agoing particular parts of the mechanism separately from the rest. But however curious such things may be to inspect, it is, perhaps, no easy task, on paper, to render them interesting to the reader. Their utility, however, in practical mechanics, should stimulate the inquirer to examine with attention a subject where ornament of style is inadmissible, and where perspicuity alone should be attempted, and which it is perhaps not always easy to attain.

The plan followed in this Essay is similar to that in Essay III., that is to say, in describing each method, and making separate observations; and the prefatory observations to that essay are equally applicable here.

Great credit is due to Buchanan in bringing into one point of view so many inventions, which will not fail to be useful to the mechanic, by enabling him more easily to compare them one with another, and more readily to select such as may be best adapted to his purpose.

357. From what has been said respecting couplings, it may easily be understood how shafts may be disconnected when at rest. But many cases in practice require that particular parts of a mill must be stopped, or set agoing*, without stopping or making any sensible alteration on the motion of the rest of the machinery. In cotton-mills, for instance, this becomes absolutely necessary; and there can be no doubt, that necessity, in this case, has given rise to many most ingenious contrivances. Previously, however, to the invention of cotton-mills, there were contrivances for this purpose in use; such, for example, as the sack-tackle in corn-mills.

358. In order to assist us in forming a judgment of the comparative merits of such improvements, it may be proper to bear in mind, a tendency attached to all matter which is intimately connected with practical mechanics, but on which daily experience shews too little attention is bestowed in the construction of machinery.

The tendency here alluded to has, by philosophers, been called *inertia*†, (or more frequently, though with less propriety, *vis inertia*,*) by which is meant, the tendency which every piece of matter, when at rest, has to remain at rest; and, when in motion, to continue in motion. In other words, the impossibility of *instantaneously* producing motion in a body, or of *instantaneously* stopping a body in motion. It is this tendency, therefore, that occasions those violent shocks in attempting to set bodies suddenly into motion; and those shocks, besides tending to destroy the machine, occasion a very great loss of power. Some part of the machine must break, or at least yield to this potent law.

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* When any particular part of machinery is set agoing, it is said among workmen to be *set on*, or *put in gear;* when stopped, *set off* or *put out of gear*.

† "A tendency to preserve in a state of rest or uniform rectilinear motion, is a property attached to all matter, and may be considered as proportional to the mass or weight of a body." (Young's Nat. Phil. Vol. I. p. 51.)

359. The practical mechanic, and perhaps also the philosopher, will find it some advantage to dismiss the term *inertia* from the place it occupies in science. When properly understood, it simply indicates that matter never changes its state, unless there be a change in the power or powers acting upon it.

If you consider inertia as a power, it must be identical with the power of gravity in one case, with the momentum of a body in another, with the power of magnetism in a third, with friction in a fourth, and so on throughout the whole of the powers in nature; but many writers write as though it were a real power, and distinct from all these, and consequently lead their readers into incorrect notions on the subject.

It is easily proved, that when a body is struck by another in motion, some time is occupied in communicating the motion from the point struck to the other parts of the body; and therefore, if the parts receiving the blow have not sufficient elasticity and cohesive power to destroy the whole momentum of the striking body till the motion be transmitted to the centre of rotation, it must necessarily break the body struck*.

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* *Vis inertia* is defined by Newton, (Def. 3. Book I.) to be a power implanted in all matter, by which it resists any change endeavoured to be made in its state; that is, by which it becomes difficult to alter its state, either of rest or motion.

Hence any part of a machine intended to be acted upon suddenly by a moving power should be strong but yielding, or it may be capable of sliding with much friction on the body to be put into motion.

360. Let us illustrate this by example. Throwing a wheel *into gear*, very often occasions the breaking of the teeth, whereas a pulley is generally put in motion by a belt so gradually, that no part of the machinery can receive any injury. In the first case, the wheel being firmly fixed on its shaft, can yield little to *inertia*, (the moving force.) In the last, there is the elasticity of the belt, as well as the liberty of slipping on the pulley to yield to the *inertia*, (moving force,) till by its friction, the belt gradually brings it into motion.

361. This subject naturally divides itself into the two following sections:

I. Of Methods used when motion is communicated by means of bands, belts or chains.

II. Of Methods when motion is communicated by means of wheel-work.

METHOD I.

THE SLIDING PULLEY.—Fig. 1, Plate IX.

362. The sliding pulley is one of the oldest contrivances for disengaging and re-engaging a machine moved by a belt or band. The pulley P, which is driven by the mill-work, and gives motion to the machine, is not fixed dead on the axle A B, but has a hollow cylindrical *bush* made of metal, accurately fitted to the axle, so that it may revolve easily upon it, and slide a little backward and forward.

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* The rules for estimating the power of materials under these circumstances may be found in Tredgold's Practical Essays on the Strength of Cast Iron, Sect. VII.

In order to make the pulley P carry round the axle A B, there is a cross piece or gland D E firmly fixed to it. On the side of the pulley which is toward the cross, there is one or more teeth T; when the pulley P is moved toward the cross D E, the teeth lay hold of it, and thereby carry round the axle. By sliding it backward, though the pulley still continues in motion, the teeth T are disengaged from the cross D E, of course the axle stops, and with it the machine to which it gives motion.

The bush C, of the pulley, projects a little upon the side opposite the teeth, and has a groove cut in its outside. Into this groove, a lever F G lies, but so as not to prevent the motion of the bush. By moving the lever F G the bush is moved along with it, and thus serves to disengage or re-engage the pulley.

OBSERVATIONS.

363. For a long time this contrivance was applied to cotton carding machines. The shock, however, proceeding from *inertia*, (Art. 358,) occasioned by the teeth striking against the fixed cross in setting the machine agoing, was very great, and apt to shake the cylinders loose from their axles. This is a very great defect, and for such purposes as have been mentioned other methods more perfect are now used.

METHOD II.

THE BAYONET.—Fig. 2.

364. The pulley or binder A, is kept in motion by a belt. It has a bush fitted to the upright axle B C, and runs always at the same height.

On its upper side, the binder A has teeth T, T projecting upward; above is a pulley, or other piece of wood or metal

DE, fixed to the shaft BC, through holes in which passes the legs F, G of the bayonet*.

The bayonet is represented by Fig. 2, No. 3. It is merely a piece of metal with two or more legs, which project downward. The part H is made hollow to receive the shaft, and has a groove into which IK fits, and the lifter moves it upward or downward, serving the same purpose as the lever FG, Fig 1.

When the bayonet is dropped downward, it lays hold of the teeth T of the binder A, which immediately carries it round, and this gives motion to the axle BC; all that is required to stop it, is merely by hand to raise the lifter IK, which has a spring or catch to keep it in that situation, until again dropped by the hand of the attendant.

OBSERVATIONS.

365. This contrivance is more perfect than the sliding pulley, Fig. 1; because the pulley does not require to be shifted in the direction of the shaft, which is often inconvenient, and requires more space. Fig. 2. has however the same defect as Fig. 1, of producing a shock at the instant when the machine is put in gear.

This method is still much used in cotton-mills. The figure represents one of the upright shafts of a spinning frame. The same contrivance is often applied to horizontal shafts.

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* Instead of the legs of the bayonet passing through holes, as here described, it is sometimes made square in the inside, so as to fit a square part of the shaft; which square answers the purpose of the piece of wood or metal, viz. that of carrying round the bayonet along with the shaft. The bayonet has sometimes one leg only; on other occasions it is made like a face-wheel, having a number of projecting teeth or legs. Indeed, it is almost obvious, that all or most of the contrivances which are mentioned in this Essay, must be variously modified, according to circumstances, or the fancy of the artisan. I have only given the way in which they are most generally applied. To give all the varieties would be an almost endless, and, perhaps, useless labour.

METHOD III.

OF THE LOCK PULLEY.—Fig. 3.

366. This method is somewhat similar to the last described, but instead of a bayonet, it has a lock at A, the bolt of which B lays hold of the cross C C, which is fixed to the axle. The pulley, when disengaged, runs on a bush. K is the key, which is turned by means of a stop, which is made to touch a cross part on the end of the key. The turning of the key throws back the bolt, and so unlocks and disengages the pulley.

OBSERVATIONS.

367. This contrivance is less simple than the bayonet. Nor indeed has the lock pulley ever been generally adopted; a proof that it has not been found of much real use. It was used at Manchester about the time that machines, called mules, for spinning cotton, were begun to be changed from being moved by hand, to receive their motion (as is now general) by power.

METHOD IV.

THE FAST AND LOOSE PULLEY.—Fig. 4, Plate X.

368. The pulley B is fixed on the axle A, and the pulley C, having a bush, is loose. The belt or band which conveys the motion, may, at pleasure, either by hand, or by a lever, be shifted from the one pulley to the other. When running on the loose pulley C, the axle stands still; when on the fast pulley B, the axle moves.

No. 1, represents the pulleys as adapted to a belt.

No. 2, represents the pulleys as made for a rope. The inner ledges are made low, in order that the rope may slide with the greater facility from the one to the other.

OBSERVATIONS.

369. It may be proper here to mention, that in order to make a belt run properly on a pulley, it is necessary to have the rim of the pulley a little rounded or swelled in the middle. The belt always inclines to that part of the pulley which is of greatest diameter. This curious property is found of great practical use. Until this property was known, it was found very troublesome to get belts prevented from slipping off the pulleys.

370. This contrivance of the fast and loose pulleys, is remarkable for its beautiful simplicity. It is attended with no shock, and is perhaps the most perfect thing yet invented for the purpose, in all cases where it can be applied.

Its application in cotton-mills is now general. Never, until it was applied, were the spinning mules found to give satisfaction, when moved by power. They are now, however, almost exclusively wrought by power. This improvement has not only produced a great saving of labour, but has been attended with another very pleasing effect. It has rendered the employment of the spinner more healthful than when moved by hand. The severity of the hand-labour in a warm and confined atmosphere, had a very unfavourable effect on the constitution.

METHOD V.—Fig. 5.

371. The undermost axle A B is kept by the mill in constant motion. The uppermost axle C D has a bridge at D,

in the form of a lever, in order to raise or lower it at pleasure. When the end D of the shaft is raised by means of the bridge, the chain or rope E F is tightened, which causes the axle B to move. But when D is lowered, the chain falls slack below the pulley, on the lowermost shaft A, and thus disengages the shafts.

OBSERVATIONS.

372. This very simple contrivance is used in some parts of England, in corn-mills, for the sack-tackle. It is evidently applicable to many other purposes; and it is obvious, that E F may be either a belt, a rope, or a chain. A chain is commonly used when applied as a sack-tackle. In that case, the rope or chain, for taking up the sacks, is wound round the barrel on the axle C D.

SECTION II.

OF THE METHODS USED WHEN MOTION IS CONVEYED BY MEANS OF WHEEL-WORK.

373. When a mill is in motion, we may, with perfect safety, lift a wheel *out of gear;* but in throwing wheels *into gear*, when a mill is in motion, there is great risk of breaking the teeth; it is, nevertheless, often done. Cotton spinners know from experience, that the risk of breaking the teeth is much lessened when the wheel to be thrown *into gear* is previously set in motion by the hand. This they do by pulling the shaft round by the belt. The reason is obvious. The inertia is lessened by this motion given by the hand to the wheel which is to be thrown *into gear*.

METHOD VI.—Fig. 6.

374. Wheels are often disengaged and re-engaged by

means of one of the bridges, A B, which carries the end of the shaft nearest the wheel, acting as a lever, having its *fulcrum* A at one end. The other end B is moveable in one direction, and is raised or allowed to fall, by turning the key C D; when *out of gear* the bridge rests securely on the end of the key, as represented in No. 2, Plate XI. No. 1, represents the wheels *in gear*.

Sometimes the bridge is raised immediately by hand, sometimes by a rope and pulley; in these cases it is held out of gear, by interposing a wedge instead of the key, or by a catch under the end of the bridge.

OBSERVATIONS.

375. This mode of disengaging wheels is sometimes inconvenient for want of room; it has, however, the advantage of having the wheels firmly fixed on the shafts. In falling into gear the ends of the teeth are apt to strike against one another, and there is often a violent shock from the inertia.

In the figures, the wheels are represented the one above the other; but the same contrivance is modified, and applied, when they happen to be in any other relative situation*.

METHOD VII.—Fig. 7, Plate XI.

376. Instead of lifting wheels out of gear, in order to disengage any piece of mechanism, one of the wheels, instead of being fast on the shaft, has a round bush like a loose pulley, and a clutch, or bayonet, which connects it with its shaft, in a similar manner with the bayonet and binder, Fig. 2.

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* The motion of the wheel, as it falls into gear, should be in the same direction as it is afterwards to move in; which will lessen the shock considerably.

Thus the wheel A, Fig. 7, No. 1, has a bush, and works on a round part of the shaft B. The clutch D, may slide on a square part of the same shaft, and is disengaged or re-engaged at pleasure, by means of the lever E F, part of which is represented as cut off, in order to shew the groove in the clutch into which it acts.

No. 2, is an elevated section of the lever E F, shewing its connexion with the clutch D, by means of two iron pins G, G, screwed into two slips of iron H, H, which fit the groove, and are not affected by any oblique position the lever may have.

OBSERVATIONS.

377. This contrivance is sometimes made more convenient than is shewn in Fig. 7, by occupying less space; and although it is subject to a shock at *setting on*, yet it is not so liable to break the teeth as in Method VI.

METHOD VIII.

THE FRICTION CLUTCH.—Fig. 8.

378. This figure represents the friction clutch; it differs from the bayonet (Method II.) in this respect, that instead of striking on a fast cross, the bayonet or clutch lays hold of the ears of a screwed hoop, which embraces a kind of drum.

A B represents part of a shaft kept in motion by the mill; C D E a bayonet, which either slips on a square part of the shaft A B, or passes through the arms of a cross P Q, (as represented in the figure,) which cross is fastened to A B.

F G is part of a shaft to be connected with the shaft A B; upon F G a kind of drum or pulley H I K is fastened; this pulley has ledges to keep the screwed hoop L M N O steady.

In *setting on* the machine, the hoop LMNO is carried round by the bayonet or clutch CDE, and by the friction of the hoop on the drum HIK, brings it into motion, in the same easy and gradual manner that a belt does a machine driven by a pulley.

The hoop, as represented in No. 3, Plate XII., from acting more perfectly as a spring, is found to answer better in practice than that represented in No. 1. and No. 2.

OBSERVATIONS.

379. There is a great deal of beauty in this ingenious contrivance, it may be applied to the largest machinery, and variously modified according to circumstances*. It is obvious that it prevents all the unpleasant and hurtful shock so common in throwing heavy machinery into gear.

It may also be the means of saving lives, for should a person's clothes be laid hold of by the wheel-work, in most cases the hoops would slip on the drum, and allow that part of the machinery to stop, without sensibly altering the general motion of the mill. It may in this way, too, prevent injury to the mill itself, which might arise from belts getting foul, or from chips falling in among the teeth of the wheel-work.

The friction clutch has been lately applied to frames for spinning flax, and it seems to me, might also, with great advantage, be applied to frames for spinning cotton water-twist.

METHOD IX.

THE FRICTION CONES.—Fig. 9, Plate XII.

380. This contrivance is similar in its principles and effects to the friction clutch.

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* Thus, for instance, it may be applied to the wheel and clutch.—Fig. 7.

On the shaft A (kept in motion by the mill) there is fixed a hollow cone H; on the shaft B is another cone E, the external part of which fits the internal part of H; E is, however, moveable like a bayonet on a square part of the shaft B, and may be moved outward and inward also like a bayonet, by a lever.

When E is moved forward, it rubs on the hollow part of H, and by friction, like the friction clutch, gradually brings the machine connected with B into motion.

OBSERVATIONS.

381. Sometimes there is added a bayonet, passing through a hollow cone, which, should occasion require, gives liberty to lock the shafts quite fast into one another.

The friction cones are sometimes applied to sack-tackles; one of them on this construction may be seen at Meux's brewery.

METHOD X.

WHEELS ACTING BY FRICTION.—Fig. 10.

382. Sometimes wheels are made to act without teeth, as represented in the figure; they move one another by contact, having their circumferences generally made of end grain of wood, which may indeed be considered as forming indefinitely small teeth.

It is evident, that wheels of this kind may work with little noise, and be put into gear, and bear against one another without risk of damage. They are commonly disengaged and re-engaged by a bridge, acting as a lever, on similar principles as described in the case of Fig. 6.

OBSERVATIONS.

383. This species of wheel-work has been used with good effect in machinery for raising coal; it is also used,

in some cases, in cotton-mills; and has, for a number of years, been employed in a saw-mill by Taylor of Southampton, the principle and method of which in transmitting mechanic power certainly deserves attention.

METHOD XI.—Fig. 11.

384. The figure represents another application of friction in transmitting mechanic force, but instead of the friction being on the hem of the wheel, as in Fig. 10, it is here applied to the sides of the wheel.

The mode here represented, is successfully put in practice, in a tackle for raising and lowering sacks in a respectable brewhouse in London. A is the axle which gives motion to the tackle, upon which is the friction wheel B; upon the axis *a* are the friction wheels C, D, and the roller E, round which the rope winds. The end of the axis *a* runs in a socket in the end of the axis A; the other end, in the brass, is in the post *f*. In raising the sacks, the wheel C is kept fast against the wheel B, by the lever *b* and catches *cc*.

For lowering sacks, the wheel D is kept against the wheel F, by the lever *b* and weight *d* going over the pulley *e*, and a man holding the line *g* in his hand, makes the friction of D upon E more or less, as is necessary.

OBSERVATIONS.

385. This is a very ingenious and simple machine, and as its principles might be applied in other cases, it is well worth the attention of the millwright.

METHOD XII.

SELF-DISENGAGING COUPLING.—Fig. 12.

386. A represents a shaft, kept in motion by the mill; B C a cast iron wheel fast on the shaft A, having four pro-

jecting teeth, D, D, &c., of wrought iron; EF, is another similar wheel, with similar teeth, G, G, &c. but is loose on the shaft H, and is made to slide on it, and to act as a kind of bayonet. (Method II.) The teeth project obliquely, as may be seen in the figure.

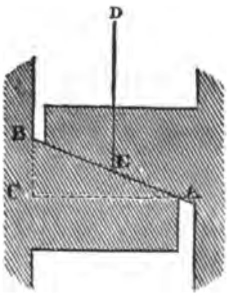
When the coupling is engaged, the teeth lay hold of one another, and the shaft H, is, by their means, carried round with the shaft A, but when any extraordinary stress comes on the shaft H, the pressure on the oblique teeth forces back the bayonet EF, and disengages the coupling. IKL, is a bended lever, having its fulcrum at K, the bayonet is kept forward by the weight of the part MK, of the lever, during the ordinary stress on H.

When the bayonet EF, is forced back, the lever is held up by a catch, until the coupling is re-engaged by the hand of the attendant. The coupling is represented in the figure as disengaged*.

* In order to succeed in producing this effect, the angle BAC must be somewhat greater than would cause the surfaces to slide upon one another, when acted upon by a pressure in the direction DE, perpendicular to AC. Then, when the machine is in motion they would actually slide apart, were it not for the friction on the shaft, and the weight of the lever.

If the angle BAC be less than the angle which would cause the bodies to slide, the coupling would not disengage itself by any force whatever.

According to Coulomb's experiments, (see Brewster's Additions to Ferguson's Lectures, vol. ii. p. 155,) the friction of iron on iron is about one fourth of the pressure, hence the angle BAC should be greater than 15 degrees; otherwise the coupling will not disengage.



OBSERVATIONS.

387. This coupling, as it prevents accidents from any sudden stress, is found very useful where turning lathes are driven by wheel-work.

Some good instances of self-disengaging apparatus may be seen in looms driven by power. Respecting these machines, the reader is referred to Duncan's "Essays on Weaving," as also to the Edinburgh Encyclopædia*.

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* It will always be found, that, in engaging by wheels, the teeth will be less liable to be stripped in small wheels than in large ones; because in small wheels the stroke will be made with a less degree of velocity, and also a small wheel requires less force to put it in motion; hence a small and light wheel with strong teeth will seldomer fail than a heavy one. Perhaps the best arrangement will be when the wheel in constant motion is small, and that to be occasionally put in motion a larger one, with elastic arms. These elastic arms might be made in the manner of coach springs.

ESSAY V.

ON MECHANISM FOR EQUALIZING THE MOTION OF MILLS, DENOMINATED LIFT-TENTERS, ENGINE GOVERNORS, AND WATER-WHEEL GOVERNORS.

INTRODUCTION.

This Essay relates to machinery not less curious in its construction than useful in practice; and as some of the apparatus is intimately connected with water-wheels, the papers which are subjoined, containing an account of some experiments* and observations on their velocity, may not be unacceptable to the reader, who will observe that when a part of the machinery of a mill is suddenly stopped, or suddenly set a-going, and the moving power remains the same, an alteration in the velocity of the mill will take place; it will move faster or slower. Every machine having a certain velocity at which it will work at greater advantage than at any other speed, the change of velocity arising from the above cause, is in all cases a disadvantage, and in delicate operations exceedingly hurtful. In the case of a cotton-mill, for instance, which is calculated to move the spindles at a certain rate, if from any cause the velocity is much increased, a loss of work immediately takes place, and an increase of waste from the breaking of the threads, &c.; on the other hand, there must be an evident loss from the machinery moving too slow.

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* An account of these experiments was originally published in the 10th volume of the Philosophical Magazine, p. 178.

388. In steam-engines this evil is remedied by a contrivance called a *governor*. (Plate XIII. Fig. 1.)—"Two balls are fixed to the ends of rods, in continual revolution, and as soon as the motion becomes a little too rapid, the balls rise considerably," and, by the intervention of a lever, act upon a *throttle-valve*[*], which diminishes the quantity of steam admitted, and of course serves to make the motion less rapid.

SECTION I.

THE STEAM-ENGINE GOVERNOR.—Fig. 1.

389. IK represents a spindle kept in motion by the engine[†]; A, B the centrifugal balls; CA and CB the rods by which the balls are suspended. These rods cross one another, and pass through the middle of the spindle at C. There is a round pin put through the spindle and the rods at C, which serves as the point of suspension for the centrifugal balls or revolving pendulum. There is a part of the spindle above C which is square, and nicely polished,

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[*] *A throttle-valve* is formed by a plate of metal, which is fixed on a spindle passing across the middle of it. When the edge of this round metal plate is in the direction of the current of steam, the aperture is at its greatest extent of opening; and as the plate becomes more oblique the opening becomes less, until it is shut by the plate being at right angles to the current.

The pressure on both sides the spindle being the same, this kind of valve is opened or shut with more ease than any other; and is, therefore, very applicable here. It is not easy to make it quite steam-tight when shut, but its tightness is not of consequence in this case.

[†] This motion is sometimes produced by a rope and pulleys, but wheelwork being more certain, is much to be preferred.

so that the piece of brass M may slide easily up and down upon it. The piece of brass M is round on the outside, and has an external groove turned upon the upper end of it to receive the lever N O, the fulcrum of which is at P. This piece of brass is connected with the ball-rods by two short pieces and joints D E, F G.

The construction of steam-engine governors sometimes differs a little from that now described; but if this particular construction be understood, there will be no difficulty in comprehending any other in use.

OPERATION.

390. When the engine goes too fast, the balls fly off from the spindle, and depress the end N of the lever, which partly shuts the throttle-valve, and thereby diminishes the quantity of steam admitted into the cylinder; and, on the other hand, when the engine goes too slow, the balls fall down toward the spindle, and elevate the end N of the lever, which partly opens the throttle-valve, and thereby increases the quantity of steam admitted into the cylinder.

391. This apparatus being of great practical use, and as it is applied to other purposes which I am about to describe, it may be proper here to give a rule for the number of revolutions which the spindle I K ought to make in proportion to the situation of the balls with regard to the centre of their suspension C.

In order to explain this rule, therefore, it is proper to observe, that "there is a great analogy between the vibration of pendulums and the revolutions of balls suspended from a fixed point. If a body suspended by a thread revolve freely in a horizontal circle, the time of the revolution will be the same whenever the height of the point of suspension above the plane of revolution is the same, whatever be the length of the thread." Thus, if the balls A, B, C, Fig. 2, be made to revolve, they will arrange themselves so as to remain very nearly in the same horizontal plane. "The time of each revolution of the balls is equal to the time occupied by a double vibration of a pendulum, of which the length is equal to the height of the point of suspension above the plane in which they revolve."*

392. Thus, for instance, if the height of the point of suspension D, Fig. 2, above the plane on which the balls revolve, be equal to the length of a pendulum which vibrates seconds, the balls, in that case, should make 30 revolutions per minute.

393. If pendulums are of the following lengths, their oscillations in one minute of time, in Britain, are as follow:

British feet and inches.				Oscillations.
"Feet	0	Inch	1·565	300
	0		9·782	120
	3		3·128	60
	13		0·512	30
	52		2·048	15

394. "Hence the oscillations of pendulums are in the subduplicate ratio, or as the square roots of their lengths; and the lengths of pendulums are in the duplicate ratio, or, as the squares of their oscillations. In order to find the length of a pendulum that will oscillate a certain number of times in a minute, make this proportion:"† as the square of the given number of oscillations is to the square of 60, or the number of seconds in a minute, so let the standard pendulum, or 39·128 inches, be to the pendulum sought.

EXAMPLE.—Required the length of a pendulum which will vibrate 20 times per minute.

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* See Young's Natural Philosophy, vol. i. p. 47.

† Anderson's Institutes of Physics, vol. i. p. 250.

Feet. Inches. Feet. Inches.

$60 \times 60 = 3600$
$20 \times 20 = 400$ 400 : 3600 : : 3 . . 3·128 : 29 . . 4·15

the length required.

SECTION I.

395. In a windmill, when the velocity is increased by the irregular action of the wind, the corn is sometimes forced rapidly through the mill without being sufficiently ground. There is an elegant contrivance for preventing this, (similar to the governor of a steam-engine,) but which I believe was much earlier in use, called in some parts of England a *Lift-Tenter*. "By means of the centrifugal force of one or more balls, which fly out as soon as the velocity is augmented, and as the rise in the arc of a circle, allow the end of a lever to rise with them, while the opposite end descends with the upper millstone, and brings it a little nearer to the lower one."*

This machine is curious, and might perhaps in other cases be usefully applied. We shall, therefore, describe two constructions, but both on the same principles.

LIFT-TENTERS FOR WINDMILLS.

First Construction.

396. This machine and part of the stone-spindle and framing with which it is connected, are represented in Fig. 3, Plate XIII.

To the stone-spindle there are fixed four arms A, A, A, A, there are four similar arms B, B, B, B firmly attached to the hollow cylinder C, which is loose on the spindle F G.

The pendulums D, D, D, D are hung above to the arms

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* Young's Natural Philosophy, vol. i. p. 233.

A, A, A, A, and through holes toward their lower extremities pass the arms of the loose cylinder.

When the mill is at rest, the pendulums hang vertically; but, by their centrifugal force, when the mill is in motion they hang obliquely; and that obliquity is increased in proportion to the velocity, and proportionately raises the loose cylinder C.

This cylinder C acts on the one end of the lever E, which has a connexion with the clove upon which the bridge of the stone-spindle rests, and accordingly raises or depresses the upper millstone in proportion as the wind is weak or strong.

Second Construction.

397. Another modification of the same principle, (applied above the *millstones,*) but having one pendulum only, is represented by Fig. 4, and will be easily understood from what has been said respecting the First Construction.

These lift-tenters are drawn from sketches taken in the neighbourhood of Liverpool in the year 1790.

SECTION III.

398. Governors are sometimes applied to water-wheels, and made on various constructions. Smiths' bellows have been applied to that use, the upper board rising or falling in proportion to the velocity of the lower board, which received its motion from the mill. But those we are about to describe, appear to me on better principles; and as they have been found of very material use, we shall proceed to describe a construction which has for several years been at work in Cartside cotton-mill, which was erected under the direction of the late Robert Burns, Esq., (of whom Pro-

fessor Robinson makes respectful mention in the Encyclopædia Britannica, Art. Water-works,) and which has there given great satisfaction*; we shall afterward describe some other similar machines for the same use.

WATER-WHEEL GOVERNOR.

First Construction.

399. The principles of this kind of water-wheel governor are nearly the same as those of the governor of a steam engine. It has a revolving pendulum which receives its motion from the mill, and in proportion as the machinery moves faster or slower, the centrifugal force acts upon the governor, and raises or depresses an iron cross, which, acting on a lever, reverses the motion by the wheel-work, which operates upon a sluice so as to enlarge or lessen the passage of the water to the water-wheel; this sluice is made on the principles of the *throttle-valve* already described, Art. 388, in order that it may be moved by a small power. So long as the machinery is moving at a proper velocity, this wheel-work of the sluice apparatus remains at rest.

Fig. *5* represents different views of this machine, and some of its parts detached. The same letter in all the figures refers to the same part.

The revolving pendulum EFGH receives its motion from the mill-work by means of a rope giving motion to a pulley I. The upright shaft MN is kept in constant motion by the wheel work OPRS. The wheel N acts constantly into the two bevelled wheels T and U, and makes them move in contrary directions. They are loose on the shaft when the mill is going at its proper speed.

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* In a letter which Buchanan had from Mr. Burns, dated February 1808, he writes to the following effect:

"The governor is the most useful thing for a water-wheel that can possibly be thought of, and I wish you would adopt it in your practice wherever you desire to make your employer prosper. I am sure it is worth a large sum to us at Cartside mill, from its keeping up the speed of the machinery, and not deviating the year round."

But if the mill moves either too fast or too slow, the one of these wheels, by means of a clutch Q, in a way to be described, is connected with and carries round the lying shaft DC, and, by a pair of bevelled wheels, communicates motion to the oblique shaft BW, which again, by a screw X, and quadrant wheel Y, moves the sluice Z, and by making it stand more or less oblique, alters the area of the passage for the water.

From inspecting Fig. 5, No. 1, it will be evident that the box *a* will be raised or depressed in proportion as the balls E and F of the revolving pendulum EFGH are further or nearer to the centre of motion; when the velocity is greatest, the balls E and F by their centrifugal force will extend themselves farthest from the centre of motion, and raise the box *a*. See also Fig. 5, No. 2, No. 3, and No. 4.

To the box *a* is fixed a cross *bc*. There is a forked lever *dqe*, the fulcrum of which is at *f*, and which turns horizontally. This forked lever has four prongs, 1, 2, 3, 4.

When the mill is at its proper speed, the cross works within the prongs 1 and 2; in this situation of the forked lever the clutch Q is disengaged from both the wheels T and U, and they move on their bushes without carrying round the lying shaft. The clutch is made to slide on a part of the shaft which is square.

When the mill goes too quick the cross gland is raised, and in turning round strikes the prong 3, which immediately causes the lever to throw the clutch into the arms of the wheel U, which then carries the clutch and shaft round with it, and by the means already described acts on the sluice, and by lessening the quantity of water falling on the wheel, diminishes its speed.

On the other hand, when the mill goes too slow, the cross is depressed, and striking the prong 4, reverses the motion of the shaft, and so produces a contrary effect on the sluice.

400. It may be proper to remark, that the train of wheel-work is so calculated as very much to reduce the motion at the sluice, and it is found from experience that this is necessary. Were the area of the aperture too suddenly changed, the effect on the water-wheel would be too violent. Every time the mill is stopped, it is proper to lift the wheel R out of gear. The centre on which the sluice turns should be one third of its height from the bottom, in order that the pressure of the water above the centre may balance that below.

At *m* there is an upright shaft, which is worked by hand when required.

WATER-WHEEL GOVERNOR.

Second Construction.

401. Fig. 6 represents a sluice regulator as executed in some parts of England. It differs little from that already described, only that the lying shaft A B receives its motion immediately from the mill, instead of from the axle of the revolving pendulum, as in the first construction. (Art. 399.) From having so minutely described that construction, it is hoped that the attentive reader will find no difficulty in comprehending Fig. 6, from inspecting the plate.

WATER-WHEEL GOVERNOR.

Third Construction.

402. Fig. 7, Plate XV. represents a water-wheel governor of a very simple construction, differing from the

foregoing in this respect, that it communicates most part of its motion by bands and pulleys instead of wheel-work. The motion is reversed by the simple means of having one of the pulleys A with an open band, and the other B with a cross band.

OBSERVATION.

It is proper to observe here, however, as was already done with regard to the governor of the steam engine, that wheel-work is much to be preferred in point of certainty, to bands and pulleys.

WATER-WHEEL GOVERNOR.

Fourth Construction.

403. This construction is represented in Fig. 8. The revolving pendulum A K L M is kept in constant motion by the water wheel.

A, A, two wheels fixed on round sockets upon the governor spindle.

B, a clutch upon a square part of the spindle, (or what might be better, a round with a feather upon one side;) C, a gland to connect the clutch with the sliding part K of the governor, which has a groove to receive it like that of a bayonet. (See Essay IV. Art. 282.) D, a piece of iron which prevents the gland from turning round, and for keeping it from flying off; E, a wheel working into the wheels A, A; F, an endless screw upon the same axle with the wheel E; G, a wheel upon the same axle with another screw H, which acts into the quadrant I, upon the sluice.

The operation of this ingenious apparatus, from what has been said of the other constructions, will, it is hoped, be sufficiently clear. This governor was designed by James Carmichael, millwright, of Dundee.

WATER-WHEEL GOVERNOR.

Fifth Construction.

404. From the inspection of Fig. 9, and what has been already said respecting water-wheel governors, this particular construction will doubtless be easily comprehended. We need only mention that the wiper A, by means of the forked lever BDC, acts on the clutch E. The rest of the movements resemble those of the first construction. (Art. 399.)

This apparatus, remarkable for its neatness and simplicity, was constructed by Hewes of Manchester*.

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* A simple and not a very expensive apparatus for equalizing the exertion of horses in thrashing-machines, is described in the Art. Agriculture, Supplement to Ency. Brit. p. 200; and in Brewster's edition of Ferguson's Lectures, p. 201, vol. ii.

APPENDIX.

ON THE

VELOCITY OF WATER-WHEELS.

No. I.

On the Velocity of Water-Wheels, by Mr. Robertson Buchanan, Engineer: communicated in a Letter to the Editor of the Philosophical Magazine.

The accompanying paper was read in May 1799, in a Philosophical Society at Edinburgh, and was afterwards published in the Philosophical Magazine.

405. There are many cases in which it is of importance to know the proportion of power necessary to give different degrees of velocity to a mill*. But as the construction of mills, and the purposes they serve are various, it is perhaps impossible to find any law of universal application. Mr. Banks, in his *Treatise on Mills*†, has drawn a conclusion which he appears to consider as invariable, namely, that "when a wheel acts by gravity, its velocity will be as the cube root of the quantity of water it receives."

But if we suppose a wheel raising water by means of cranks and pumps, on Mr. Banks's principle, Buchanan thought it might easily be demonstrated, that by reducing the velocity of the wheel to a certain degree, the wheel would raise more water than would be necessary to move it at that velocity; a thing evidently impossible.

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* It was a scarcity of water for the Rothesay mills which directed my attention particularly to this subject.

† See Banks on Mills, pp. 17, 18, 144, 145, 146.

In this view it would seem there is no actual case in which Mr. Banks's conclusions will hold true. But, however they may apply to other mills, the experiments of Buchanan seem to prove at least that they do not apply to cotton-mills. On the ground of these experiments, made at different times, and with all the attention in our author's power, (and not from any abstract consideration,) did he presume to call in question an authority for which we entertained the highest respect.

406. In January 1796 he measured the quantity of water the Rothesay old cotton-mill required: 1st. When going at its common velocity; and 2dly, when going at half that velocity. The result was, that the last required just half the quantity of water which the first did. It is to be observed, that in these experiments the quantities of water were calculated from the heads of water and apertures of the sluices.

From these experiments he inferred, "That the quantity of water necessary to be employed in giving different degrees of velocity to a cotton-mill, must be nearly as that velocity."

He was satisfied with this experiment, and the inference drawn from it, till some gentlemen well acquainted with the theory and practice of mechanics expressed their doubts on the subject. He had then recourse to another experiment, which he considered as less liable to error than the former.

407. The water which drives the old cotton-mill falls, a little below it, into a perpendicular-sided pond, which serves as a dam for a corn-mill at some distance below it. To ascertain, therefore, the proportional quantities of

water used by the old mill, nothing more was necessary than to measure the time the water took to rise to a certain height in that pond; and accordingly, on the first of May 1798, he made the experiments noted in the following table:

Number of experiments.	1	2	3	4
Revolutions of one of the upright shafts per minute.	46	46	24	23
Rise of water in the pond in inches.	5	5	5	5
Time in minutes and seconds.	6·58	6·57	14·45	15·0

The first and second experiments were made with the mill at its common velocity; the third and fourth at nearly half that velocity.

The time which the mill required to use the same quantity of water in these experiments may be taken in round numbers; the proper velocity at 7 minutes, and half that velocity at 15 minutes.

The result of these experiments approaches very nearly to that of 1796. The difference may be accounted for by the small degree of leakage which must have taken place at the sluices on the lower end of the pond; and the time being greater in the third and fourth experiments, the leakage would of course be greater.

408. Smeaton* and others have proved, in a very satisfactory manner, that "the mechanic power, that must of necessity be employed in giving different degrees of velocity to the same body, must be as the square of that velocity." But it appeared to Buchanan, that the result of the above experiments may be easily reconciled to this proposition, by considering what Smeaton says immediately

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* See Smeaton on Mills, p. 18. See his Miscellaneous Papers, p. 92.

afterwards:—"If the converse of this proposition (says he) did not hold true, viz., that if a body in motion, in being stopped, would not produce a mechanical effect equal or proportional to the square of its velocity, or to the mechanical power employed in producing it, the effect would not correspond with its producing cause."* Now it is to be observed, that Smeaton's experiments were made on the *velocity* of heavy bodies *free from friction* and *other causes of resistance;* but in mills there is not only friction, but obstacles to be removed: and experiments made on friction have proved that the frictions of many kinds of bodies increase in direct proportion to their velocity. But the velocity of a cotton-mill at work may be considered as a mechanical effect; and, if so, must correspond with its producing cause.

409. The preceding experiments on the Rothesay mill, are undoubtedly correct and consistent with the principles of motion and power, and also with the experiments of Smeaton on Mills and Mechanic Power.

It is shewn in the additions to this essay that the mechanical power is as the quantity of water on the wheel, multiplied into its velocity when the wheel, fall, and other circumstances remain the same, and since the mechanical effect is measured by the resistance multiplied into the velocity of the working point when the friction is constant; if the quantity of water be diminished by its half, either half the resistance, or half the velocity with which it is overcome, must be taken away, otherwise there will not be an equilibrium between the power and effect. But at the same time it is to be observed, that an increased velocity lessens the friction of the intermediate machinery, and consequently a greater effect would be produced by the greater velocity, as appears to be the case by the experiments. There is not, however, in the detail of these experiments, sufficient data to enable us to arrive at any useful conclusions.

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* Smeaton on Mechanic Powers applied to Bodies at rest. Miscellaneous Papers, p. 92.

410. Roberton, an engineer of some eminence, made observations on the foregoing paper, alleging that the conclusions of Banks give most satisfactory evidence that particular care and judgment are necessary in making experiments.

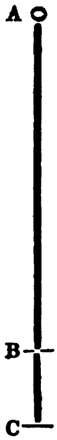
It appeared to Roberton that the wrong conclusions which have been drawn by this and other writers on this subject have wholly arisen from misapprehending some of Sir Isaac Newton's fundamental principles of mechanics, and from a love of establishing theoretical expressions rather than strict observations of the invariable laws of nature; expressions such as these: viz., *Quantity of Motion, Instantaneous Impulse.*

Taking a constant portion of time (viz., a second) to be the measure of the velocity of a body, and an instant to be the measure of the effect it produces; or by taking time as the measure of the cause, and space as the measure of the effect. As to an *instantaneous* effect, Roberton argues that it is an absurdity in itself as well as in mechanics. We can form no idea of a body put into motion, without the acting power or body act upon the body put into motion for some *time*, and also over some *space;* and to suppose otherwise leads us entirely out of the sound principles of mechanics.

In mechanics every effect is equal to its producing cause. In the case of a power acting on a body producing motion, and also this body acting against another power which retards its motion: if the causes of action and resistance are each measured by the *time* the motions are produced and retarded, the result will be equal.

Or if they be measured by the *space* over which they act, the results will be equal; and this is an universal principle, whether applied to accelerating power and motion, as gravity, &c., or to machines which act constantly and uniformly. Yet in the case of uniform motion, space or time may be used at pleasure; as from the uniformity of space and time they become a common measure.

To illustrate this, suppose the body A acted upon by the power of gravity through the space AB, in a portion of time which we will call one. When it arrives at B, it meets with another medium of resistance, which is ten times greater than the former; the body A will be resisted in proportion to the cause of action and resistance, that is to say, if the *time* of action were one second, the *time* of resistance will be one tenth of a second, and the distance AB will be to the distance BC as ten to one; so that whether *space* or *time* be taken as the measure of action, the same must be taken for the measure of the effect, to have the results proportionate and equal. But if the cause be measured by *time*, and the effect by *space*, the results will be as the squares of the times, or, which is the same thing, as the squares of the velocity.



Thus, suppose a body in motion, with a velocity of one, has a power to penetrate into a bank of earth one foot. If the same body, with a velocity of two, strike the bank, it will penetrate to the depth of four feet: for the velocity is double, and the time of action is double, and therefore the results will be compounded of both, that is as the square of the velocity.

From the above it may be inferred, that if equal bodies be acted upon by unequal powers, the times requisite to produce an equal motion will be reciprocally proportionate to the powers; that is to say, if a power of ten act upon a body for one second of time, and the power of one act upon an equal for ten seconds, they will produce equal velocities. But the spaces through which the bodies are carried are very unequal, being as ten to one: and if the square roots of the powers producing the effects be taken, that will give the times they take in carrying the body acted upon through equal spaces.

But it is obvious this doctrine has no more to do with the operation of machines, than simply their first starting from rest to the motion necessary for working. When this is acquired, the power applied and the power of resistance balance each other, and whatever be the motion the machine moves at, the same power will carry it on, (if it be upheld,) provided the machine act in such a manner as not to accumulate resistance by the accumulation of motion, which is the case in forcing fluids through pipes, &c. In cases of this kind, the nature of the machine must be particularly kept in view, and not to adopt any law to explain the resistance the acting body meets with, but what is simply deduced from the very machine we have under consideration. But in most cases, any machine may be considered as acting purely on a statical principle. The raising of weights, or overcoming friction, Roberton considers purely as acting on that principle; and when the power of action is equal to the resisting power, the machine is indifferent to motion or rest. If the machine be at rest, the power will not move it, being a balance to the resistance. If the machine be set in motion, the power will keep it in the same motion, (provided the power be upheld,) the same as equal weights hung over a pulley, or in the opposite scales of a beam. If they be at rest, they will remain so; and if they be put in motion, they will endeavour to persevere in the same.

The above doctrine of a statical principle is proved in the most satisfactory manner by the experiments made at the old mill of Rothesay, the motion of the water-wheel being exactly proportioned to the quantity of water expended, and therefore an exact and equal load upon the wheel; that is to say, the *buckets were equally full* when the mill moved at its *ordinary motion*, or at *half that motion.*

The effect, therefore, of letting more water on a wheel, is not to lodge a greater quantity in the buckets, but to supply the same quantity when the wheel is in a greater motion.

Banks, however, made his experiments agree with his theory, yet Roberton took no trouble in enquiring into them, alleging it would be to little purpose to have done so.

"Suffice it to say," he adds, "that the very small quantities of water which Banks made use of, and the slowness of the motion of his wheel in his experiments, gives no ground for placing the smallest dependence on them, and when compared with the more judicious and accurate experiments of Smeaton, they dwindle into contempt."

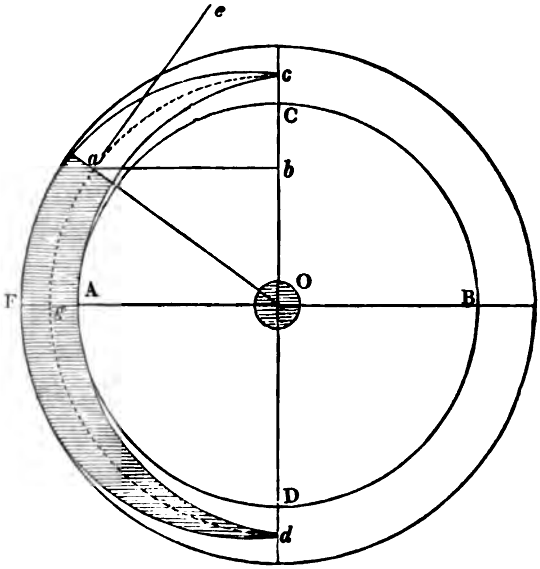
411. Roberton further says that "Smeaton, in running his wheel at nearly three feet in the second, brought it nearly to a maximum, and lost but about one fourth or one fifth of the original effect (alluding to his *overshot* wheels). Banks, at his highest motion, run his wheel about one foot in the second, and reducing it to one half of that motion, the same quantity of water then expended, was capable of performing four times the work; and by deduction from thence, it appears plain that his wheel, (from his own theory,) would perform about twenty times the quantity of work which Smeaton's could perform with the same quantity of water, and about 16 times more than nature; so that the observation, (alluding to the theory of Banks,) is very just in saying that, by reducing the motion of the wheel, it is demonstrable it would raise more water than supply itself."

ON OVERSHOT WATER-WHEELS.

412. The best water-wheel is that which is calculated to produce the greatest effect when it is supplied by a stream, furnishing a given quantity of water, with a given fall.

The mechanical effect depends on the proportion of the wheel's diameter to the height of the fall; and on the velocity of the circumference of the wheel. These are the two principal parts to be considered in the theory of wheels; but there are also some other points which ought to be attended to, because the effect is much decreased when they are neglected.

Of the proportion of the radius of the wheel to the height of the fall.—Let ACBD be the wheel, and EA the depth of the buckets; then, according to experiments on water-wheels, it appears that the rotary force of the water in the buckets is nothing at *c* and *d*, and that it increases nearly, if not accurately, in the direct ratio of the distance



from *c* or *d*, and is greatest at A. That is, the force at any point *a* in a direction *ea*, or perpendicular to the radius, is as *ac*.

A slight consideration of the figure is sufficient to inform us, that the wheel will not produce the greatest effect when it receives the water at the upper point C, and that there must be considerable advantage in making the wheel of a greater diameter, so that it may receive the water at some point between A and C. The point which will ensure the greatest effect we are now to calculate.

Put $c =$ that portion of the circumference which is to be loaded with water; and $x =$ the arc comprehended between the point where the water flows upon the wheel and the horizontal line EA; also make $b =$ the area of the stream supplying the buckets. Then the solid which represents the effective force, will be $\frac{1}{2}\, b \times \left(\frac{c^2 - 2x^2}{c - x}\right)$; which is to be the greatest possible; or $\frac{c^2 - 2x^2}{c - x} =$ a maximum. By the principles of maxima and minima, this takes place when $x = c\,(1 - \sqrt{\tfrac{1}{2}})$ or $x = {\cdot}2929\, c$. Accordingly the arc $c - x$ must be the quadrant *dg* or 90°, and the arc $x = 37{\cdot}27°$.

Hence we have this important practical maxim. A water-wheel will produce the greatest effect when the diameter of the wheel is proportioned to the height of the fall, so that the water flows upon the wheel at a point about $52\frac{3}{4}$ degrees distant from the summit of the wheel.

If r be the radius of the wheel to the extreme part of the bucket, and h the effective height of the fall, then $h = r$ $(1 + \sin. 37\frac{1}{4},)$ or $h = 1{\cdot}605\, r$; for the sin. $37\frac{1}{4} = {\cdot}605$. Also ${\cdot}623\, h = r$. Therefore when the effective height of the fall is determined, the radius of the wheel is easily calculated. When the effective fall is $\frac{8}{9}$ of the whole fall, if we make h the whole fall, $r = {\cdot}554\, h$, or $1{\cdot}108\, h =$ the diameter of the wheel.

The effective height of the fall is less than the true height, by as much as is necessary for giving the water the same velocity as the wheel before it flows upon it.

In low falls a wheel would work with advantage in a considerable depth of tail water, provided the buckets were of a suitable form for moving through the water, and the effective fall made through a very accurate sweep, so that the sweep and not the form of the bucket should confine the water upon the wheel.

413. *Of the velocity of the circumference of the wheel to produce a maximum effect.*—It is necessary to premise, that the velocity with which the water flows upon the float-boards or buckets, is considered to be equal to the velocity of the wheel, and to strike against the floats as nearly as possible in the direction of the motion of the wheel.

Let x be that part of the fall which gives the necessary velocity v to the water, when the effect is a maximum; v will then be the velocity of the circumference of the wheel. Also, make $a =$ that part of the fall which would correspond to the velocity of the circumference of the wheel when the power would be equal to the friction of the loaded machine only; or when the useful effect would be nothing. Now if h be the whole fall, the effective force of the water on the wheel will always be proportional to $h - x$, when the effect is a maximum; and to $h - a$, when the useful effect, or work done, is nothing.

Hence, $v\ (\overline{h - x} - \overline{h - a})$ must be a maximum; or, $v\ (a - x) =$ a max., but $v = x^{\frac{1}{2}}$, therefore $x^{\frac{1}{2}}\ (a - x) =$ a max., which according to the rules of maxima and minima takes place when $a = 3x$.

It is evident that the value of a must entirely depend on the nature of the machine, for if there be many moving parts between the power and the resistance, the friction will be greater, and consequently a will be less. The machine must be very simple indeed, if the friction be less than one half the moving power, and it will often amount to two thirds of it. If we suppose it to be two thirds, then $a = \frac{h}{3}$, and consequently $x = \frac{h}{9}$, and $v = \sqrt{\frac{64\frac{1}{3}h}{9}} = 2{\cdot}67\sqrt{h}$.

Hence, when the friction amounts to two thirds of the moving power, the velocity of the circumference of an overshot wheel in feet per second, should be 2·67 times the square root of the whole height of the fall in feet.

Again, that part of the fall is to be determined, which will give the water the same velocity as the wheel, and since $a = \frac{h}{3}$, and $3x = a$, we have $x = \frac{h}{9}$. Hence, when the friction is two thirds of the power, that part of the fall which will give the water the proper velocity, is one ninth of the whole height.

These results may now be usefully compared with the experiments of Smeaton; at the same time it is obvious that his experiments were not adapted for arriving at general conclusions, because the water was always delivered upon the same wheel; for it is clear, from the preceding investigation, that every particular wheel must have its particular maximum.

In Smeaton's experiments on overshot wheels, the wheel was 2 feet in diameter, therefore the height of the fall should be 2¼ feet. Now the square root of 2¼ is 1·5; and $1{\cdot}5 \times 2{\cdot}67 = 4{\cdot}005$, that is, the velocity of the wheel should be 4 feet per second; or it should make 38 turns per minute. Smeaton infers that "the best velocity for practice" will be when a wheel of 2 feet diameter makes 30 revolutions per minute. (Miscellaneous Papers, p. 51.) But his model had much more friction in proportion to the effective force of water on the wheel than two thirds, that we have here calculated upon. When the calculation is made according to the friction of Smeaton's model, $v = 2{\cdot}4\sqrt{h}$; and the velocity of the model wheel would come out 3·6 feet per second, or 34 turns per minute. This velocity will perhaps apply correctly enough to overshot wheels, where the water flows on at the summit, and to rough made machinery; but the former calculation is that which I consider most correct, for the improved kind of wheels here pointed out. It is to be understood, that the friction allowed for, includes all the kinds of resistance and loss of force which lessen the useful effect, as well as the resistance of the rubbing surfaces, properly called friction. Many readers will think that two thirds of the effective force, is greatly too much to be lost; it will be well if it draw their attention to lessening the stress on every part of machinery, and to the importance of having few rubbing surfaces, and other causes of resistance.

414. *On computing the power of overshot water-wheels.*—In determining the proportion of the radius of the wheel to the height of the fall, an equation is given for the effective force. Resuming that equation, we have $\frac{1}{2}b\left(\frac{c^2 - 2x^2}{c - x}\right)$ = the effective force of the water, and $\frac{1}{2}bv\left(\frac{c^2 - 2x^2}{c - x}\right)$ = its mechanical power. But the quantity of water expended in maintaining this power, will be bv. Hence, the quantity of water expended, is, to its mechanical power, as $1 : \frac{1}{2}\left(\frac{c^2 - 2x^2}{c - x}\right)$.

When the wheel is supplied at the summit, $x = \frac{1}{2}c$; and therefore, the quantity of water expended, is to its mechanical power, as $1 : \frac{1}{2}c$. Or the power is equal to half the weight of water supplied to the wheel.

The same relation takes place when $x = o$; that is, when the wheel is supplied at the height of the axis. Hence when the radius of a breast wheel is equal to the effective height of the fall, its power will be the same as that of an overshot wheel supplied at the summit.

When the wheel is supplied at the point which produces the greatest effect, $x = \cdot 2929\,c$; and consequently the quantity of water expended is to its mechanical power as $1 : 0{\cdot}5857\,c$. This effect is greater than when the wheel is supplied at the summit in the ratio of $1{\cdot}1714 : 1$.

These comparisons will convey some useful information to many readers; and they may sometimes suggest to scientific writers the advantage of studying the actual nature of machines; for relations so extremely obvious and simple could never have been overlooked by any one who might have condescended to examine the subject.

The power of a water-wheel may be considered under two points of view; each of which has its peculiar use. If we wish to compare it with any other first mover, then we shall have to calculate its mechanical power. But when it is desirable to compute the resistance it will overcome at the working point, the effective force should be calculated.

415. When the water flows upon the wheel, either at or above the axis, the mechanical power is $\frac{1}{2}\,bv\,\frac{c^2 - 2x^2}{c - x}$ cubic feet of water, or $31{\cdot}25\ bv\,\frac{c^2 - 2x^2}{c - x}$ lbs. Where bv is the quantity of water expended in a second, in cubic feet, c the part of the circumference between the lowest point of the wheel, and the place where the water flows upon it in feet, and x the part of the circumference between the point which is level with the axis, and that where the water flows upon the wheel in feet.

Throughout these Essays, the mechanical power of a horse is estimated at 200 lbs. moving with a velocity of $3\frac{2}{3}$ feet per second. Then a water-wheel will be equal $\frac{31{\cdot}25\,bv\,(c^2 - 2x^2)}{200 \times 3\frac{2}{3}\,(c - x)}$ horses; $= \frac{\cdot 00426\,bv\,(c^2 - 2x^2)}{c - x}$ horses.

When the water flows on either at the summit or at the level of the axis, the mechanical power is $31 \cdot 25\ bvc$ lbs. or it is $= 0 \cdot 00426\ bvc$ horses.

When the water flows on at $52\frac{3}{4}$ degrees distant from the summit, the mechanical power is $37 \cdot 192\ bvc$ lbs. or $= \cdot 005\ bvc$ horses. Since in this case, $c = 127\frac{1}{4}$ degrees of the circumference, we have $c = 127\frac{1}{4} \times \cdot 0174533\, r$; and as $r = \cdot 554\, h$; and $v = 2 \cdot 67\ \sqrt{h}$; by substituting these quantities, we have $122 \cdot 176\ bh^{\frac{3}{2}}$ lbs. = the mechanical power; or $\cdot 0164\, bh^{\frac{3}{2}}$ = the number of horses, where h = the whole height of the fall in feet, and b the area of the aperture through which the water flows upon the wheel in feet.

416. The effective force is $31 \cdot 25\ bc$ lbs. when the water flows on either at the summit, or at the level of the axis.

When the water flows on at $52\frac{3}{4}$ degrees distant from the summit of the wheel, the effective force is $37 \cdot 192\ bc$ lbs. or $45 \cdot 746\ bh$ lbs.

OF THE POWER OF BREAST WHEELS.

417. When the water flows on below the level of the axis of the wheel, it may be termed a breast wheel.

Let y be the distance below the axis measured on the circumference, then $\dfrac{c^2\, bv}{2\,(c + y)}$ equal the mechanical power in cubic feet of water, or $\dfrac{31 \cdot 25\, c^2\, bv}{c + y}$ lbs. When $y = c$ the power will be reduced one half, and when $y = 2\,c$ it will be reduced two thirds, and so on.

If we assume that the mechanical power of an undershot wheel is half that of an overshot one "under the same circumstances of quantity and fall;"* then it will be an advantage to employ an undershot wheel whenever the fall is less than three tenths of the radius of the wheel. But since the radius of the wheel may in many cases be diminished, it does not appear to be desirable to employ an undershot wheel in any case, except where the quantity of water is great and the fall inconsiderable.

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* Smeaton's Experiments, Miscellaneous Papers, p. 49.

ESSAY VI.

ON

CHANGING THE VELOCITY OF MACHINERY WHILE IN MOTION.

INTRODUCTION.

The machinery employed in manufactures may be divided into two classes: 1st. Millwork. 2nd. Smaller Machinery. The mechanism described in this Essay belongs to the latter class, and has hitherto been chiefly used in cotton-mills; but useful hints may perhaps be taken for its application to other valuable purposes.

It would be satisfactory to be able to record the names of the inventors of many of the ingenious contrivances which are described in these Essays. But the secrecy which interest prompts in the machinery used in manufactures—the same difficulties giving rise in different minds, without any communication of ideas, to the same means of overcoming them, and the very gradual steps by which improvements are usually made, render it, in most cases, impracticable to trace the inventions to their true sources.

It may be taken for granted that the reader is acquainted with the common modes of altering the velocity of any particular part of machinery, by changing the wheels or pulleys. This change, however, requires that the machinery be stopped for some time until the alteration be made.

But many cases occur in which it is desirable to change the velocity without such loss of time. Some of those cases shall now be considered, beginning with one of the most simple,—that of changing the speed of a turning-lathe, according as the nature of the substance to be turned, or its diameter may require.

SECTION I.

I.—TURNING LATHE MOTIONS.

419. A series of pulleys gradually increasing in size upon an axle, is moved by the mill, and on the spindle of the lathe is a similar series, but in an opposite order, so that the same length of belt will work on all the opposite pulleys, according to the speed required.

These series resemble two truncated cones, having the smaller diameter of the one opposite the greater diameter of the other, so that the same belt is equally tight on whatever pair of pulleys it may work.

This contrivance is represented by Fig. 1, Plate XVI. applied to the spindle A B of a turning lathe. C D is part of a shaft driven by the mill at a certain regular velocity. When a slow motion is required, the belt works at E F; when a greater velocity is required, the belt is shifted by pressing it to one side, to another pair of opposite pulleys. It is hoped that the figure will make this so clear, that all further explanation will be unnecessary.

OBSERVATION.

420. This contrivance, very simple in its construction, is found of important practical use in the turning of various substances.

II.—ALTERNATE CONES.

421. There is another contrivance on similar principles to that above described, which has been found very useful where a motion constantly varying is required. Instead of the opposite series of pulleys, there are two opposite cones.

The one of these cones gives motion to the other by a belt which by the machinery is gradually moved from one end toward the other of the cones.

This piece of machinery is represented by Fig. 2. A B is the belt; C is the guide, which, receiving its motion from the machinery, traverses the belt at pleasure, with any velocity which the case may require.

Thus the one cone moving at a uniform motion, communicates a varying velocity to the other.

OBSERVATION.

422. This piece of mechanism, remarkable for its simplicity, I have had occasion to put extensively in practice, and have found it give great satisfaction.

III.—ALTERATION OF VELOCITY BY WHEELS MOVING ONE ANOTHER BY FRICTION.

423. The same effect as the alternate cones is sometimes produced by the rim of one wheel moving on the face of another, by means of the roughness of their surfaces, the inequalities of which may be considered as indefinitely small teeth.

Thus A B, Fig. 3, is a face-wheel, moving at a uniform rate.

C D is another wheel, which, from the face of A B, receives a vertical motion. Accordingly as it is required to move C D slower or faster, it is by proper contrivances made to act nearer or further from the centre of A B.

OBSERVATIONS.

424. As there must a twisting motion, similar to that of edge-stones for bruising various substances, take place here, and as it is only properly applicable to cases in which the strain is exceedingly small, I apprehend that the alternate cones is a much more perfect manner of producing a change of velocity. These wheels, however, work very well for regulating the taking up motions of the bobbins in machines for roving cotton by spindles. In this case, the force required is very small.

SECTION II.

425. Certain cases in practice require an instantaneous change of velocity; as, for instance, when those machines for spinning cotton, called *Mules*, are moved by power.

The *mule* is a machine different in its construction from that brought to a high state of perfection by Sir R. Arkwright.

The mule is better adapted than the *water-twist frame* (Arkwright's Machine) for spinning all kinds of *weft*, and produces finer yarn than can be spun by any other machine. For the invention of the *mule* we are indebted to James Crompton, formerly of Hall-in-the-Wood, near Bolton-le-Moors, Lancashire. This machine was, for many years, worked by hand only, the variety of its movements rendering it difficult to accomplish the moving of it by power of water or of steam sufficiently simple to be of common use.

William Kelly at Lanark, early obtained a patent for a mode of working this machine by power, but it was not until a considerable time afterward that power was generally adopted. The plans which were tried were very various, and the improvement was progressive. One happy consequence of this improvement has been experienced; the spinners are now found to enjoy better health than they did when they had to labour hard, while they breathed in warm and confined apartments.

In order to save time after the carriage of the mule is brought to its furthest extent, it is necessary to increase the velocity of the spindles. This increase of velocity is called the *double speed.* Various contrivances have been adopted for this purpose, but three only shall be described; one being performed by *ropes*, another by *belts*, the third employing the aid of *wheels*. This last, indeed, from its greater certainty, is what is most generally adopted. These contrivances will, however, serve to shew the progress of improvement in this species of machinery.

DOUBLE SPEED.

First Construction.

426. The axle A B, Fig. 4, Plate XVII. is suspended by a cast iron frame from the ceiling of the room. This axle is kept in motion by means of the fixed pulley C, which is moved by a belt from the mill-work. On the same axle are two loose pulleys D and E. (Essay IV. Art. 280.) Ropes from these pulleys communicate with the fast pulleys F and G, on the axle X Y of the fly wheel of the mule.

The loose pulleys have catches on their sides; while these are disengaged the mule is at rest. In order to put the mule in motion, the smaller pulley E, by means of the sliding guide I K L, is slipped to one side, so as to lay hold of the gland H, which is fixed on the axle, and carries the pulley round along with it, and thus moves the mule at its slower motion.

When the fly wheel W has made a sufficient number of revolutions at this rate, the slider is moved by means of wheel-work and a wiper toward the fast pulley A, which motion disengages the small pulley from the gland, and engages the larger pulley with C, which produces a quick motion in the fly wheel.

OBSERVATIONS.

427. This apparatus was in use in Manchester in the year 1797, but the shocks produced by the catches (Essay IV. Art. 281.) and other imperfections, soon occasioned its disuse. But there is often much to be learnt from the examination of machines which have been abandoned. It is but by comparison of things of the same species that we are able to appreciate their true value.

DOUBLE SPEED.

Second Construction.

428. This apparatus differs from the First Construction, principally in having belts instead of ropes for communicating motion.

On the axle A B, there are five pulleys, E, D, C, G, H, all of them loose but C, which is fast. When the belt from the mill-work is on C, the mule is at rest, because the axle revolves without carrying round any of the loose pulleys.

In order to put the mule in motion, the belt is, by means of a sliding guide, shifted on to the pulley D, which carries round the pulley E along with it; and by another belt, moves the pulley F on the fly wheel axle X Y, and thus moves the mule at its slower motion; afterward (as was described of the *First Construction*) the sliding guide shifts the belt from D to G, which, by carrying round H in a similar manner, produces a quick motion in the fly wheel W.

OBSERVATION.

429. This construction was in use in Manchester in the year 1799; and as the *shocks* complained of in the *First Construction* did not occur in this, it was found a material step in the improvement of working mules by power.

DOUBLE SPEED.

Third Construction.

430. This construction differs from the *second*, in having the whole apparatus attached to the framing of the mule, and in having the aid of toothed wheels for producing the change of velocity.

On the axle A B, Fig. 6, are three pulleys, C, D, E. The pulley C is fast on the axle. D and E are loose, but on the side of E is fixed the small spur-wheel F. The larger spur-wheel G is fast on the axle A B.

On the axle X Y are fixed other two spur-wheels, H and I, of the same size as those on the axle A B, but placed so as that the larger wheel on the one axle shall be constantly in gear with the smaller on the other.

When the belt (put in motion by the mill-work) is on D, the mule is at rest; when shifted on to E it carries the smaller wheel F round with it, which being *in gear* with the larger wheel H, moves the fly wheel axle X Y at its slower motion.

On the other hand, when the belt is shifted to the pulley C, which is fast on the axle, it carries round the larger wheel G, which is also fast. G being *in gear* with the smaller wheel I, moves the fly wheel at the greater velocity, or, as it is termed, at *double speed.*

OBSERVATIONS.

431. This piece of mechanism was first adopted in Manchester about the year 1800, and although sometimes its parts may be somewhat differently arranged, it continues, I believe, still in general use.

While it is free from the *shocks* produced by catches, it is also (owing to having the change of velocity produced by wheels) free from the uncertainties of motion arising from any change in the tightness of the belts as employed in the *Second Construction.*

ESSAY VII.

ON THE

FRAMING OF MILL-WORK.

PREFACE.

THE preceding Essays relate principally to the *moving parts* of machinery, but as it seemed essential to a system of mill-work, to say somewhat on the subject of the *framing* which supports the moving parts, Buchanan was induced to commit to paper the following ideas on that head.

SECTION I.

432. The general principles of carpentry must obviously be applicable to the framing of mill-work. These principles I shall not here repeat, but beg leave to refer to what Professor Robison has written on this subject, in the Encyclopædia Britannica, and to Mr. Peter Nicholson's various writings on the subject; I shall here consider only the peculiarities of the framing of mill-work*.

433. Mill-work, from its motion, occasions a tremor on

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* See also Art. Carpentry, New Supplement to the Encyclopædia Britannica. Tredgold's Elementary Principles of Carpentry, 4to. 1820; and Practical Essay on Cast Iron, 8vo. 1822.

all the parts of its framing, which subjects it to much more speedy decay than the mere pressure upon carpentry.

Besides this general tremor, it is often subjected to violent sudden thrusts, from the bad action of the wheels, or from reciprocating motions.

It ought, therefore, not only to be sufficiently *strong* and *stiff*, but sufficiently *heavy*, to give solidity and steadiness.

Where the framing of machinery is not firm and well bound, a vibratory motion in its parts, of course, takes place; which vibratory motion expends a considerable portion of the power applied. This loss of power is very difficult of investigation. It is certain, however, that whatever motion of a vibratory nature is communicated to the framing and objects in contact with it, (abstracting from the elasticity of the parts,) must be lost to the effect the machine would produce, were the parts sufficiently strong and well bound together; and it is to be observed, that firm and well-bound framing is much preferable to heavy framing not so well connected in its parts. It is as certain, that though the framing in either case may be constructed so as to be equally strong; yet the heavy framing, from its vibration, will expend more of the original power than that which is less heavy but firmly connected.

434. Besides *strength*, *stiffness*, and *solidity*, the framing of mill-work requires to be constructed so as *to be easy of repair;* and so contrived, that *any particular part may be repaired or renewed* with the least possible derangement to the other parts of the framing.

435. There is another circumstance in this species of framing which demands great attention. *The shafts often require to be restored to their true situations*, from which they may have deviated by the wearing of the parts. Now the framing ought to be so constructed as easily to admit of this *restoration of the shafts*, as also of any other shifting of them which may in practice become necessary.

436. But though the framing which supports the parts of mills and machines should be firm, it is an advantage that the part on which any axis rests should have a small degree of elastic tremor when the machine is in motion. Such tremor has considerable power in diminishing the friction. It may further be observed, that framing to support machinery should be as independent of the building as possible, because the tremor it always communicates is exceedingly injurious.

Before proceeding further into the subject, it may be proper to consider the *bearings* of shafts.

SECTION II.

OF THE BEARINGS OF SHAFTS.

437. The bearings on which gudgeons and journals rest and revolve, are sometimes termed *Pillows*, and frequently *Brasses*, from being often made of that substance.

The *bearings* for pivots, at the lower extremity of upright shafts, are denominated *Steps;* the parts where the journals of vertical shafts or spindles turn and bear against are called *Bushes;* and for small spindles, such as those used in the manufactures of flax and cotton, *Breasts*.

It has become general to fix pillows in blocks of cast iron. Hence the term *Pillow Block*, and sometimes, corruptly, *Plumber Block*. In Manchester they are called *Pedestals*.

438. The SUBSTANCES used for *Pillows*, &c., are various, but brass is the most common*. Other substances, however, which are cheaper, have in many instances been found equal, at least, in durability.

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* The metal our author terms brass, is usually the composition of copper and tin, called gun metal. Gun metal is much harder than common brass, and much more durable. Common brass is a compound of copper and zinc, and is now rarely used for bearings.

At the cotton works of Deanston, near Down, a water wheel has run nearly 30 years on pillows of cast iron, with little sensible wear on the gudgeons, nor were they ever found liable to heat*.

The outer skin of cast iron, particularly when cast in metallic moulds, is remarkably hard, and it is reasonable to suppose that it would make a durable pillow, as we have seen is the case in the above instance.

Mr. Murray of Leeds was enabled to bore the hardest cast iron, some cylinders of which, from the whiteness of the grain at the places broken off by a chisel, denoted its superior quality. Such iron is equally hard throughout.

A patent was granted long ago for wheel bushes of metal, of a peculiar hardness, which proved to be nothing more than very hard cast iron, but the patentee had discovered a mode of boring it, which Murray imitated.

Stone has often been used with good effect for pillows for gudgeons and journals. The great objection to stones for this purpose, is the difficulty, arising from their hardness, of forming them into proper shapes.

At Sheffield, where the journeymen grinders are obliged to keep this part of their machinery in oil, and in repair, they have found from long experience, that a piece of green (unseasoned) thorn tree is exceedingly durable†. But in general they prefer using brown paper, adding always one ply some time after another, so as to form a kind of pasteboard; this substance they find less liable to heat, and much more durable than brass‡.

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* Heating generally takes place from the surfaces of the journal and pillow being too small, and sometimes from the journal having worn too deep into the pillow, in which last case, in particular, a great friction takes place.

† Green oak soaked in boiling oil, is said to be used with advantage. Brewster's Additions to Ferguson, vol. ii. p. 179.

‡ If the bearings for gudgeons were made by screwing many thicknesses of pasteboard together, in the same manner as the rollers of calendars are made, they would be extremely durable, and have very little friction.

Wooden pillows are often used. Box wood and lignum vitæ were long in use. The latter has been found an improper substance for the purpose. Beech is preferable to either, and has been used with great success for steps. Holly has also been found to answer well for the same purpose*.

FORMS OF STEPS AND OF PIVOTS OF UPRIGHT SHAFTS.

439. The most usual form is to terminate vertical shafts somewhat like an egg, leaving the pivot rather smaller upwards, to prevent it from binding in the step. (Plate XVIII. Fig. 1.)

440. Another mode, (see Fig. 2,) is to terminate the shaft flat at the lower end, and make a small oblong recess in it. To this end is fitted a round piece of steel A, flat below, excepting a groove *a* across it, something like that in the head of a screw nail. The use of the groove is regularly to feed in the oil. The step into which the pivot works is made square or octagon, so as to supply oil from the angles. Nos. 2, 3, and 4, Fig. 2, are different views of the piece of steel A.

441. A pivot and step on this construction was deposited with the Society for the Improvement of Arts, and was said to have been 20 years in use in a horse-mill, without being perceptibly worn. Buchanan applied this contrivance in a case of heavy upright shafts, where other means

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* Friction rollers are sometimes employed to diminish the quantity of friction, but not with much advantage in heavy machinery, because they are liable to get out of order, and require very accurate workmanship. Their effect depends on converting a sliding into a rolling motion, with only the slide of the small axis which retains the roller in its place. Friction wheels are also used for the same purpose; they differ from rollers in this—the stress on the axis is borne by the axes of the friction wheels, but the stress is on the surface of the rollers. The advantage of friction wheels is very trifling.

were found inadequate to prevent the action of the pivot, and it ever after gave satisfaction in its use. It is to be observed, however, that it does not answer well in cases having much lateral pressure, and a better mode (see Fig. 3) is now in use in several mills. The steel pivot is made cylindrical, and fixed into the foot of the upright shaft. The bottom of the step is convex, and there is interposed one or more pieces of steel formed like a double convex lens. These pieces of steel having a little motion, make the relative motion of the pivot less, and consequently lessen the friction.

442. The egg-formed pivot is sometimes made separate, of wrought iron and steel, and inserted into the lower end of the cast iron shaft*.

443. The late Mr. Bramah, in the specification of his patent machinery for surface planing, includes a mode of running pivots entirely on a fluid, and raising or depressing them at pleasure, by means of a small forcing-pump and stop cock. See Gregory's Mechanics, vol. ii. p. 418, 2d edition.

444. The journals of upright shafts are supported sometimes by breasts, (Fig. 4,) and sometimes by bushes, in passing through a floor.

445. The spindles of millstones usually run in wooden bushes. A block of cross elm, A B C, Fig. 5, about 9 inches diameter, and 3 inches thick, forms the principal part of it, and is lodged in the eye of the millstone. In order that the spindle may at all times run steadily, there are three pieces of hard wood, D, D, D, let into grooves in the block,

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* The rubbing surfaces of cast iron pivots should not have a greater pressure upon them than one ton upon a square inch, or they will be very subject to heat, and the friction and wear will be increased. Large vertical shafts may often be made to revolve on conical rollers, on the same principle as friction rollers; when they are well made, the motion is very smooth, and the friction much reduced.

so that their three ends embrace the spindle E. These pieces are of equal breadth throughout, so that they may easily be wedged forward when they wear. This simple and ingenious contrivance has been very long in use. Some use a piece of cast iron in preference to the block of elm, to answer the same purpose, and some a greased rope to run the spindle in, instead of the three pieces of wood.

SECTION III.

OF WOODEN FRAMING.

Headstock Framing.

446. The framing used for supporting the gudgeons of a water wheel is denominated the headstock framing.

A, Fig. 6, represents the headstock, which contains the pillow-block, B C D E is the frame.

The frame is supported by the top of the building of the arch or wheel-pit.

The headstock rests on the frame, and is moveable horizontally on all sides; when in its proper situation, it is kept there by a key or wedge G at each end of the headstock, which is supported by the dovetail form of the lower part of the headstock.

FRAMING FOR LYING SHAFTS.

447. Lying shafts are usually supported by bridges between posts. A B, Fig. 7, represents part of a lying shaft, C D the bridge which carries the shaft. It is raised or lowered by means of a key below, and another above it, in the mortice of the posts E G and H I, through which it passes.

The bridge is also moveable on end, and is kept in its proper situation by means of small keys K, K, K, K.

448. The posts, instead of being each made of one solid piece of timber, are sometimes framed of separate pieces, as represented in Fig. 8.

Lying shafts, instead of being supported by posts, are sometimes suspended from a ceiling, as shewn in Fig. 9. The bridge is tempered by keys, &c., as when posts are used.

FRAMING FOR UPRIGHT SHAFTS.

449. Upright shafts are generally supported by bridges adjusted endwise, and upward and downward, like those of the lying shaft; but in order that they may be moved horizontally in every direction, the pedestal is contrived to receive keys at its ends, similar to a headstock. (See Fig. 10.)

Screws are frequently used instead of wedges for adjusting the step.

450. Fig. 11 represents the framing of an upright and lying shaft connected by bevelled wheels.

451. Sometimes a bridge is not immediately supported by posts, but by intermediate pieces, which are called *cloves*. This construction is common in single corn-mills. Thus, AB, Fig. 12, is a bridge; CD and EF are cloves.

452. Respecting the decay of timber, and the means of preventing it, Buchanan refers to Dr. Parry's paper, in the Transactions of the Bath Agricultural Society, and reprinted in Nicholson's Journal, Vol. XX. Nos. 85, 86, 87, and Repertory, No. LXIII. That paper appears well worthy the attention of those who have occasion to construct works of timber*. Kyan's mode of preserving timber, till a better shall be proposed, now supersedes all the schemes heretofore promulgated.

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* See also "Elementary Principles of Carpentry," Sect. X. Art. 327—365.

SECTION IV.

OF CAST IRON FRAMING.

453. In a previous part of this work, mention has been made of the great increase of late years of the use of cast iron in mill-work.

Cast iron possesses great superiority over timber, for constructing the framing of mill-work. It is not only much more durable, but from the uniformity of its texture, may be converted into any shape, so as to give it great advantage in arranging the materials with respect to strength, and proportioning it to the stress it has to sustain. Timber, on the other hand, being stronger in some directions than others, and of very limited breadth, is confined in its arrangement, and requires, in certain cases, much workmanship; whereas, after the patterns for cast iron are once made, any number of castings may be formed from them with very little labour or trouble.

Those who have scientifically considered the strength and stress of materials, know that when timber is broken by any lateral pressure*, it is owing in a considerable degree to the compression of the beam on the hollow side, which puts the fulcrum of the ideal lever much nearer the point of resistance than it would be in a substance less liable to compression. Cast iron is much less liable to compression than timber, which gives it an advantage in withstanding lateral pressure, greater than might be expected from the mere comparative absolute cohesion of the substances*.

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* See Emerson's Mechanics, Sect. VIII. p. 93, and Gregory's Mechanics, Vol. I. Book I. Chap. V.

These references must have been made without consulting the works quoted, as the investigation of the strength of beams is conducted by both these writers on the supposition that the virtual fulcrum is an incompressible arris at one of the surfaces of a beam; and the one considers the materials to be extensible, the other inextensible.

454. "Iron is generally much more uniform in its strength than wood; yet experiments shew that there is some difference occasioned by different kinds of ore: the difference is not only found in iron from different furnaces, but from the same furnace and the same melting; this may arise in a great measure from the different degrees of heat which it has when it is poured into the mould."†

455. Banks concludes that a bar of the weakest cast iron, 1 inch square, and 1 foot long, will break with about 2190 lbs., and that cast iron is at an average 4 times as strong as *oak*, and 5½ times as strong as *deal;* the weight being in all cases applied in the middle; the beam lying horizontally, and supported by props‡.

456. The strength of any beam, to withstand any weight, being as the breadth and the square of the depth, (see Essay II. on the Shafts of Mills, &c., Chap. IV. Emerson, p. 93,) it is evident that a bar of cast iron of the same length, must be much stronger when its tranverse section is like Fig. 14, than when like Fig. 13. The form represented in Fig. 14 has a further advantage, that of greater *stiffness*. The distinction between *strength* and *stiffness* is not in practice generally understood or attended to. This distinction is most easily comprehended by considering their limits. The limit of *strength*, is well known to be *fracture*, or breaking. The limit of stiffness, is *flexure*, or bending. Now stiffness increases in a much higher ratio

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* The idea that the virtual fulcrum is nearer the compressed side in cast iron than in wood, when the pieces are similarly strained, is at best an assertion without a proof, either from theory or experience. And at the time our author wrote, the resistance of cast iron to compression was greatly overrated. See Essay on Cast Iron, Art. 63.

† Gregory's Mechanics, Vol. I. Art. A. 190.

‡ Banks on Power of Machines, &c., p. 94.

ratio than strength, viz., as the cube of the depth*. For example, if we double the depth of a beam, we increase its stress only 4 times, whereas we increase its stiffness 8 times. (See Essay II. on the Shafts of Mills.)

457. For these reasons, the advantage is evident of making cast iron framing in thin broad plates, at right angles to one another, instead of imitating the solid forms of wooden framing. This practice is called by millwrights *feathering*. The plate is sometimes on one side, as represented in Fig. 15, and sometimes its section is like the letter T, see Fig. 14, the whole being one solid mass.

The common practice in making cast iron framing now, is to imitate wooden framing, which has been found from experience sufficiently strong in giving the same breadth and depth of the several pieces at their point of greatest stress. Thus, suppose Fig. 16 to be the section of the timber at the place of greatest stress, the section of the cast iron is made like Fig. 17, or like Fig. 18; advantage is also taken of the nature of the material, to give it a breadth varying in proportion to the stress. This variation in shape is not always in practice judiciously done; by attending to what is said in Essay II., Chap. IV., the millwright, it is hoped, will be better enabled to proportion the parts to the stress they have to sustain†. In addition to what is said respecting the making of patterns, in Essay I. on the Teeth of Wheels, Buchanan says it is a good practice to give the patterns a thin coat of oil paint; as, while it preserves the pattern, the paint makes it rise more freely out of the sand.

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* Young's Nat. Phil. Vol. ii. Art. 333.

† The most advantageous forms for different purposes have been considered in the *Practical Essay on Cast Iron*, Sect. III. and IV., where extensive tables of the strength and stiffness of cast iron will be found, which may frequently save the millwright much trouble in calculation; for he cannot always have examples of the same construction to refer to, either executed in wood or iron.

458. To give an instance of this variation of form in the framing of mill-work, bridges of wood for sustaining the shafts are usually made as represented in Fig. 19, those of cast iron as shewn in Fig. 20.

459. In cast iron framing, advantage is also taken of the properties of the hollow cylinder, of the economical application of which form in nature we have so many beautiful examples. (See Essay II.)

460. A headstock of cast iron for a water-wheel is represented in Fig. 21, Plate XIX.

461. Various modes are used of suspending shafts from a ceiling. Fig. 22 represents a construction in very general practice.

462. Fig. 23 represents the cast-iron framing of a flour mill having three pair of mill-stones, and to Fig. 24, a machine used in bleaching, called *Squeezers.*

After the process of washing by the dash-wheel, the water is compressed from the cloth by means of this machine.

Squeezers consist of a pair of wooden rollers, which, in moving, draw the cloth through between them. The lower roller receives its motion from a mill, and the uppermost is pressed down upon it by means of levers. Till of late these rollers were fixed in strong wooden frames; but the framing is now generally made of cast iron, which makes a neater and more durable piece of work.

A represents the lower roller, B the upper roller, C D a lever which presses upon the brass of the upper roller, E F another lever to increase the power connected with C D. The extremity of F is kept down by a pin; in some cases a weight is used in place of the pin.

ESSAY VIII.

GEOMETRICAL AND PRACTICAL METHODS

FOR FINDING THE

CENTRES OF GRAVITY OF MILL WHEELS;

ILLUSTRATED BY EXAMPLES,

IN WHICH TWO, THREE, AND FOUR WHEELS COMPRISE THE SYSTEM UPON ONE AND THE SAME SHAFT.

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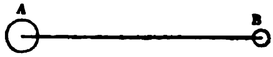
463. THERE is one branch of mechanical science which belongs essentially to mill-work that must be here added to these Essays of Buchanan. We allude to Methods of finding the Centre of Gravity of two or more bodies connected together by straight inflexible rods passing through their respective centres.

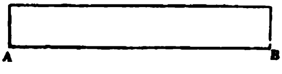
Suppose A and B to be two bodies connected together by the straight inflexible bar A B passing through their centres, and it were required to find the centre of gravity of those bodies.

FIG. 1.



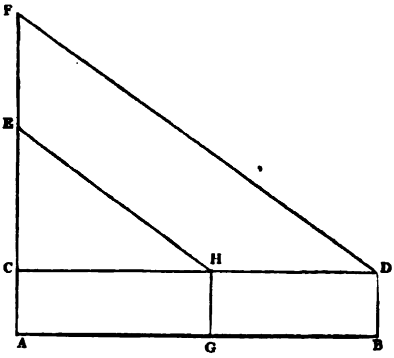
At the points A and B, Fig. 2, we should erect the per-

FIG. 2.



pendiculars AC and BD of any convenient length, and through C draw CD parallel to AB; then we should produce AC to F, Fig. 3, and make CE to EF as the body B is to the body A. Then joining FD, and through E drawing EH parallel to FD, and

FIG. 3.



from H dropping the perpendicular HG, the point G would indicate the centre of gravity of the two bodies A and B;

for A : B :: BG : AG

hence equating the products of the extreme and mean terms

$$A \cdot AG = B \cdot BG$$

From which we infer, that when two bodies connected together by a straight inflexible bar, are in equilibrio, the products of their masses multiplied by their respective distances, are equal.

Let a = the mass of A
b = the mass of B
d = the distance AG
δ = the distance BG
and D = the distance AB

Then according to the foregoing proportion,

$$ad = b\delta; \text{ but } \delta = \mathrm{D} - d$$

$$\text{consequently, } ad = b\mathrm{D} - bd; \text{ and}$$

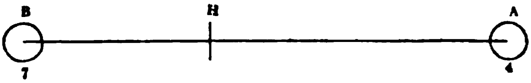
$$d = \frac{b\mathrm{D}}{a + b}; \text{ also } \delta = \frac{a\mathrm{D}}{a + b}.$$

Consequently, the places of the centre of gravity is known in terms of the masses, and the distance between their respective centres. Hence the following practical rule:

464. *Multiply either body by the whole distance between their centres; divide the product by the sum of the bodies; the quotient will be the distance from the centre of gravity of that body opposite to the one by which the whole distance is multiplied.*

Example. Let the two bodies be respectively 4 and 7 cwt.; and their distance asunder 24 feet.

Fig. 4.



Then we have $a = 4$, $b = 7$, and $D = 24$, or $\frac{7 \times 24}{4 + 7} = \frac{168}{11} = 15\frac{3}{11}$ feet, being the distance of the centre of gravity from the body A.

Also $\frac{4 \times 24}{4 + 7} = \frac{96}{11} = 8\frac{8}{11}$ feet, being the distance of the centre of gravity from the body B.

465. The example supposes the connecting rod to be void of weight; but in mechanics this is never the case. The same law must obtain, with respect to the portions of the connecting rod, that we saw existing in the mass of each body multiplied into its distance from the common centre of gravity. The centre of gravity of an uniform connecting bar must be at the middle of its length when that bar is prismatic or cylindrical.

If p = mass or weight of one unit or length of the bar, then is $\frac{pd^2}{2}$ = effective energy of one portion, and $\frac{p\delta^2}{2}$ = the effective energy of the other; and these, together with the effective energies of the bodies a and b referred to opposite sides of the centre of gravity, must still be in equilibrio; hence

$$a\,d + \frac{p\,d^2}{2} = b\,\delta + \frac{p\,\delta^2}{2}$$

But $\delta = \mathrm{D} - d$, and therefore by substitution we obtain

$$(a + b + p\mathrm{D})\,d = (b + \frac{p\,\mathrm{D}}{2})\,\mathrm{D},$$

which being reduced gives the following equations

$$d = \frac{(2\,b + p\,\mathrm{D})\,\mathrm{D}}{2\,(a + b + p\,d)}$$

$$\delta = \frac{(2\,a + p\,\mathrm{D})\,\mathrm{D}}{2(a + b + \mathrm{P}\,\mathrm{D})}.$$

Hence the following practical rule:

466. *To twice the weight of either body, add the whole weight of the lever or connecting bar, and multiply the sum by the central distance; then divide the product by twice the mass compounded of the bodies and the bar, and the quotient will be the distance of the centre of gravity from that body opposite to the one whose double is employed in the first step of the operation.*

EXAMPLE 1.—The bar is 24 feet, and weighs 1 cwt., the bodies 4 and 7 cwt. respectively as before:

Then $a = 4$; $b = 7$; $\mathrm{D} = 24$ feet, and $p = \frac{1}{24}$ cwt. $\therefore\ d = \frac{(2 \times 7 + \frac{1}{24} \times 24)\,24}{2\,(4 + 7 + \frac{1}{24} \times 24)} = \frac{15 \times 24}{2\,(4 + 7 + 1)} = \frac{360}{24} = 15$ feet, being the distance of the centre of gravity from A, and therefore $24 - 15 = 9$ feet, the distance of B from the centre of gravity; for $9 + 15 = 24$ feet.

EXAMPLE 2.—Let the bar be of cast iron, 42 feet long and 252 lbs. weight; the bodies at its extremities weighing 13440 and 17920 lbs. respectively.

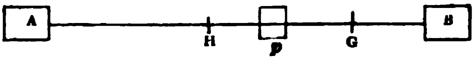
It will be found by calculation that A = 13440 and B = 17920, are respectively $23\frac{1102}{1129}$ feet, and $18\frac{27}{1129}$ feet from the common centre of gravity of the bodies.

467. When three bodies connected together by a straight

inflexible bar, are in equilibrio, the product of one body multiplied by its distance from the common centre of gravity of the system, is equal to the product which arises when the sum of the other two bodies is multiplied by the distance between their common centre, and that to which the whole system is referred*.

If a = mass of the body A;
b = mass of the body B;
p = mass of p concentrated in p;
d = distance between a and b;
δ = distance between a and p,
and x = AH the distance between a and the common centre H.

FIG. 5.



Then if H falls between a and p, pH $= \delta - x$, and HB $= d - x$; but if the common centre falls between b and p, we have pH $= x - \delta$, and BH $= d - x$; and in either case we have

$$(a + b + p)x = bd + p\delta$$

$$\text{or } x = \frac{(bd + p\delta)}{(a + b + p)}.$$

468. The practical rule is the following:

Multiply each of the bodies b and p by the respective distances from a; then divide the sum of the products by the aggregate of the three masses for the distance of the centre of gravity from the first body a, to which the distance of the other bodies b and p are referred.

EXAMPLE.—Let the bodies be 15, 20, 25 tons respectively; and their distance 12 and 16 feet from each other; then it will be found that

$$bd = 700;\ p\delta = 240;\ \text{and } bd + p\delta = 940$$

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* Dr. Jamieson's Mechanics for Practical Men. London, 1837, 8vo.

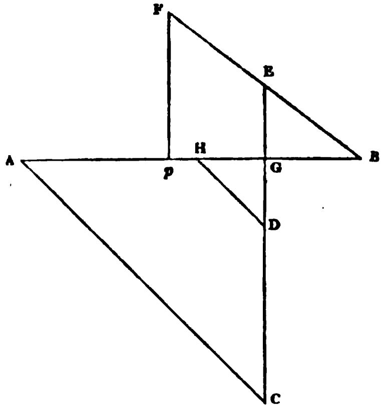
but $(a + b + p) = 60$; therefore x is distant from a by $\frac{940}{60} = 15\frac{2}{3}$ feet; $x - \delta = 3\frac{2}{3}$ = the distance of x from p; also $d - x = 12\frac{1}{3}$ feet = distance from b: that is to say;

AH = $15\frac{2}{3}$ feet, or the distance of a from H
pH = $3\frac{2}{3}$ feet, or the distance of p from H
and BH = $12\frac{1}{3}$ feet, or the distance of b from H.

Hence AH + HB = $15\frac{2}{3} + 12\frac{1}{3} = 28 = 16 + 12$ feet.

469. These results Dr. Jamieson verifies by the following construction in his "*Mechanics for Practical Men.*"*

FIG. 6.



Draw the straight line AB, and from a scale of equal parts make Ap = 12 and pB = 16 feet; through the point B draw the straight line BF in any direction with respect to AB; make BE = 20, and EF = 25, the numbers which respectively express the magnitudes of the bodies p and b acting on the straight line AB, at the points p and B; join Fp, and through the point E draw EG parallel to Fp, which produce to C, and make GD = 15, the number which expresses the magnitude of the body a acting at A, and make DC = 45, the number = sum of p and b acting at G: join

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* Article, Centre of Gravity, pp. 19, 20.

CA, and through D draw DH parallel to CA; then is H the place of the centre of gravity of the three forces a, p, and b, acting at the points A, p, and B of the bar AB; and AH, pH, and BH, their respective distances, which if measured from the scale will be found equal to $15\frac{2}{3}$, $3\frac{2}{3}$, and $12\frac{1}{3}$ feet respectively.

Workmen may be informed, that in constructions of this kind it is not necessary to take the numbers which express the magnitudes of the bodies from the same scale as those which express their relative distances; for since they are magnitudes dissimilar to one another, they cannot be compared; consequently the ratio or proportion of the numbers is all that we require: all the magnitudes of the same kind must however be taken from the same scale.

This remark is made because some of the foregoing numbers express weight, others lineal measure; in setting off their relations we used the same scale for all; but this is not necessary.

470. The cases of utility consistent with this theorem are only three; viz.

1. When p is less than $a + b$, but such that $a + p$ is greater than b, and $b + p$ greater than a;
2. When p is equal to $a + b$;
3. When p is greater than $a + b$.

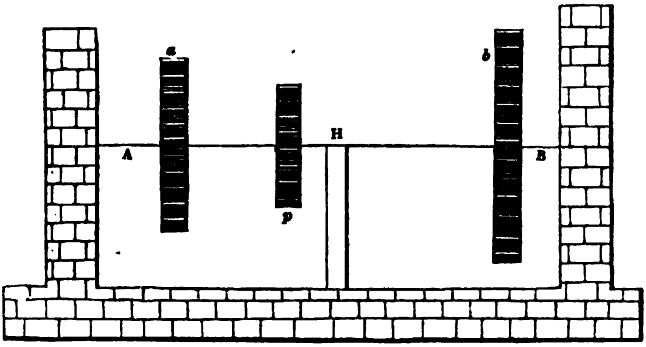
The equation of equilibrium is the following, which we borrow from the "Mechanics for Practical Men."

$$\delta = \frac{d}{2p}(a + p - b).$$

The following examples are given to show persons unacquainted with algebra how they may apply the principle now before them.

EXAMPLE 1.—At the extremities of an iron shaft 22 feet long are fixed two wheels, a and b, respectively 2 and $2\frac{1}{2}$ cwt.; and somewhere between these another wheel, p, is fixed, $1\frac{1}{2}$ cwt.; at what distance from each of the extreme wheels must the intermediate one be fixed, so that the whole weight may come upon the middle of the shaft, when it is supported by a transverse bearer H?

FIG. 7.



Here $a = 2$; $p = 1{\cdot}5$; $b = 2{\cdot}5$, and $d = 22$ feet, supposed to be the distance between the centres of the extreme wheels; then since the shaft is supported on its gudgeons at the extremities, and on the journal at the transverse bearer, we may consider it as having no effect upon the system of wheels as regards the place of the centre of gravity; therefore by substituting the above numbers in the foregoing equation, we have

$$\delta = \frac{22}{2 \times 1{\cdot}5}(2 + 1{\cdot}5 - 2{\cdot}5) = \frac{22}{3} \times 1 = 7\tfrac{1}{3} \text{ feet};$$

being the distance of the lighter wheel a from p; but the middle of the shaft is 11 feet from a or b; therefore we have $11 - 7\frac{1}{3} = 3\frac{2}{3}$ for the distance of p from the journal; and $11 + 3\frac{2}{3} = 14\frac{2}{3}$ for its distance from the greater wheel b.

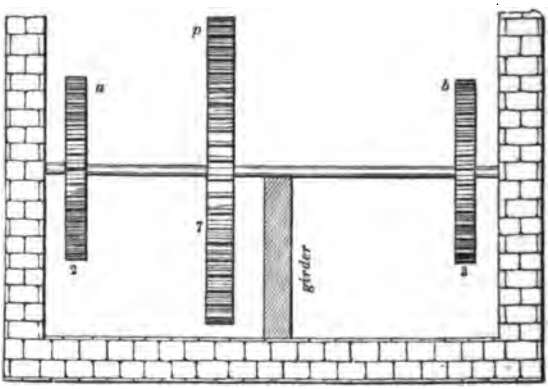
471. When the distance is known or limited by situation, as in practice frequently happens to be the case, the equation becomes

$$d = \frac{2p\,\delta}{a + p - b}.$$

Here we have given, as is plain from the terms on the right hand side of the equation, the magnitudes of the three bodies *a*, *p* and *b*, acting in the same straight line, and the distance between the middle body *p*, and the first extreme *a*; and we are required to find *d*, the distance between the extreme bodies *a* and *b*, and that the common centre of gravity, or the centre of the system shall fall at the middle of that distance.

472. Suppose then for illustration of the case we take the following

FIG. 8.



EXAMPLE.—The shaft of a mill-wheel has to sustain three wheels of the weights of 2, 7, and 3 cwts.; what must be the length of the shaft from centre to centre of the extreme wheels, in order that a transverse girder placed at the middle of its length shall release the gudgeons from the pressure and sustain the system at rest, the distance between the first extreme and the intermediate wheels being 12 feet.

Here we have given by the question, $a = 2$, $p = 7$, $b = 3$, and $\delta = 12$; then writing these numbers for their constituents in the equation, we have

$$d = \frac{2 \times 7 \times 12}{2 + 7 - 3} = \frac{168}{6} = 28 \text{ feet,}$$

for the distance between centre and centre of the extreme wheels a and b; consequently the place we must assign as the common centre of gravity is 14 feet from either extreme end, and 2 feet from the place of p the intermediate wheel.

473. To verify this result, we may compute the place of the common centre of gravity of the two wheels p and b by the first problem, in which case we have

$$\frac{16 \times 3}{7 + 3} = 4{\cdot}8 \text{ feet},$$

as the distance from p, consequently the distance between the centre of the system and that of the two bodies p and b is $4{\cdot}8 - 2 = 2{\cdot}8$ feet; then reasoning by the previous illustration, we have

$$14\,a = 2{\cdot}8\,(p + b), \text{ that is}$$
$$14 \times 2 = 2{\cdot}8 \times 10 = 28, \text{ as before.}$$

Or, to numerous machinists who are masters of algebra, if we put $x =$ the distance of each extreme wheel from the centre of the shaft, then $2\,x =$ the whole length of the shaft, and $x - 12 =$ the distance of the intermediate wheel; hence

$$2\,x + (x - 12) \times 7 = 3\,x$$
$$\text{or } 7\,x - 84 = x\,;\ i.e.\ 6\,x = 84\,;$$

therefore $x = 14$, and $14 \times 2 = 28$ as before.

474. When the weight of the axle of the wheels is given, we may adopt w to express that element. Then, if the bar be of uniform shape and density, the centre of gravity of each segment made by the centre of the system, will occur at the middle of its length, and the weight of the segment will be expressed by $w\,x$, and $w\,(d-x)$ respectively. The effective strength of the energies is then

$$\tfrac{1}{2}\,w x^2, \text{ and } \tfrac{1}{2}\,w\,(d-x)^2,$$

consequently in the case of an equilibrium, we shall have

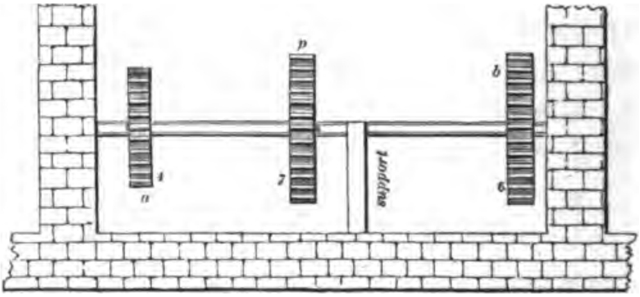
$$a x + p\,(x-\delta) + \tfrac{1}{2}\,w x^2 = b\,(d-x) + \tfrac{1}{2}\,w\,(d-x)^2,$$
$$\text{or } a x + \tfrac{1}{2}\,w x^2 = b\,(d-x) + p\,(\delta-x) + \tfrac{1}{2}\,w\,(d-x)^2;$$

but in either case, when the equations are properly reduced, we find generally that

$$x = \frac{(2\,b + w\,d)\,d + 2\,p\,\delta}{2\,(a + b + p + w\,d)}.$$

The following practical example will bring this complicated equation into a readable form, better than could be expressed by a rule.

Fig. 9.



Example.—A cast iron shaft, 4 inches square, and 36 feet long between gudgeon and gudgeon, is required to sustain three wheels, whose weights are 4, 7, and 6 cwt. respectively, placed at the distance of 14 and 22 feet from each other. At what point of the shaft must an upright be placed to remove the pressure entirely from the gudgeons, and balance the shaft with all its apparatus.

Here we have given the wheels $a = 4$, $p = 7$, and $b = 6$, and the shaft $d = 36$, also $\delta = 14$. Writing these numbers for their correspondent symbols in the foregoing equation, we shall have

$$x = \frac{(2 \times 6 + 36\,w)\,36 + 2 \times 7 \times 14}{2\,(4 + 6 + 7 + 36\,w)}.$$

Now since the material of which the shaft is made is cast iron, the weight of 1 foot in length, or the value of w is $4 \times 4 \times 3{\cdot}2 = 51{\cdot}2$ lbs.*; consequently, by substituting 51·2 instead of w in the foregoing value of x, we shall obtain

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* The weight of a bar of cast iron one inch square and 12 inches long, is 3·2 lbs., the multiplier used in the question.

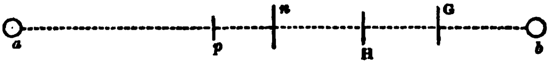
$$x = \frac{(\overline{2 \times 6} + \overline{36 \times 51{\cdot}2}) \times 36 + 2 \times 7 \times 14}{2\,(4 + 6 + 7 + \overline{36 \times 51{\cdot}2})} =$$

18 feet very nearly.

Therefore, the place of the support is at 18 feet from each of the extreme wheels, and 4 feet from the intermediate one; but if the weight of the shaft had not been taken into the estimate, we should have had $x = 18\,\frac{8}{17}$ feet; hence the effect which this element produces is, to place the support $\frac{8}{17}$ of a foot, or very nearly 6 inches more one way than the other, a quantity which in large constructions may be disregarded. But it was necessary to shew that we should not consider the axle void of weight in our calculations, especially where their accuracy may be tested by other persons who would not allow this element to be thrown out of the equation of equilibrium.

Of the centre of gravity of four or more bodies situated in the same right line.

475. This is but an extension of the previous case: indeed the law of continuation is so obvious, that we shall make one example suffice for its illustration; but to make the way smooth, let a, p, n, b, represent the four bodies taken in order, from a the first, to b the last.



Let δ denote the distance from a to n, and δ' the distance from a to n, and d the distance from a to b. Also let x denote the distance from a to the place of the common centre of the whole mass.

If then the bodies a and p are on one side of the common centre, while the other two bodies n and b are situate on the other side of that centre, we shall have x; $(x - \delta)$;

$(\delta' - x)$, and $(d - x)$ for the respective distances of the bodies from the centre of gravity; consequently, by the principle already indicated, we have

$$a\,x + p\,(x - \delta) = n\,(\delta' - x) + b\,(d - x);$$

which by transposition and division gives

$$x = \left(\frac{p\,\delta + n\,\delta' + b\,d}{a + p + n + b}\right)$$

And we may write this equation thus, for the benefit of such readers as may require its meaning in words at length.

476. RULE.—*Multiply the magnitude or density of each body by its respective distance from the beginning of the system, and divide the sum of the products by the sum of the bodies for the distance of the centre of gravity sought.*

EXAMPLE.—Four bodies connected by a straight inflexible bar, have their weights respectively, 18, 26, 12, and 30 cwt.; and their distances from each other are,

From a to p, 17 feet,
" a to n, 23 ditto,
" a to b, 40 ditto.

At what point in the length of the bar shall the common centre of gravity be marked?

Here we have given

$$\begin{array}{l} a = 18;\ n = 12;\ \delta = 17; \\ p = 26;\ b = 30;\ \delta' = 23; \end{array} \text{ and } d = 40;$$

Let these values of the elements of the system be substituted in lieu of the symbols in the foregoing equation, and it reads

$$x = \frac{\overline{26 \times 17} + \overline{12 \times 23} + \overline{30 \times 40}}{18 + 26 + 12 + 3} = 22{\cdot}3$$

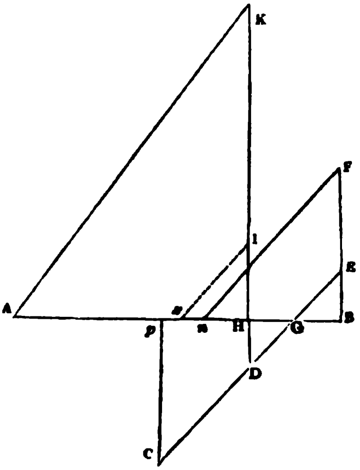
feet from a; 5·3 feet from p; $\frac{1}{10}$ from n; and 17·7 from b.

477. We shall now exhibit the principle of continuation

by a geometrical construction, in which the reader may trace with great facility the combinations involved in the equation we have just worked out for him.

Let A B be a straight line passing through the centres of the four bodies, *a*, *p*, *n*, *b*.

FIG. 10.



Make A B = 40 feet, taken from a scale of equal parts. On the straight line A B set off A *p* and A *n* equal respectively to 17 and 23 feet, taken from the same scale as A B. Then are the points A, *p*, *n*, and B, the positions of the four bodies, the weights of which constitute the elements of the question, and whose common centre of gravity we shall now trace by completing the construction of the diagram.

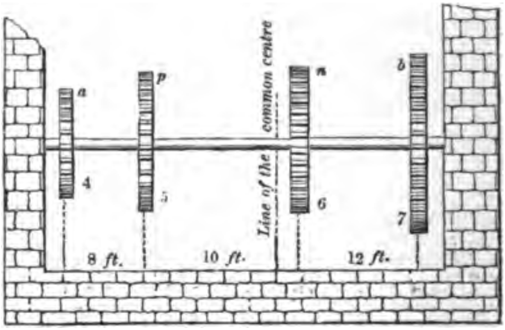
Through the point B draw the straight line B F in any direction at pleasure; make B E proportional to the weight of the body *n*, and E F to that of the body *b*; join F *n*; and through the point E draw E G parallel to F *n*; then is the point G the common centre of gravity of the bodies *b* and *n*.

Next produce E G to C, making G D proportional to the weight of the body *p*, and D C = B F, or proportional to the sum of the bodies *b* and *n*; join C *p*, and through the point D, draw DH parallel to Cp. Then is the point H the common centre of gravity of the three bodies, p, n, b.

Finally, produce DH to the point K, making HI proportional to the weight of the body a, and IK equal to GC, or proportional to the sum of the bodies p, n, and b; join KA, and through the point I, draw Ix parallel to KA; then shall the point x on the line AB be the common centre of gravity sought*.

For we have BF $= n + b$; B$n = d - \delta$; EF $= b$; GC $= p + n + b$; G$p = \frac{b\,(d - \delta) + n\,(\delta' - \delta)}{n + b}$; DC $= n + b$; HK $= a + p + n + b$; HA $= \frac{bd + p\delta + n\delta'}{n + p + b}$; KI $= p + n + b$; and A$x = \left\{\frac{bd + p\delta' + n\delta}{a + p + n + b}\right\}$ an equation which is identical with that from which we demonstrated the example, and which we shall now turn to account in the solution of another bearing immediately on the subject of this essay, and with which also it is our intention to bring it to a close.

FIG. 11.



EXAMPLE.—Four cast iron wheels, the weights of which are respectively 4, 5, 6, and 7 cwts., are fixed upon a shaft at the several distances of 8, 10, and 12 feet apart, taken in order; then, if the shaft have no influence upon its position, what is the distance of each wheel from the common centre of gravity of the system?

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* In Dr. Jamieson's "Mechanics for Practical Men," there is an elegant demonstration of this construction.

There are here supplied by the question, seven terms of its elements, and the eighth is to be found thus, agreeably to the foregoing equation; viz.,

$a = 4$; $p = 5$; $n = 6$; $b = 7$; $\delta = 8$; $\delta' = 18$; and $d = 30$; hence, if we substitute these numerical values for their corresponding symbols, we shall have

$$x = \frac{\overline{5 \times 8} + \overline{6 \times 18} + \overline{7 \times 30}}{4 + 5 + 7} = 16\tfrac{3}{11}$$

feet from the body a; $8\tfrac{3}{11}$ from p; $1\tfrac{8}{11}$ from n; and $13\tfrac{8}{11}$ from b.

The numerical operation deducible from the geometrical construction, furnishes three elegant proportions. For by similar triangles, BEG and BFn, we have

$$\text{BF} : \text{B}n :: \text{EF} : n\text{G}; \text{ that is}$$

$$(n + b) : (d - \delta') :: b : n\text{G} = \frac{b\,(d - \delta')}{(n + b)}; \text{ which}$$

arithmetically becomes

$$6 + 7 : 30 - 18 :: 7 : n\text{G} = \frac{84}{13} = 6\frac{6}{13}.$$

Again, in the similar triangles HDG and pCG, we have

$$\text{GC} : \text{G}p :: \text{DC} : p\text{H}; \text{ that is}$$

$$(p + n + b) : \frac{b\,(d - \delta) + n\,(\delta' - \delta)}{n + b} :: n + b : p\text{H}$$

$$= \frac{b\,(d - \delta) + n\,(\delta' - \delta)}{p + n + b}, \text{ which}$$

arithmetically is written

$$5 + 6 + 7 : \frac{84}{13} + (18 - 8) :: 6 + 7 : p\text{H} = \frac{214}{18} = 11\frac{8}{9}.$$

Finally, in the similar triangles HIx and HKA, we have the following proportion,

$$\text{HK} : \text{HA} :: \text{KI} : \text{A}x; \text{ that is,}$$

$$(a+p+n+b : \left\{\frac{bd+p\delta+n\delta'}{p+n+b}\right\} :: (p+n+b) : \text{A}x;$$

which by equating the product of the extremes and means gives the elegant equation that preceded the example, and which arithmetically is written out thus:

$$(4+5+6+7) : \left(\frac{214}{18}+8\right) :: (5+6+7) : \text{A}x = \frac{358}{22};$$

consequently by division the fraction becomes $\text{A}x = 16\frac{3}{11}$ feet, the same as before.

478. Thus we have traced from principles of the greatest simplicity the theory of the common centre of gravity for so much of mill-work as has reference to systems of wheels arming the same axle. We shall close this article by remarking in reference to bevel gear, that

1. The centre of gravity of the surface of a cone is the same as the centre of gravity of its triangular section. And the centre of gravity of a right cone is situated at $\frac{3}{4}$ths of the axis from the vertex: or $\frac{1}{4}$th from the base of the cone. The same holds good of any pyramid whose base is a polygon.

2. The centre of gravity of the surface of a conic frustum is the same as the centre of gravity of the trapezoid formed by a plane passing along the axis. And the centre of gravity of the conic frustum, when its height and the diameters of the two ends are given, is determined by the following equation,

$$\delta = \frac{h\,(3\,\text{R}^2 + 2\,\text{R}r + r^2)}{4\,(r^2 + \text{R}r + r^2)}$$

Where h is the height of the conic frustum; R is the radius of the less end; r the radius of the greater; and δ represents the distance of the centre from the less end.

The practical rule is this:

To the sum of the squares of the radii of the two ends add their product, then multiply the sum by 4, and reserve the result for a divisor.

To three times the square of the radius of the greater end, add the square of the radius of the less end, together with twice the product of the radii, and multiply the sum by the height of the frustum for a dividend.

Then, divide the dividend by the reserved divisor, and the quotient will express the distance between the centre of magnitude of the less end, and the centre of gravity of the frustum.

3. The centre of gravity of the surface of a cylinder is the same as the centre of gravity of the parallelogram made by the plane passing through the axis.

4. The distance of the centre of gravity of a circular arc from the centre of the circle is a fourth proportional to the length of the arc, the radius of the circle, and the chord of the arc.

5. The ordinate of a common parabola is a mean proportional between the abscissa and the parameter of the axis; and the position of the centre of gravity is in the axis of the figure, and at the distance of three fifths of the abscissa from the vertex.

6. The centre of gravity of any semiparabola occurs in the ordinate of the axis, passing through the centre of gravity of the whole parabola.

7. The distance between the vertex and the centre of gravity of a parabolic conoid, is equal to two thirds of the axis.

There are several other figures that occur in mill-work, but the discussion we have entered into has spun out much beyond the limit we had assigned to it, and we are therefore compelled to refer the reader to other treatises, which enter into the composition, revolution, and properties of bodies in motion, for all such matter as should be known to complete the education of a sound millwright.

479. The tables of squares and cubes which are annexed will be acceptable, as also the square roots and cube roots of all numbers from 1 to 1000, which have been taken from Hutton's "Course of Mathematics," and will be found very useful on many occasions.

Number.	Square.	Cube.	Square Root.	Cube Root.
1	1	1	1·0000000	1·000000
2	4	8	1·4142136	1·259921
3	9	27	1·7320508	1·442250
4	16	64	2·0000000	1·587401
5	25	125	2·2360680	1·709976
6	36	216	2·4494897	1·817121
7	49	343	2·6457513	1·912931
8	64	512	2·8284271	2·000000
9	81	729	3·0000000	2·080084
10	100	1000	3·1622777	2·154435
11	121	1331	3·3166248	2·223980
12	144	1728	3·4641016	2·289428
13	169	2197	3·6055513	2·351335
14	196	2744	3·7416574	2·410142
15	225	3375	3·8729833	2·466212
16	256	4096	4·0000000	2·519842
17	289	4913	4·1231056	2·571282
18	324	5832	4·2426407	2·620741
19	361	6859	4·3588989	2·668402
20	400	8000	4·4721360	2·714418
21	441	9261	4·5825757	2·758923
22	484	10648	4·6904158	2·802039
23	529	12167	4·7958315	2·843867
24	576	13824	4·8989795	2·884499
25	625	15625	5·0000000	2·924018
26	676	17576	5·0990195	2·962496
27	729	19683	5·1961524	3·000000
28	784	21952	5·2915026	3·036589
29	841	24389	5·3851648	3·072317
30	900	27000	5·4772256	3·107232
31	961	29791	5·5677644	3·141381
32	1024	32768	5·6568542	3·174802
33	1089	35937	5·7445626	3·207534
34	1156	39304	5·8309519	3·239612
35	1225	42875	5·9160798	3·271066
36	1296	46656	6·0000000	3·301927
37	1369	50653	6·0827625	3·332222
38	1444	54872	6·1644140	3·361975
39	1521	59319	6·2449980	3·391211
40	1600	64000	6·3245553	3·419952
41	1681	68921	6·4031242	3·448217
42	1764	74088	6·4807407	3·476027
43	1849	79507	6·5574385	3·503398
44	1936	85184	6·6332496	3·530348
45	2025	91125	6·7082039	3·556893
46	2116	97336	6·7823300	3·583048
47	2209	103823	6·8556546	3·608826
48	2304	110592	6·9282032	3·634241
49	2401	117649	7·0000000	3·659306
50	2500	125000	7·0710678	3·684031
51	2601	132651	7·1414284	3·708430

Number.	Square.	Cube.	Square Root.	Cube Root.
52	2704	140608	7·2111026	3·732511
53	2809	148877	7·2801099	3·756286
54	2916	157464	7·3484692	3·779763
55	3025	166375	7·4161985	3·802953
56	3136	175616	7·4833148	3·825862
57	3249	185193	7·5498344	3·848501
58	3364	195112	7·6157731	3·870877
59	3481	205379	7·6811457	3·892996
60	3600	216000	7·7459667	3·914867
61	3721	226981	7·8102497	3·936497
62	3844	238328	7·8740079	3·957892
63	3969	250047	7·9372539	3·979057
64	4096	262144	8·0000000	4·000000
65	4225	274625	8·0622577	4·020726
66	4356	287496	8·1240384	4·041240
67	4489	300763	8·1853528	4·061548
68	4624	314432	8·2462113	4·081656
69	4761	328509	8·3066239	4·101566
70	4900	343000	8·3666003	4·121285
71	5041	357911	8·4261498	4·140818
72	5184	373248	8·4852814	4·160168
73	5329	389017	8·5440037	4·179339
74	5476	405224	8·6023253	4·198336
75	5625	421875	8·6602540	4·217163
76	5776	438976	8·7177979	4·235824
77	5929	456533	8·7749644	4·254321
78	6084	474552	8·8317609	4·272659
79	6241	493039	8·8881944	4·290841
80	6400	512000	8·9442719	4·308870
81	6561	531441	9·0000000	4·326749
82	6724	551368	9·0553851	4·344481
83	6889	571787	9·1104336	4·362071
84	7056	592704	9·1651514	4·379519
85	7225	614125	9·2195445	4·396830
86	7396	636056	9·2736185	4·414005
87	7569	658503	9·3273791	4·431047
88	7744	681472	9·3808315	4·447960
89	7921	704969	9·4339811	4·464745
90	8100	729000	9·4868330	4·481405
91	8281	753571	9·5393920	4·497942
92	8464	778688	9·5916630	4·514357
93	8649	804357	9·6436508	4·530655
94	8836	830584	9·6953597	4·546836
95	9025	857375	9·7467943	4·562903
96	9216	884736	9·7979590	4·578857
97	9409	912673	9·8488578	4·594701
98	9604	941192	9·8994949	4·610436
99	9801	970299	9·9498744	4·626065
100	10000	1000000	10·0000000	4·641589
101	10201	1030301	10·0498756	4·657010
102	10404	1061208	10·0995049	4·672330

Number.	Square.	Cube.	Square Root.	Cube Root.
103	10609	1092727	10·1488916	4·687548
104	10816	1124864	10·1980390	4·702669
105	11025	1157625	10·2469508	4·717694
106	11236	1191016	10·2956301	4·732624
107	11449	1225043	10·3440804	4·747459
108	11664	1259712	10·3923048	4·762203
109	11881	1295029	10·4403065	4·776856
110	12100	1331000	10·4880885	4·791420
111	12321	1367631	10·5356538	4·805896
112	12544	1404928	10·5830052	4·820284
113	12769	1442897	10·6301458	4·834588
114	12996	1481544	10·6770783	4·848808
115	13225	1520875	10·7238053	4·862944
116	13456	1560896	10.7703296	4·876999
117	13689	1601613	10·8166538	4·890973
118	13924	1643032	10·8627805	4·904868
119	14161	1685159	10·9087121	4·918685
120	14400	1728000	10·9544512	4·932424
121	14641	1771561	11·0000000	4·946088
122	14884	1815848	11·0453610	4·959675
123	15129	1860867	11·0905365	4·973190
124	15376	1906624	11·1355287	4·986631
125	15625	1953125	11·1803399	5·000000
126	15876	2000376	11·2249722	5·013298
127	16129	2048383	11·2694277	5·026526
128	16384	2097152	11·3137085	5·039684
129	16641	2146689	11·3578167	5·052774
130	16900	2197000	11·4017543	5·065797
131	17161	2248091	11·4455231	5·078753
132	17424	2299968	11·4891253	5·091643
133	17689	2352637	11·5325626	5·104469
134	17956	2406104	11·5758369	5·117230
135	18225	2460375	11·6189500	5·129928
136	18496	2515456	11·6619038	5·142563
137	18769	2571353	11·7046999	5·155137
138	19044	2628072	11·7473444	5·167649
139	19321	2685619	11·7898261	5·180101
140	19600	2744000	11·8321596	5·192494
141	19881	2803221	11·8743421	5·204828
142	20164	2863288	11·9163753	5·217103
143	20449	2924207	11·9582607	5·229321
144	20736	2985984	12·0000000	5·241482
145	21025	3048625	12·0415946	5·253588
146	21316	3112136	12·0830460	5·265637
147	21609	3176523	12·1243557	5·277632
148	21904	3241792	12·1655251	5·289572
149	22201	3307949	12·2065556	5·301459
150	22500	3375000	12·2474487	5·313293
151	22801	3442951	12·2882057	5·325074
152	23104	3511808	12·3288280	5·336803
153	23409	3581577	12·3693169	5·348481

Number.	Square.	Cube.	Square Root.	Cube Root.
154	23716	3652264	12·4096736	5·360108
155	24025	3723875	12·4498996	5·371685
156	24336	3796416	12·4899960	5·383213
157	24649	3869893	12·5299641	5·394690
158	24964	3944312	12·5698051	5·406120
159	25281	4019679	12·6095202	5·417501
160	25600	4096000	12·6491106	5·428835
161	25921	4173281	12·6885775	5·440122
162	26244	4251528	12·7279221	5·451362
163	26569	4330747	12·7671453	5·462556
164	26896	4410944	12·8062485	5·473703
165	27225	4492125	12·8452326	5·484806
166	27556	4574296	12·8840987	5·495865
167	27889	4657463	12·9228480	5·506879
168	28224	4741632	12·9614814	5·517848
169	28561	4826809	13·0000000	5·528775
170	28900	4913000	13·0384048	5·539658
171	29241	5000211	13·0766968	5·550499
172	29584	5088448	13·1148770	5·561298
173	29929	5177717	13·1529464	5·572054
174	30276	5268024	13·1909060	5·582770
175	30625	5359375	13·2287566	5·593445
176	30976	5451776	13·2664992	5·604079
177	31329	5545233	13·3041347	5·614673
178	31684	5639752	13·3416641	5·625226
179	32041	5735339	13·3790882	5·635741
180	32400	5832000	13·4164079	5·646216
181	32761	5929741	13·4536240	5·656652
182	33124	6028568	13·4907376	5·667051
183	33489	6128487	13·5277493	5·677411
184	33856	6229504	13·5646600	5·687734
185	34225	6331625	13·6014705	5·698019
186	34596	6434856	13·6381817	5·708267
187	34969	6539203	13·6747943	5·718479
188	35344	6644672	13·7113092	5·728654
189	35721	6751269	13·7477271	5·738794
190	36100	6859000	13·7840488	5·748897
191	36481	6967871	13·8202750	5·758965
192	36864	7077888	13·8564065	5·768998
193	37249	7189057	13·8924440	5·778996
194	37636	7301384	13·9283883	5·788960
195	38025	7414875	13·9642400	5·798890
196	38416	7529536	14·0000000	5·808786
197	38809	7645373	14·0356688	5·818648
198	39204	7762392	14·0712473	5·828476
199	39601	7880599	14·1067360	5·838272
200	40000	8000000	14·1421356	5·848035
201	40401	8120601	14·1774469	5·857766
202	40804	8242408	14·2126704	5·867464
203	41209	8365427	14·2478068	5·877130
204	41616	8489664	14·2828569	5·886765

Number.	Square.	Cube.	Square Root.	Cube Root.
205	42025	8615125	14·3178211	5·896368
206	42436	8741816	14·3527001	5·905941
207	42849	8869743	14·3874946	5·915481
208	43264	8998912	14·4222051	5·924992
209	43681	9129329	14·4568323	5·934473
210	44100	9261000	14·4913767	5·943922
211	44521	9393931	14·5258390	5·953341
212	44944	9528128	14·5602198	5·962731
213	45369	9663597	14·5945195	5·972092
214	45796	9800344	14·6287388	5·981426
215	46225	9938375	14·6628783	5·990727
216	46656	10077696	14·6969385	6·000000
217	47089	10218313	14·7309199	6·009244
218	47524	10360232	14·7648231	6·018463
219	47961	10503459	14·7986486	6·027650
220	48400	10648000	14·8323970	6·036811
221	48841	10793861	14·8660687	6·045943
222	49284	10941048	14·8996644	6·055048
223	49729	11089567	14·9331845	6·064126
224	50176	11239424	14·9666295	6·073177
225	50625	11390625	15·0000000	6·082201
226	51076	11543176	15·0332964	6·091199
227	51529	11697083	15·0665192	6·100170
228	51984	11852352	15·0996689	6·109115
229	52441	12008989	15·1327460	6·118032
230	52900	12167000	15·1657509	6·126925
231	53361	12326391	15·1986842	6·135792
232	53824	12487168	15·2315462	6·144634
233	54289	12649337	15·2643375	6·153449
234	54756	12812904	15·2970585	6·162239
235	55225	12977875	15·3297097	6·171005
236	55696	13144256	15·3622915	6·179747
237	56169	13312053	15·3948043	6·188463
238	56644	13481272	15·4272486	6·197154
239	57121	13651919	15·4596248	6·205821
240	57600	13824000	15·4919334	6·214464
241	58081	13997521	15·5241747	6·223084
242	58564	14172488	15·5563492	6·231678
243	59049	14348907	15·5884573	6·240251
244	59536	14526784	15·6204994	6·248800
245	60025	14706125	15·6524758	6·257324
246	60516	14886936	15·6843871	6·265826
247	61009	15069223	15·7162336	6·274304
248	61504	15252992	15·7480157	6·282760
249	62001	15438249	15·7797338	6·291194
250	62500	15625000	15·8113883	6·299604
251	63001	15813251	15·8429795	6·307992
252	63504	16003008	15·8745079	6·316359
253	64009	16194277	15·9059737	6·324704
254	64516	16387064	15·9373775	6·333025
255	65025	16581375	15·9687194	6·341325

Number.	Square.	Cube.	Square Root.	Cube Root.
256	65536	16777216	16·0000000	6·349602
257	66049	16974593	16·0312195	6·357859
258	66564	17173512	16·0623784	6·366096
259	67081	17373979	16·0934769	6·374310
260	67600	17576000	16·1245155	6·382504
261	68121	17779581	16·1554944	6·390676
262	68644	17984728	16·1864141	6·398827
263	69169	18191447	16·2172747	6·406958
264	69696	18399744	16·2480768	6·415068
265	70225	18609625	16·2788206	6·423157
266	70756	18821096	16·3095064	6·431226
267	71289	19034163	16·3401346	6·439276
268	71824	19248832	16·3707055	6·447305
269	72361	19465109	16·4012195	6·455314
270	72900	19683000	16·4316767	6·463304
271	73441	19902511	16·4620776	6·471274
272	73984	20123648	16·4924225	6·479224
273	74529	20346417	16·5227116	6·487153
274	75076	20570824	16·5529454	6·495064
275	75625	20796875	16·5831240	6·502956
276	76176	21024576	16·6132477	6·510829
277	76729	21253933	16·6433170	6·518684
278	77284	21484952	16·6733320	6·526519
279	77841	21717639	16·7032931	6·534335
280	78400	21952000	16·7332005	6·542132
281	78961	22188041	16·7630546	6·549911
282	79524	22425768	16·7928556	6·557672
283	80089	22665187	16·8226038	6·565415
284	80656	22906304	16·8522995	6·573139
285	81225	23149125	16·8819430	6·580844
286	81796	23393656	16·9115345	6·588531
287	82369	23639903	16·9410743	6·596202
288	82944	23887872	16·9705627	6·603854
289	83521	24137569	17·0000000	6·611488
290	84100	24389000	17·0293864	6·619106
291	84681	24642171	17·0587221	6·626705
292	85264	24897088	17·0880075	6·634287
293	85849	25153757	17·1172428	6·641851
294	86436	25412184	17·1464282	6·649399
295	87025	25672375	17·1755640	6·656930
296	87616	25934336	17·2046505	6·664443
297	88209	26198073	17·2336879	6·671940
298	88804	26463592	17·2626765	6·679419
299	89401	26730899	17·2916165	6·686882
300	90000	27000000	17·3205081	6·694328
301	90601	27270901	17·3493516	6·701759
302	91204	27543608	17·3781472	6·709172
303	91809	27818127	17·4068952	6·716569
304	92416	28094464	17·4355958	6·723950
305	93025	28372625	17·4642492	6·731316
306	93636	28652616	17·4928557	6·738665

Number.	Square.	Cube.	Square Root.	Cube Root.
307	94249	28934443	17·5214155	6·745997
308	94864	29218112	17·5499288	6·753313
309	95481	29503629	17·5783958	6·760614
310	96100	29791000	17·6068169	6·767899
311	96721	30080231	17·6351921	6·775168
312	97344	30371328	17·6635217	6·782422
313	97969	30664297	17·6918060	6·789661
314	98596	30959144	17·7200451	6·796884
315	99225	31255875	17·7482393	6·804091
316	99856	31554496	17·7763888	6·811284
317	100489	31855013	17·8044938	6·818461
318	101124	32157432	17·8325545	6·825624
319	101761	32461759	17·8605711	6·832771
320	102400	32768000	17·8885438	6·839903
321	103041	33076161	17·9164729	6·847021
322	103684	33386248	17·9443584	6·854124
323	104329	33698267	17·9722008	6·861211
324	104976	34012224	18·0000000	6·868285
325	105625	34328125	18·0277564	6·875343
326	106276	34645976	18·0554701	6·882388
327	106929	34965783	18·0831413	6·889419
328	107584	35287552	18·1107703	6·896435
329	108241	35611289	18·1383571	6·903436
330	108900	35937000	18·1659021	6·910423
331	109561	36264691	18·1934054	6·917396
332	110224	36594368	18·2208672	6·924355
333	110889	36926037	18·2482876	6·931300
334	111556	37259704	18·2756669	6·938232
335	112225	37595375	18·3030052	6·945149
336	112896	37933056	18·3303028	6·952053
337	113569	38272753	18·3575598	6·958943
338	114244	38614472	18·3847763	6·965819
339	114921	38958219	18·4119526	6·972682
340	115600	39304000	18·4390889	6·979532
341	116281	39651821	18·4661853	6·986369
342	116964	40001688	18·4932420	6·993191
343	117649	40353607	18·5202592	7·000000
344	118336	40707584	18·5472370	7·006796
345	119025	41063625	18·5741756	7·013579
346	119716	41421736	18·6010752	7·020349
347	120409	41781923	18·6279360	7·027106
348	121104	42144192	18·6547581	7·033850
349	121801	42508549	18·6815417	7·040581
350	122500	42875000	18·7082869	7·047298
351	123201	43243551	18·7349940	7·054003
352	123904	43614208	18·7616630	7·060696
353	124609	43986977	18·7882942	7·067376
354	125316	44361864	18·8148877	7·074043
355	126025	44738875	18·8414437	7·080698
356	126736	45118016	18·8679623	7·087341
357	127449	45499293	18·8944436	7·093970

Number.	Square.	Cube.	Square Root.	Cube Root.
358	128164	45882712	18·9208879	7·100588
359	128881	46268279	18·9472953	7·107193
360	129600	46656000	18·9736660	7·113786
361	130321	47045881	19·0000000	7·120367
362	131044	47437928	19·0262976	7·126935
363	131769	47832147	19·0525589	7·133492
364	132496	48228544	19·0787840	7·140037
365	133225	48627125	19·1049732	7·146569
366	133956	49027896	19·1311265	7·153090
367	134689	49430863	19·1572441	7·159599
368	135424	49836032	19·1833261	7·166095
369	136161	50243409	19·2093727	7·172580
370	136900	50653000	19·2353841	7·179054
371	137641	51064811	19·2613603	7·185516
372	138384	51478848	19·2873015	7·191966
373	139129	51895117	19·3132079	7·198405
374	139876	52313624	19·3390796	7·204832
375	140625	52734375	19·3649167	7·211247
376	141376	53157376	19·3907194	7·217652
377	142129	53582633	19·4164878	7·224045
378	142884	54010152	19·4422221	7·230427
379	143641	54439939	19·4679223	7·236797
380	144400	54872000	19·4935887	7·243156
381	145161	55306341	19·5192213	7·249504
382	145924	55742968	19·5448203	7·255841
383	146689	56181887	19·5703858	7·262167
384	147456	56623104	19·5959179	7·268482
385	148225	57066625	19·6214169	7·274786
386	148996	57512456	19·6468827	7·281079
387	149769	57960603	19·6723156	7·287362
388	150544	58411072	19·6977156	7·293633
389	151321	58863869	19·7230829	7·299893
390	152100	59319000	19·7484177	7·306143
391	152881	59776471	19·7737199	7·312383
392	153664	60236288	19·7989899	7·318611
393	154449	60698457	19·8242276	7·324829
394	155236	61162984	19·8494332	7·331037
395	156025	61629875	19·8746069	7·337234
396	156816	62099136	19·8997487	7·343420
397	157609	62570773	19·9248588	7·349596
398	158404	63044792	19·9499373	7·355762
399	159201	63521199	19·9749844	7·361917
400	160000	64000000	20·0000000	7·368063
401	160801	64481201	20·0249844	7·374198
402	161604	64964808	20·0499377	7·380322
403	162409	65450827	20·0748599	7·386437
404	163216	65939264	20·0997512	7·392542
405	164025	66430125	20·1246118	7·398636
406	164836	66923416	20·1494417	7·404720
407	165649	67419143	20·1742410	7·410794
408	166464	67917312	20·1990099	7·416859

Number.	Square.	Cube.	Square Root.	Cube Root.
409	167281	68417929	20·2237484	7·422914
410	168100	68921000	20·2484567	7·428958
411	168921	69426531	20·2731349	7·434993
412	169744	69934528	20·2977831	7·441018
413	170569	70444997	20·3224014	7·447034
414	171396	70957944	20·3469899	7·453039
415	172225	71473375	20·3715488	7·459036
416	173056	71991296	20·3960781	7·465022
417	173889	72511713	20·4205779	7·470999
418	174724	73034632	20·4450483	7·476966
419	175561	73560059	20·4694895	7·482924
420	176400	74088000	20·4939015	7·488872
421	177241	74618461	20·5182845	7·494810
422	178084	75151448	20·5426386	7·500740
423	178929	75686967	20·5669638	7·506660
424	179776	76225024	20·5912603	7·512571
425	180625	76765625	20·6155281	7·518473
426	181476	77308776	20·6397674	7·524365
427	182329	77854483	20·6639783	7·530248
428	183184	78402752	20·6881609	7·536122
429	184041	78953589	20·7123152	7·541986
430	184900	79507000	20·7364414	7·547842
431	185761	80062991	20·7605395	7·553688
432	186624	80621568	20·7846097	7·559525
433	187489	81182737	20·8086520	7·565355
434	188356	81746504	20·8326667	7·571173
435	189225	82312875	20·8566536	7·576984
436	190096	82881856	20·8806130	7·582786
437	190969	83453453	20·9045450	7·588579
438	191844	84027672	20·9284495	7·594363
439	192721	84604519	20·9523268	7·600138
440	193600	85184000	20·9761770	7·605905
441	194481	85766121	21·0000000	7·611662
442	195364	86350888	21·0237960	7·617411
443	196249	86938307	21·0475652	7·623151
444	197136	87528384	21·0713075	7·628883
445	198025	88121125	21·0950231	7·634606
446	198916	88716536	21·1187121	7·640321
447	199809	89314623	21·1423745	7·646027
448	200704	89915392	21·1660105	7·651725
449	201601	90518849	21·1896201	7·657414
450	202500	91125000	21·2132034	7·663094
451	203401	91733851	21·2367606	7·668766
452	204304	92345408	21·2602916	7·674430
453	205209	92959677	21·2837967	7·680085
454	206116	93576664	21·3072758	7·685732
455	207025	94196375	21·3307290	7·691371
456	207936	94818816	21·3541565	7·697002
457	208849	95443993	21·3775583	7·702624
458	209764	96071912	21·4009346	7·708238
459	210681	96702579	21·4242853	7·713844

Number.	Square.	Cube.	Square Root.	Cube Root.
460	211600	97336000	21·4476106	7·719442
461	212521	97972181	21·4709106	7·725032
462	213444	98611128	21·4941853	7·730614
463	214369	99252847	21·5174348	7·736187
464	215296	99897344	21·5406592	7·741753
465	216225	100544625	21·5638587	7·747310
466	217156	101194696	21·5870331	7·752860
467	218089	101847563	21·6101828	7·758402
468	219024	102503232	21·6333077	7·763936
469	219961	103161709	21·6564078	7·769462
470	220900	103823000	21·6794834	7·774980
471	221841	104487111	21·7025344	7·780490
472	222784	105154048	21·7255610	7·785992
473	223729	105823817	21·7485632	7·791487
474	224676	106496424	21·7715411	7·796974
475	225625	107171875	21·7944947	7·802453
476	226576	107850176	21·8174242	7·807925
477	227529	108531333	21·8403297	7·813389
478	228484	109215352	21·8632111	7·818845
479	229441	109902239	21·8860686	7·824294
480	230400	110592000	21·9089023	7·829735
481	231361	111284641	21·9317122	7·835168
482	232324	111980168	21·9544984	7·840594
483	233289	112678587	21·9772610	7·846013
484	234256	113379904	22·0000000	7·851424
485	235225	114084125	22·0227155	7·856828
486	236196	114791256	22·0454077	7·862224
487	237169	115501303	22·0680765	7·867613
488	238144	116214272	22·0907220	7·872994
489	239121	116930169	22·1133444	7·878368
490	240100	117649000	22·1359436	7·883735
491	241081	118370771	22·1585198	7·889095
492	242064	119095488	22·1810730	7·894446
493	243049	119823157	22·2036033	7·899791
494	244036	120553784	22·2261108	7·905129
495	245025	121287375	22·2485955	7·910460
496	246016	122023936	22·2710575	7·915784
497	247009	122763473	22·2934968	7·921100
498	248004	123505992	22·3159136	7·926408
499	249001	124251499	22·3383079	7·931710
500	250000	125000000	22·3606798	7·937005
501	251001	125751501	22·3830293	7·942293
502	252004	126506008	22·4053565	7·947573
503	253009	127263527	22·4276615	7·952847
504	254016	128024064	22·4499443	7·958114
505	255025	128787625	22·4722051	7·963374
506	256036	129554216	22·4944438	7·968627
507	257049	130323843	22·5166605	7·973873
508	258064	131096512	22·5388553	7·979112
509	259081	131872229	22·5610283	7·984344
510	260100	132651000	22·5831796	7·989569

Number.	Square.	Cube.	Square Root.	Cube Root.
511	261121	133432831	22·6053091	7·994788
512	262144	134217728	22·6274170	8·000000
513	263169	135005697	22·6495033	8·005205
514	264196	135796744	22·6715681	8·010403
515	265225	136590875	22·6936114	8·015595
516	266256	137388096	22·7156334	8·020779
517	267289	138188413	22·7376340	8·025957
518	268324	138991832	22·7596134	8·031129
519	269361	139798359	22·7815715	8·036293
520	270400	140608000	22·8035085	8·041451
521	271441	141420761	22·8254244	8·046603
522	272484	142236648	22·8473193	8·051748
523	273529	143055667	22·8691933	8·056886
524	274576	143877824	22·8910463	8·062018
525	275625	144703125	22·9128785	8·067143
526	276676	145531576	22·9346899	8·072262
527	277729	146363183	22·9564806	8·077374
528	278784	147197952	22·9782506	8·082480
529	279841	148035889	23·0000000	8·087579
530	280900	148877000	23·0217289	8·092672
531	281961	149721291	23·0434372	8·097758
532	283024	150568768	23·0651252	8·102839
533	284089	151419437	23·0867928	8·107912
534	285156	152273304	23·1084400	8·112980
535	286225	153130375	23·1300670	8·118041
536	287296	153990656	23·1516738	8·123096
537	288369	154854153	23·1732605	8·128144
538	289444	155720872	23·1948270	8·133186
539	290521	156590819	23·2163735	8·138223
540	291600	157464000	23·2379001	8·143253
541	292681	158340421	23·2594067	8·148276
542	293764	159220088	23·2808935	8·153293
543	294849	160103007	23·3023604	8·158305
544	295936	160989184	23·3238076	8·163309
545	297025	161878625	23·3452351	8·168309
546	298116	162771336	23·3666429	8.173302
547	299209	163667323	23·3880311	8·178289
548	300304	164566592	23·4093998	8·183269
549	301401	165469149	23·4307490	8·188244
550	302500	166375000	23·4520788	8·193212
551	303601	167284151	23·4733892	8·198175
552	304704	168196608	23·4946802	8·203131
553	305809	169112377	23·5159520	8·208082
554	306916	170031464	23·5372046	8·213027
555	308025	170953875	23·5584380	8·217965
556	309136	171879616	23·5796522	8·222898
557	310249	172808693	23·6008474	8·227825
558	311364	173741112	23·6220236	8·232746
559	312481	174676879	23·6431808	8·237661
560	313600	175616000	23·6643191	8·242570
561	314721	176558481	23·6854386	8·247474

Number.	Square.	Cube.	Square Root.	Cube Root.
562	315844	177504328	23·7065392	8·252371
563	316969	178453547	23·7276210	8·257263
564	318096	179406144	23·7486842	8·262149
565	319225	180362125	23·7697286	8·267029
566	320356	181321496	23·7907545	8·271903
567	321489	182284263	23·8117618	8·276772
568	322624	183250432	23·8327506	8·281635
569	323761	184220009	23·8537209	8·286493
570	324900	185193000	23·8746728	8·291344
571	326041	186169411	23·8956063	8·296190
572	327184	187149248	23·9165215	8·301030
573	328329	188132517	23·9374184	8·305865
574	329476	189119224	23·9582971	8·310694
575	330625	190109375	23·9791576	8·315517
576	331776	191102976	24·0000000	8·320335
577	332929	192100033	24·0208243	8·325147
578	334084	193100552	24·0416306	8·329954
579	335241	194104539	24·0624188	8·334755
580	336400	195112000	24·0831892	8·339551
581	337561	196122941	24·1039416	8·344341
582	338724	197137368	24·1246762	8·349125
583	339889	198155287	24·1453929	8·353904
584	341056	199176704	24·1660919	8·358678
585	342225	200201625	24·1867732	8·363446
586	343396	201230056	24·2074369	8·368209
587	344569	202262003	24·2280829	8·372966
588	345744	203297472	24·2487113	8·377718
589	346921	204336469	24·2693222	8·382465
590	348100	205379000	24·2899156	8·387206
591	349281	206425071	24·3104916	8·391942
592	350464	207474688	24·3310501	8·396673
593	351649	208527857	24·3515913	8·401398
594	352836	209584584	24·3721152	8·406118
595	354025	210644875	24·3926218	8·410832
596	355216	211708736	24·4131112	8·415542
597	356409	212776173	24·4335834	8·420246
598	357604	213847192	24·4540385	8·424944
599	358801	214921799	24·4744765	8·429638
600	360000	216000000	24·4948974	8·434327
601	361201	217081801	24·5153013	8·439009
602	362404	218167208	24·5356883	8·443687
603	363609	219256227	24·5560583	8·448360
604	364816	220348864	24·5764115	8·453028
605	366025	221445125	24·5967478	8·457690
606	367236	222545016	24·6170673	8·462347
607	368449	223648543	24·6373700	8·466999
608	369664	224755712	24·6576560	8·471647
609	370881	225866529	24·6779254	8·476289
610	372100	226981000	24·6981781	8·480926
611	373321	228099131	24·7184142	8·485557
612	374544	229220928	24·7386338	8·490184

Number.	Square.	Cube.	Square Root.	Cube Root.
613	375769	230346397	24·7588368	8·494806
614	376996	231475544	24·7790234	8·499423
615	378225	232608375	24·7991935	8·504035
616	379456	233744896	24·8193473	8·508641
617	380689	234885113	24·8394847	8·513243
618	381924	236029032	24·8596058	8·517840
619	383161	237176659	24·8797106	8·522432
620	384400	238328000	24·8997992	8·527018
621	385641	239483061	24·9198716	8·531600
622	386884	240641848	24·9399278	8·536178
623	388129	241804367	24·9599679	8·540750
624	389376	242970624	24·9799920	8·545317
625	390625	244140625	25·0000000	8·549879
626	391876	245314376	25·0199920	8·554437
627	393129	246491883	25·0399681	8·558990
628	394384	247673152	25·0599282	8·563537
629	395641	248858189	25·0798724	8·568080
630	396900	250047000	25·0998008	8·572618
631	398161	251239591	25·1197134	8·577152
632	399424	252435968	25·1396102	8·581680
633	400689	253636137	25·1594913	8·586204
634	401956	254840104	25·1793566	8·590723
635	403225	256047875	25·1992063	8·595238
636	404496	257259456	25·2190404	8·599747
637	405769	258474853	25·2388589	8·604252
638	407044	259694072	25·2586619	8·608752
639	408321	260917119	25·2784493	8·613248
640	409600	262144000	25·2982213	8·617738
641	410881	263374721	25·3179778	8·622224
642	412164	264609288	25·3377189	8·626706
643	413449	265847707	25·3574447	8·631183
644	414736	267089984	25·3771551	8·635655
645	416025	268336125	25·3968502	8·640122
646	417316	269586136	25·4165301	8·644585
647	418609	270840023	25·4361947	8·649043
648	419904	272097792	25·4558441	8·653497
649	421201	273359449	25·4754784	8·657946
650	422500	274625000	25·4950976	8·662391
651	423801	275894451	25·5147016	8·666831
652	425104	277167808	25·5342907	8·671266
653	426409	278445077	25·5538647	8·675697
654	427716	279726264	25·5734237	8·680123
655	429025	281011375	25·5929678	8·684545
656	430336	282300416	25·6124969	8·688963
657	431649	283593393	25·6320112	8·693376
658	432964	284890312	25·6515107	8·697784
659	434281	286191179	25·6709953	8·702188
660	435600	287496000	25·6904652	8·706587
661	436921	288804781	25·7099203	8·710982
662	438244	290117528	25·7293607	8·715373
663	439569	291434247	25·7487864	8·719759

Number.	Square.	Cube.	Square Root.	Cube Root.
664	440896	292754944	25·7681975	8·724141
665	442225	294079625	25·7875939	8·728518
666	443556	295408296	25·8069758	8·732891
667	444889	296740963	25·8263431	8·737260
668	446224	298077632	25·8456960	8·741624
669	447561	299418309	25·8650343	8·745984
670	448900	300763000	25·8843582	8·750340
671	450241	302111711	25·9036677	8·754691
672	451584	303464448	25·9229628	8·759038
673	452929	304821217	25·9422435	8·763380
674	454276	306182024	25·9615100	8·767719
675	455625	307546875	25·9807621	8·772053
676	456976	308915776	26·0000000	8·776382
677	458329	310288733	26·0192237	8·780708
678	459684	311665752	26·0384331	8·785029
679	461041	313046839	26·0576284	8·789346
680	462400	314432000	26·0768096	8·793659
681	463761	315821241	26·0959767	8·797967
682	465124	317214568	26·1151297	8·802272
683	466489	318611987	26·1342687	8·806572
684	467856	320013504	26·1533937	8·810868
685	469225	321419125	26·1725047	8·815159
686	470596	322828856	26·1916017	8·819447
687	471969	324242703	26·2106848	8·823730
688	473344	325660672	26·2297541	8·828009
689	474721	327082769	26·2488095	8·832285
690	476100	328509000	26·2678511	8·836556
691	477481	329939371	26·2868789	8·840822
692	478864	331373888	26·3058929	8·845085
693	480249	332812557	26·3248932	8·849344
694	481636	334255384	26·3438797	8·853598
695	483025	335702375	26·3628527	8·857849
696	484416	337153536	26·3818119	8·862095
697	485809	338608873	26·4007576	8·866337
698	487204	340068392	26·4196896	8·870575
699	488601	341532099	26·4386081	8·874809
700	490000	343000000	26·4575131	8·879040
701	491401	344472101	26·4764046	8·883266
702	492804	345948408	26·4952826	8·887488
703	494209	347428927	26·5141472	8·891706
704	495616	348913664	26·5329983	8·895920
705	497025	350402625	26·5518361	8·900130
706	498436	351895816	26·5706605	8·904336
707	499849	353393243	26·5894716	8·908538
708	501264	354894912	26·6082694	8·912736
709	502681	356400829	26·6270539	8·916931
710	504100	357911000	26·6458252	8·921121
711	505521	359425431	26·6645833	8·925307
712	506944	360944128	26·6833281	8·929490
713	508369	362467097	26·7020598	8·933668
714	509796	363994344	26·7207784	8·937843

Number.	Square.	Cube.	Square Root.	Cube Root.
715	511225	365525875	26·7394839	8·942014
716	512656	367061696	26·7581763	8·946180
717	514089	368601813	26·7768557	8·950343
718	515524	370146232	26·7955220	8·954502
719	516961	371694959	26·8141754	8·958658
720	518400	373248000	26·8328157	8·962809
721	519841	374805361	26·8514432	8·966957
722	521284	376367048	26·8700577	8·971100
723	522729	377933067	26·8886593	8·975240
724	524176	379503424	26·9072481	8·979376
725	525625	381078125	26·9258240	8·983508
726	527076	382657176	26·9443872	8·987637
727	528529	384240583	26·9629375	8·991762
728	529984	385828352	26·9814751	8·995883
729	531441	387420489	27·0000000	9·000000
730	532900	389017000	27·0185122	9·004113
731	534361	390617891	27·0370117	9·008222
732	535824	392223168	27·0554985	9·012328
733	537289	393832837	27·0739727	9·016430
734	538756	395446904	27·0924344	9·020529
735	540225	397065375	27·1108834	9·024623
736	541696	398688256	27·1293199	9·028714
737	543169	400315553	27·1477439	9·032802
738	544644	401947272	27·1661554	9·036885
739	546121	403583419	27·1845544	9·040965
740	547600	405224000	27·2029410	9·045041
741	549081	406869021	27·2213152	9·049114
742	550564	408518488	27·2396769	9·053183
743	552049	410172407	27·2580263	9·057248
744	553536	411830784	27·2763634	9·061309
745	555025	413493625	27·2946881	9·065367
746	556516	415160936	27·3130006	9·069422
747	558009	416832723	27·3313007	9·073472
748	559504	418508992	27·3495887	9·077519
749	561001	420189749	27·3678644	9·081563
750	562500	421875000	27·3861279	9·085603
751	564001	423564751	27·4043792	9·089639
752	565504	425259008	27·4226184	9·093672
753	567009	426957777	27·4408455	9·097701
754	568516	428661064	27·4590604	9·101726
755	570025	430368875	27·4772633	9·105748
756	571536	432081216	27·4954542	9·109766
757	573049	433798093	27·5136330	9·113781
758	574564	435519512	27·5317998	9·117793
759	576081	437245479	27·5499546	9·121801
760	577600	438976000	27·5680975	9·125805
761	579121	440711081	27·5862284	9·129806
762	580644	442450728	27·6043475	9·133803
763	582169	444194947	27·6224546	9·137797
764	583696	445943744	27·6405499	9·141788
765	585225	447697125	27·6586334	9·145774

Number.	Square.	Cube.	Square Root.	Cube Root.
766	586756	449455096	27·6767050	9·149757
767	588289	451217663	27·6947648	9·153737
768	589824	452984832	27·7128129	9·157713
769	591361	454756609	27·7308492	9·161686
770	592900	456533000	27·7488739	9·165656
771	594441	458314011	27·7668868	9·169622
772	595984	460099648	27·7848880	9·173585
773	597529	461889917	27·8028775	9·177544
774	599076	463684824	27·8208555	9·181500
775	600625	465484375	27·8388218	9·185452
776	602176	467288576	27·8567766	9·189401
777	603729	469097433	27·8747197	9·193347
778	605284	470910952	27·8926514	9·197289
779	606841	472729139	27·9105715	9·201228
780	608400	474552000	27·9284801	9·205164
781	609961	476379541	27·9463772	9·209096
782	611524	478211768	27·9642629	9·213025
783	613089	480048687	27·9821372	9·216950
784	614656	481890304	28·0000000	9·220872
785	616225	483736625	28·0178515	9·224791
786	617796	485587656	28·0356915	9·228706
787	619369	487443403	28·0535203	9·232618
788	620944	489303872	28·0713377	9·236527
789	622521	491169069	28·0891438	9.240433
790	624100	493039000	28·1069386	9·244335
791	625681	494913671	28·1247222	9·248234
792	627264	496793088	28·1424946	9·252130
793	628849	498677257	28·1602557	9·256022
794	630436	500566184	28·1780056	9·259911
795	632025	502459875	28·1957444	9·263797
796	633616	504358336	28·2134720	9·267679
797	635209	506261573	28·2311884	9·271559
798	636804	508169592	28·2488938	9·275435
799	638401	510082399	28·2665881	9·279308
800	640000	512000000	28·2842712	9·283177
801	641601	513922401	28·3019434	9·287044
802	643204	515849608	28·3196045	9·290907
803	644809	517781627	28·3372546	9·294767
804	646416	519718464	28·3548938	9·298623
805	648025	521660125	28·3725219	9·302477
806	649636	523606616	28·3901391	9·306327
807	651249	525557943	28·4077454	9·310175
808	652864	527514112	28·4253408	9·314019
809	654481	529475129	28·4429253	9·317859
810	656100	531441000	28·4604989	9·321697
811	657721	533411731	28·4780617	9·325532
812	659344	535387328	28·4956137	9·329363
813	660969	537366797	28·5131549	9·333191
814	662596	539353144	28·5306852	9·337016
815	664225	541343375	28.5482048	9·340838
816	665856	543338496	28·5653137	9·344657

Number.	Square.	Cube.	Square Root.	Cube Root.
817	667489	545338513	28·5832119	9·348473
818	669124	547343432	28·6006993	9·352285
819	670761	549353259	28·6181760	9·356095
820	672400	551368000	28·6356421	9·359901
821	674041	553387661	28·6530976	9·363704
822	675684	555412248	28·6705424	9·367505
823	677329	557441767	28·6879766	9·371302
824	678976	559476224	28·7054002	9·375096
825	680625	561515625	28·7228132	9·378887
826	682276	563559976	28·7402157	9·382675
827	683929	565609283	28·7576077	9·386460
828	685584	567663552	28·7749891	9·390241
829	687241	569722789	28·7923601	9·394020
830	688900	571787000	28·8097206	9·397796
831	690561	573856191	28·8270706	9·401569
832	692224	575930368	28·8444102	9·405338
833	693889	578009537	28·8617394	9·409105
834	695556	580093704	28·8790582	9·412869
835	697225	582182875	28·8963666	9·416630
836	698896	584277056	28·9136646	9·420387
837	700569	586376253	28·9309523	9·424142
838	702244	588480472	28·9482297	9·427893
839	703921	590589719	28·9654967	9·431642
840	705600	592704000	28·9827535	9·435388
841	707281	594823321	29·0000000	9·439130
842	708964	596947688	29·0172363	9·442870
843	710649	599077107	29·0344623	9·446607
844	712336	601211584	29·0516781	9·450341
845	714025	603351125	29·0688837	9·454071
846	715716	605495736	29·0860791	9·457799
847	717409	607645423	29·1032644	9·461524
848	719104	609800192	29·1204396	9·465247
849	720801	611960049	29·1376046	9·468966
850	722500	614125000	29·1547595	9·472682
851	724201	616295051	29·1719043	9·476395
852	725904	618470208	29·1890390	9·480106
853	727609	620650477	29·2061637	9·483813
854	729316	622835864	29·2232784	9·487518
855	731025	625026375	29·2403830	9·491219
856	732736	627222016	29·2574777	9·494918
857	734449	629422793	29·2745623	9·498614
858	736164	631628712	29·2916370	9·502307
859	737881	633839779	29·3087018	9·505998
860	739600	636056000	29·3257566	9·509685
861	741321	638277381	29·3428015	9·513369
862	743044	640503928	29·3598365	9·517051
863	744769	642735647	29·3768616	9·520730
864	746496	644972544	29·3938769	9·524406
865	748225	647214625	29·4108823	9·528079
866	749956	649461896	29·4278779	9·531749
867	751689	651714363	29·4448637	9·535417

Number.	Square.	Cube.	Square Root.	Cube Root.
868	753424	653972032	29·4618397	9·539081
869	755161	656234909	29·4788059	9·542743
870	756900	658503000	29·4957624	9·546402
871	758641	660776311	29·5127091	9·550058
872	760384	663054848	29·5296461	9·553712
873	762129	665338617	29·5465734	9·557363
874	763876	667627624	29·5634910	9·561010
875	765625	669921875	29·5803989	9·564655
876	767376	672221376	29·5972972	9·568298
877	769129	674526133	29·6141858	9·571937
878	770884	676836152	29·6310648	9·575574
879	772641	679151439	29·6479342	9·579208
880	774400	681472000	29·6647939	9·582839
881	776161	683797841	29·6816442	9·586468
882	777924	686128968	29·6984848	9·590093
883	779689	688465387	29·7153159	9·593716
884	781456	690807104	29·7321375	9·597337
885	783225	693154125	29·7489496	9·600954
886	784996	695506456	29·7657521	9·604569
887	786769	697864103	29·7825452	9·608181
888	788544	700227072	29·7993289	9·611791
889	790321	702595369	29·8161030	9·615397
890	792100	704969000	29·8328678	9·619001
891	793881	707347971	29·8496231	9·622603
892	795664	709732288	29·8663690	9·626201
893	797449	712121957	29·8831056	9·629797
894	799236	714516984	29·8998328	9·633390
895	801025	716917375	29·9165506	9·636981
896	802816	719323136	29·9332591	9·640569
897	804609	721734273	29·9499583	9·644154
898	806404	724150792	29·9666481	9·647736
899	808201	726572699	29·9833287	9·651316
900	810000	729000000	30·0000000	9·654893
901	811801	731432701	30·0166620	9·658468
902	813604	733870808	30·0333148	9·662040
903	815409	736314327	30·0499584	9·665609
904	817216	738763264	30·0665928	9·669176
905	819025	741217625	30·0832179	9·672740
906	820836	743677416	30·0998339	9·676301
907	822649	746142643	30·1164407	9·679860
908	824464	748613312	30·1330383	9·683416
909	826281	751089429	30·1496269	9·686970
910	828100	753571000	30·1662063	9·690521
911	829921	756058031	30·1827765	9·694069
912	831744	758550528	30·1993377	9·697615
913	833569	761048497	30·2158899	9·701158
914	835396	763551944	30·2324329	9·704698
915	837225	766060875	30·2489669	9·708236
916	839056	768575296	30·2654919	9·711772
917	840889	771095213	30·2820079	9·715305
918	842724	773620632	30·2985148	9·718835

Number.	Square.	Cube.	Square Root.	Cube Root.
919	844561	776151559	30·3150128	9·722363
920	846400	778688000	30·3315018	9·725888
921	848241	781229961	30·3479818	9·729410
922	850084	783777448	30·3644529	9·732930
923	851929	786330467	30·3809151	9·736448
924	853776	788889024	30·3973683	9·739963
925	855625	791453125	30·4138127	9·743475
926	857476	794022776	30·4302481	9·746985
927	859329	796597983	30·4466747	9·750493
928	861184	799178752	30·4630924	9·753998
929	863041	801765089	30·4795013	9·757500
930	864900	804357000	30·4959014	9·761000
931	866761	806954491	30·5122926	9·764497
932	868624	809557568	30·5286750	9·767992
933	870489	812166237	30·5450487	9·771484
934	872356	814780504	30·5614136	9·774974
935	874225	817400375	30·5777697	9·778461
936	876096	820025856	30·5941171	9·782946
937	877969	822656953	30·6104557	9·785428
938	879844	825293672	30·6267857	9·788908
939	881721	827936019	30·6431069	9·792386
940	883600	830584000	30·6594194	9·795861
941	885481	833237621	30·6757233	9·799333
942	887364	835896888	30·6920185	9·802803
943	889249	838561807	30·7083051	9·806271
944	891136	841232384	30·7245830	9·809736
945	893025	843908625	30·7408523	9·813198
946	894916	846590536	30·7571130	9·816659
947	896809	849278123	30·7733651	9·820117
948	898704	851971392	30·7896086	9·823572
949	900601	854670349	30·8058436	9·827025
950	902500	857375000	30·8220700	9·830475
951	904401	860085351	30·8382879	9·833923
952	906304	862801408	30·8544972	9·837369
953	908209	865523177	30·8706981	9·840812
954	910116	868250664	30·8868904	9·844253
955	912025	870983875	30·9030743	9·847692
956	913936	873722816	30·9192497	9·851128
957	915849	876467493	30·9354166	9·854561
958	917764	879217912	30·9515751	9·857992
959	919681	881974079	30·9677251	9·861421
960	921600	884736000	30·9838668	9·864848
961	923521	887503681	31·0000000	9·868272
962	925444	890277128	31·0161248	9·871694
963	927369	893056347	31·0322413	9·875113
964	929296	895841344	31·0483494	9·878530
965	931225	898632125	31·0644491	9·881945
966	933156	901428696	31·0805405	9·885357
967	935089	904231063	31·0966236	9·888767
968	937024	907039232	31·1126984	9·892174
969	938961	909853209	31·1287648	9·895580

Number.	Square.	Cube.	Square Root.	Cube Root.
970	940900	912673000	31·1448230	9·898983
971	942841	915498611	31·1608729	9·902383
972	944784	918330048	31·1769145	9·905781
973	946729	921167317	31·1929479	9·909177
974	948676	924010424	31·2089731	9·912571
975	950625	926859375	31·2249900	9·915962
976	952576	929714176	31·2409987	9·919351
977	954529	932574833	31·2569992	9·922738
978	956484	935441352	31·2729915	9·926122
979	958441	938313739	31·2889757	9·929504
980	960400	941192000	31·3049517	9·932883
981	962361	944076141	31·3209195	9·936261
982	964324	946966168	31·3368792	9·939636
983	966289	949862087	31·3528308	9·943009
984	968256	952763904	31·3687743	9·946379
985	970225	955671625	31·3847097	9·949747
986	972196	958585256	31·4006369	9·953113
987	974169	961504803	31·4165561	9·956477
988	976144	964430272	31·4324673	9·959839
989	978121	967361669	31·4483704	9·963198
990	980100	970299000	31·4642654	9·966554
991	982081	973242271	31·4801525	9·969909
992	984064	976191488	31·4960315	9·973262
993	986049	979146657	31·5119025	9·976612
994	988036	982107784	31·5277655	9·979959
995	990025	985074875	31·5436206	9·983305
996	992016	988047936	31·5594677	9·986648
997	994009	991026973	31·5753068	9·989990
998	996004	994011992	31·5911380	9·993328
999	998001	997002999	31·6069613	9·996665

⁂ In this edition some errors in the table of Dr. Hutton have been corrected. The most extensive and accurate tables of the powers of numbers, are those published by Professor Barlow, of the Royal Military Academy, entitled *New Mathematical Tables*, which also contain tables of prime numbers and factors, so useful in arranging the numbers of wheel work.

APPENDIX B.

REMARKS

ON THE

INTRODUCTION OF THE SLIDE PRINCIPLE

IN

TOOLS AND MACHINES EMPLOYED IN THE PRODUCTION OF MACHINERY.

BY JAMES NASMYTH.

480. THE striking and rapid progress which has within the last thirty years taken place in the perfection of all descriptions of machinery, not only as regards a more complete and sound knowledge of the principles of mechanical or constructive science, as exhibited in the general arrangement of the parts, but more especially in respect to the *increased perfection of the workmanship*, which is now so generally met with in the vast variety of machines which are yearly sent forth, as it were to proclaim new triumphs over matter, cannot but lead us to endeavour to find a cause for so remarkable and important a feature in the history of mechanism.

481. In pursuing this inquiry, we shall find that the accumulated experience and skill in constructive science, which has resulted from a continually increasing demand for machinery, will only throw light on one portion of this interesting subject; inasmuch as increased experience alone will not sufficiently account for the almost mathematical

accuracy and precision which we find existing in, and conferred on, the forms of the various details, whether of the most delicate or ponderous machines; to have produced which, were it even *possible* by manual dexterity and labour, would have entailed so vast an expense in construction, that neither in respect to quantity or price could we have ever hoped to be able (even with our present mechanical population increased tenfold) to have kept pace with the demand which has resulted from the increased perfection and facilities of production realized by improved mechanism.

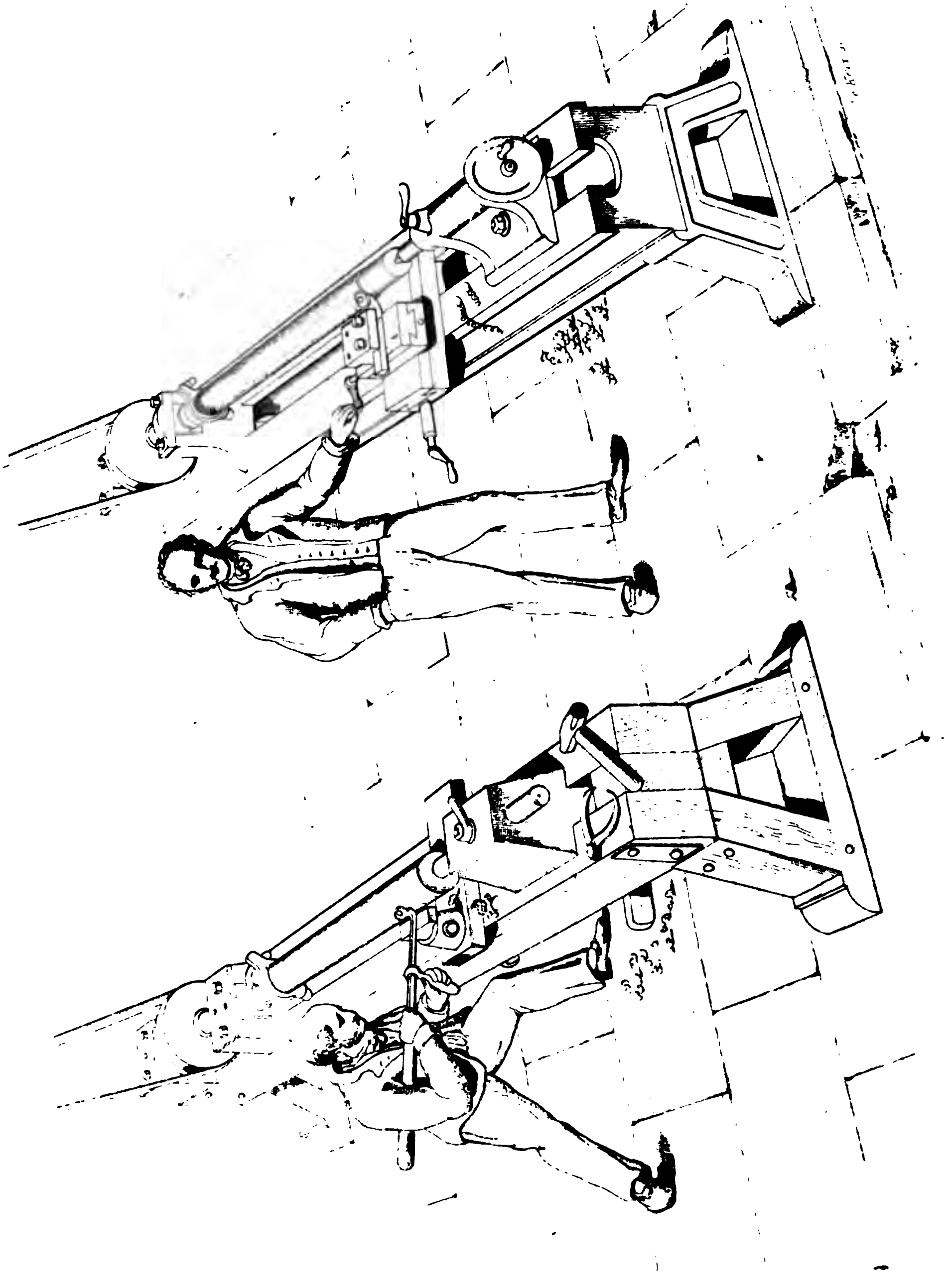
482. Viewing abstractedly the forms of the various details of which every machine is composed, we shall find that they consist of certain combinations of six primitive or elementary geometrical figures, namely, *the line*, *the plane*, *the circle*, *the cylinder*, *the cone*, and *the sphere;* and that, however complex the arrangement, and vast the number of the parts of which a machine consists, we shall find that all may be as it were decomposed and classed under these *six* forms; and that, in short, every machine, whatever be its purpose, simply consists of a combination of these forms, more or less complex, for the attainment of certain objects and performance of required duties. It therefore follows, that the more near to absolute mathematical truth we can have the forms of those parts, the more perfectly will the machine perform its duties.

483. Up to within the last thirty years, nearly every part of a machine had to be made and finished to its required form, by mere manual labour; that is, on the dexterity of the *hand* of the workman, and the correctness of his *eye*, had we entirely to depend for accuracy and precision in the execution of such machinery as was then required; consequently, the enormous expense which was incurred in such attempts, even in the production of comparatively simple machines, in most cases proved a formidable barrier

to the supply of such as the increasing wants of civilization rendered desirable, and when at length the successful efforts of Watt and Arkwright produced such an entire revolution in the steam engine and cotton manufacture, and so disclosed to mankind such vast mines of wealth in the latent powers of production, and capabilities of every country, and as the only obstacle to the attainment of so desirable an end consisted in our almost entire dependence upon manual dexterity for the formation and production of such machines as were required, the necessity of more trustworthy and productive agents rendered some change in the system imperative. In short, a sudden demand for machinery of unwonted accuracy arose, while the stock of workmen then existing were neither adequate in respect to number or ability to meet the wants of the time, and but for the introduction of the principle which I am about to describe, we never could have attained to one-thousandth part of the bright objects which were then disclosed to view, and which have since been so wonderfully and amply realized.

484. The *principle* to which I allude consists in the substitution of a mechanical contrivance in place of the human hand, for *holding*, *applying*, and *directing* the motions of a cutting tool to the surface of the work to be cut, by which we are enabled to constrain the tool to move along or across the surface of the object with *such absolute precision*, that with scarce any expenditure of force, and indeed, in most cases, none at all on the part of the workman, (as shall be seen presently,) we are enabled to produce any of the before-named elementary geometrical forms with a degree of accuracy, ease, and rapidity, as compared with the old, imperfect, *hand system*, as may well be considered a mighty triumph over matter; and the more justly so, when we behold the vast results which improved machinery is enabling us to bring about, all of which may, in a

more or less direct manner, be traced back to the accession of power which we have acquired by means of ~~the~~ general introduction and application of *the slide rest principle.*



485. How it has happened that the inestimable merits of this contrivance have not been more justly appreciated, and lain as it were unobserved, it is difficult to account; it may be that its beautiful simplicity has been overlooked in the glare of dazzling results which it has produced; it is only by considering how we could "get on" without its important help, that the real value of this admirable contrivance appears before us in its true light.

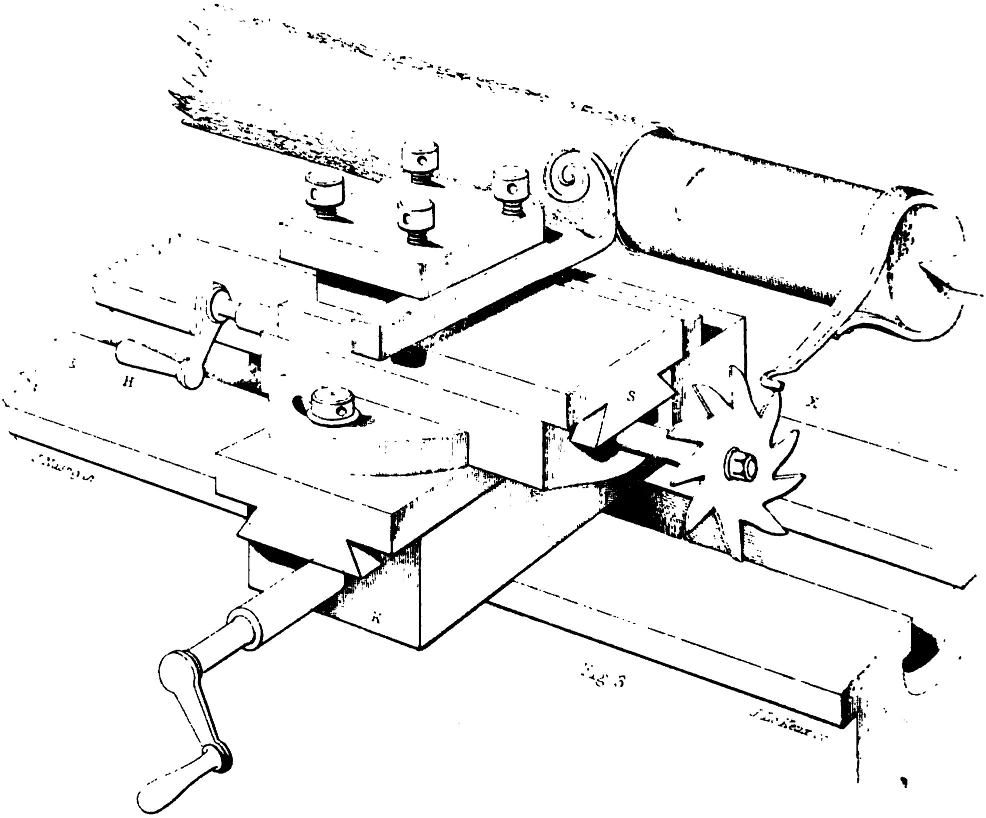
486. It is not indeed saying at all too much to state, that its influence in improving and so extending the use of machinery, has been as great as that produced by the improvement of the steam engine in respect to perfecting manufactures and extending commerce, inasmuch as without the aid of the vast accession to our power of producing perfect mechanism, which it at once supplied, we could never have worked out into practical and profitable forms the conceptions of those master-minds who, during the last half century, have so successfully pioneered the way for mankind ever after attaining the otherwise latent treasures of the material world, even although opposed by time, space, and the elements! I regret much that my limits will not permit me to trace in detail, through all their ramifications, the almost infinite benefits which have been conferred on mankind by our having (through means of this admirable slide rest principle) obtained a most complete and signal triumph over the material world. *The steam engine itself*, which supplies us with such unbounded *power*, owes its present perfection to this admirable means of giving to metallic objects the most precise and perfect geometrical forms. How could we, for instance, have good steam engines, if we had not the means of boring out a true cylinder, or turning a true piston rod, or planing a valve face? It is this alone which has furnished us with the means of carrying into practice the accumulated results of scientific investigation in mechanical subjects.

487. With a view to render the preceding remarks more generally understood, I have given the annexed sketch, in order to illustrate the advantages of the *slide* rest principle, as a substitute for manual labour and dexterity, in the case of the turning lathe; the more so, as it was in this form and application in which its admirable merits became first known to the mechanical world.

488. Fig. 1. represents the system of hand turning in general practice previous to the introduction of the slide rest. Here it will be seen that the workman has no other means of applying and guiding his tool to the work in the lathe, than his mere unaided muscular strength, the expenditure of which, in the case of turning large objects, would be so great, that he could stand it for no length of time; and even if he were able, he would have to depend on his strength and dexterity alone for producing even the humblest and most plain class of work. By such means of this nature as were generally practised before the introduction of the slide rest, we could only attain to any thing like true work, by an almost infinite expenditure of labour, for with the utmost care on his part he could not avoid occasionally cutting a little too deep, the consequence of which would be that he would require to go all over the rest of the surface, in order as it were to lower it to the level of the accidentally too deep cut just named; in most cases, in so doing, he would make the work too small, or have occasion either to leave the mark in the bar, or else alter all his measures to suit the bad results of depending on the chance of his dexterity. It will be seen that the workman in Fig. 1. rests, or obtains support for, the end of his tool, so as to resist the force of the cut, by placing it upon "the rest" R.

489. Now let us just suppose that instead of holding his tool with his hands, that he had it bolted firm to this same rest, and that while it was cutting a shaving from the bar

in the lathe, that he had means of *sliding the rest with its tool* along the bed of the lathe, parallel to the axis of the work, it is evident that, in so doing, we should be able to turn the bar quite true; and if a screw was provided for the purpose of giving this sliding motion, we should then have *a slide rest;* exactly in such manner was this truly admirable tool introduced to the mechanical world. On reference to Fig. 2, it will be at once seen that these objects are attained in a very simple manner. The tool is in this case held fast and firm by a species of iron hand or vice, while it is constrained to move in a definite direction by means of the slide S, (see Fig. 3,) the sliding motion being communicated by the hand of the workman to the screw handle H, the required depth of cut being regulated by the under slide K, operated upon in like manner by a screw and handle; so that by the separate or combined motion of these two slides, the point of the tool can be made to traverse along or across the work as required, with an expenditure of power on the part of the workman so trifling as scarce to be appreciated; and with such a degree of definite and precise accuracy will the tool by these means move, that, after setting the tool to work, he needs not to look at it so long as he simply keeps turning the screw handle; and by a very simple contrivance, which we have endeavoured to exhibit in Fig. 3, X, the attendance of the workman is entirely dispensed with by the introduction of the *self-acting principle,* by which the revolution of the work in the lathe is made to supply the place of the hand of the workman. As may be seen at X, Fig. 3, by simply fixing to the work in the lathe a piece of iron as at O, and placing on the end of the screw S of the upper slide a star wheel X, it is evident that at each revolution of the work in the lathe the end of the iron finger X will come in contact with one of the teeth of the wheel, and move it round a tooth at each turn, bringing the next in succession into a situation so as in like manner, at each revolution of the work, the screw wheel x is moved round, and the tool by that simple means slid by successive steps along the surface of the work; here, then, by this simple adaptation, we have not only done away with necessity for a dexterous workman, but have entirely removed all necessity of attendance whatsoever during the progress of the tool over the slide length of the surface of the work.



This will in some degree convey an idea of the nature of the *self-acting principle*, by the adoption of which we are enabled to elevate to so high a degree the productive powers of our workmen and machinery. There are a vast variety of modes of attaining this self-acting motion, but the one above alluded to will be sufficient, the more so as it is the most generally employed, and most simple.

490. It was this holding of a tool by means of an iron hand, and constraining it to move along the surface of the work in so certain a manner, and with such definite and precise motion, which formed the great era in the history of mechanism, inasmuch as we thenceforward became possessed, by its means, of the power of operating alike on the most ponderous or delicate pieces of machinery with a degree of minute precision, of which language cannot convey an adequate idea; and in many cases we have, through its agency, equal facility in carrying on the most perfect workmanship in the interior parts of certain machines, where neither the hand nor eye can reach, and nevertheless we can give to these parts their required form with a degree of accuracy as if we had the power of transforming ourselves into pigmy workmen, and so apply our labour to the innermost holes and corners of our machinery.

491. It would be blamable indeed (after having endeavoured to set forth the vast advantages which have been conferred on the mechanical world, and therefore on mankind generally, by the invention and introduction of the slide rest) were I to suppress the name of that admirable individual to whom we are indebted for this powerful agent towards the attainment of mechanical perfection. I allude to the late Henry Maudslay, engineer, of London, whose useful life was enthusiastically devoted to the grand object of improving our means of producing perfect workmanship and machinery; *to him* we are certainly indebted for *the slide rest*, and consequently, to say the least, we are indirectly so for the vast benefits which have resulted from the introduction of so powerful an agent in perfecting our machinery and mechanism generally. The indefatigable care

which he took in inculcating and diffusing among his workmen, and mechanical men generally, sound ideas of practical knowledge and refined views of construction, has rendered and ever will continue to render his name identified with all that is noble in the ambition of a lover of mechanical perfection. The vast results which have sprung from his admirable mind, is his best monument and eulogium.

492. The vast practical advantage which resulted from the substitution of "the slide rest" in place of the hand in the process of turning, had its natural effect in causing its adoption and application to other important processes in constructive science. So striking and certain were the effects and advantages as respects the superior quality and cheapness of the work produced by its means, that it soon induced a very marked change in mechanical designs, inasmuch as this, that many improved arrangements in mechanism had been kept back from the vast expense attendant on the employment of certain forms in the parts, such as perfectly true cylindrical rods or circular or flat surfaces, which the important aid of the slide rest now renders so cheap, (comparatively speaking,) that every practical engineer, in making out his design in detail, had only to keep in mind the vast capabilities and powers of the slide rest, to enable his fancy to luxuriate in the introduction of the most perfect geometrical forms, as not only attainable in practice, but actually *the cheapest* forms through whose agency he could attain his object. I have every reason, indeed, to call the introduction of the slide rest a great era in the history of mechanism, as every piece of machinery which was produced by its agency, bore such evident marks of superiority, as very rapidly and extensively proclaimed to the mechanical world that a great step (leap forward, I should rather say) had been made, and in proof of it, we have only to look around us at this day to see what is doing by improved machinery, to place beyond doubt what I have

stated as to this era in mechanism—"the introduction of the slide rest."

493. Were I to attempt to trace in detail the almost infinite application of the slide rest principle, I should require to describe almost every machine which is employed in giving definite forms to materials; but as such would be incompatible with my limits, I shall confine myself to one or two of the more generally used and important applications; and in endeavouring to do so, I shall, for the sake of clearness, avoid those minute details which, although most frequently combined with the slide principle, yet are so subordinate, and so frequently varied according to the taste of the engineer, that it is best to strip them from the simple illustrations I have endeavoured to give, so as to leave, as it were, more prominent and conspicuous *the principle* of the machine.

494. I cannot properly introduce to the attention of my readers a more worthy and truly important immediate descendant of "the slide rest" than *the planing machine*, which has done more within the last 10 or 15 years for reducing the cost, and for extending the use of perfect machinery, than had been the case by all the improvements in mechanism for the last century.

495. There is no form which is so frequently required and essential to any piece of mechanism as the plane surface, or rectangular prismatic forms generally.

496. The vast expense attendant on the production of such, by the tedious and unsatisfactory process of chipping and filing, caused every engineer to avoid by all means any arrangements which rendered such forms necessary, however essential they might be to the perfect action of the machine. It is quite laughable to observe, in any old piece of mechanism, the niggardly use of those important forms arising from the above obstacle. The introduction of the planing machine at once altered the entire system, inas-

much as forms and arrangements became practically possible, which formerly the engineer dared not think of using. This was simply following out in the plane surface, what the slide rest had produced in the turning lathe as regards cylindrical forms; and the result was, that not only was the machinery produced by its agency most strikingly superior, by its direct influence, but also as the planing machine enabled us to produce *improved tools* at so very much reduced cost, that mighty principle in all affairs, (namely, cause and effect tearing each other alternately.) The first planing machine enabled us to produce the second still better; that again produced a better still; and now slide rests of the most perfect kind came streaming forth from them, and they, again, assisted in making better still; so that in a very short time a most important branch of engineering business, namely, tool-making, arose, which had its existence not merely owing to the demand preexisting for such improved tools, but in fact, raised upon a demand as it were of its own creating, and all this caused by the slide rest, and its offspring, the planing machine. One has only to go into any of those vast establishments, which within the last 10 years have sprung up for the purpose of supplying the demand for machinery, and we shall find that nine-tenths of all the fine mechanism in use, and in process of production, is through the agency, more or less direct, of *the slide rest and planing machine.*

497. Figure 4 represents the general arrangement of parts existing in most planing machines. It consists of two principal parts, namely, the bed B on which the table T slides by certain mechanism backward and forward, so that any piece of work, W, being bolted to it, partakes of the same, as if it were a part of the table T; the table T being constrained to move in a perfectly straight line to and fro, by its sliding on the two angular ridges, C C.

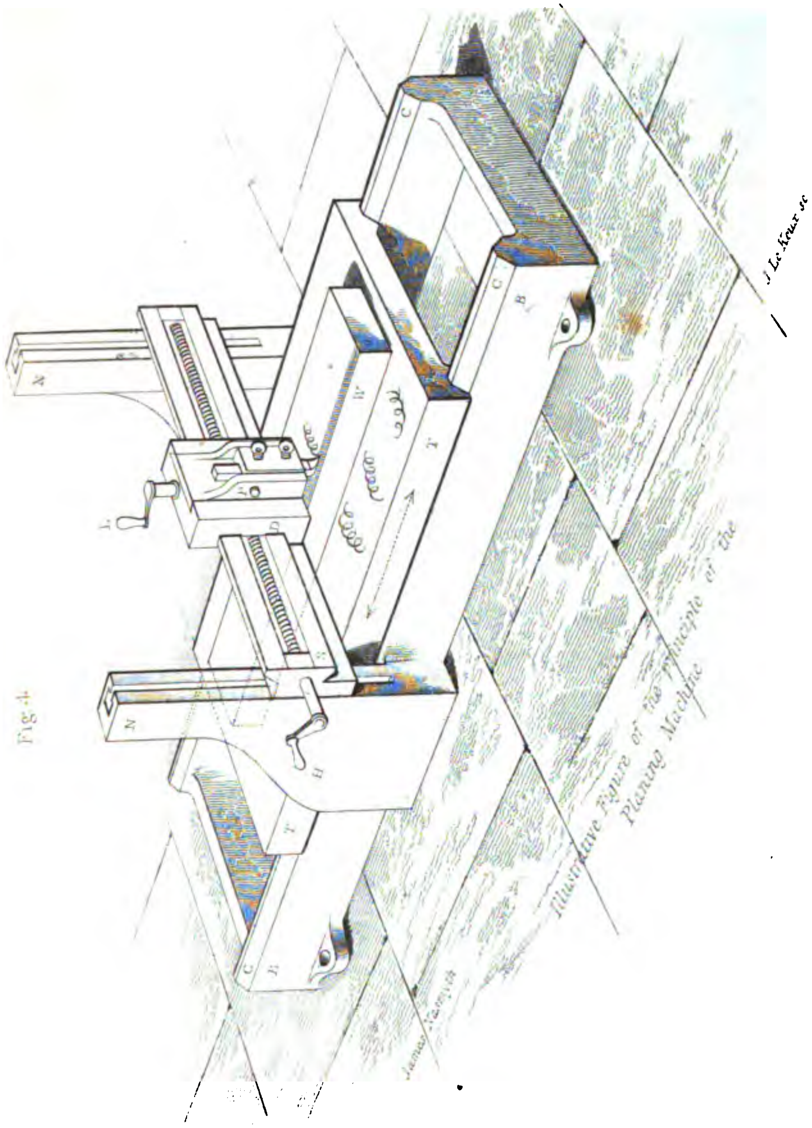


Fig. 4

Illustrative Figure of the Principle of the Planing Machine

498. Over the table T is fixed "a slide rest" S, which is held fast by being bolted to the two upright standards N N. This slide S has a transverse slide D, which serves to hold the tool in such a manner that it may be lowered down and adjusted so as to cause the tool to take a cut more or

less deep as desired, which adjustment is performed by the handle L, so that every time the table and the work fixed to it moves to and fro, the tool in the down slide D, is by certain apparatus moved each time a little way across the table, so that by a repeated series of sliding backwards and forwards of the table, the tool is made to traverse the surface of the work, and in so doing it transfers the perfectly true figure of the slide S, on to that of the surface of the work W, and so produces a perfect plane surface. I trust an inspection of the figure will do more to render this clear, than any further attempt at description.

499. As to the means of giving motion to the table, as also to the screw of the slide S, it is not required here to enter into such details, as they vary so much according to the fancy of different makers, who have each their peculiar fancy as to the best arrangement.

500. An inspection of the figure will, I trust, satisfy any one that this machine is derived from the slide rest, for the slide S is nothing more than a slide rest, held to its work by the two standards N N, while the work W represents a surface on the lathe, which is made to move in a straight line, in place of a revolving motion, as it would have done had it been a cylindrical surface being turned in the lathe. This, indeed, is my main object in giving this figure, as it serves to show that it is to the slide rest system that we are indebted for the planing machine, however varied the constructive details of such planing machines as we meet with may be, yet we shall find that they all embody the above principal arrangements, and are all slide rests for turning, i. e. planing *flat* work.

501. Again, in the case of the screw-cutting machine, we shall find (Fig. 5) that it consists simply of a slide rest, which receives its sliding motion from the revolution of the spindle or work in the lathe. I have chosen the latter, as it tends to render the arrangement more distinct.

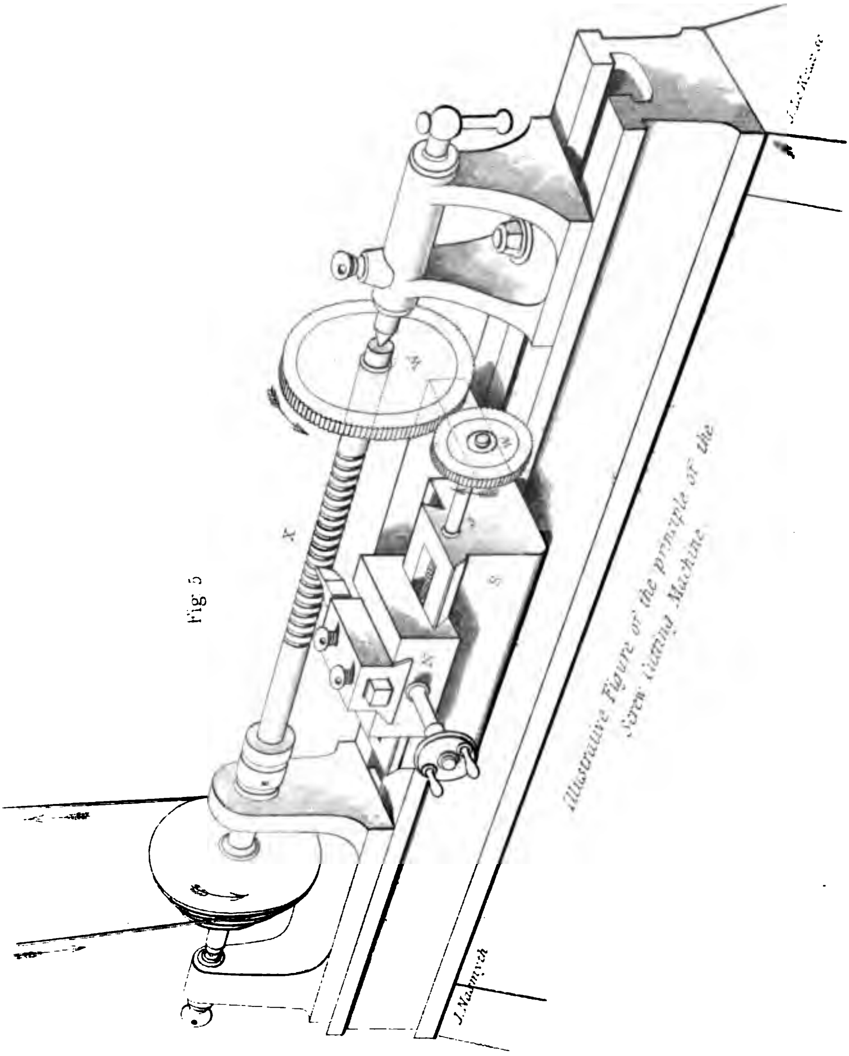


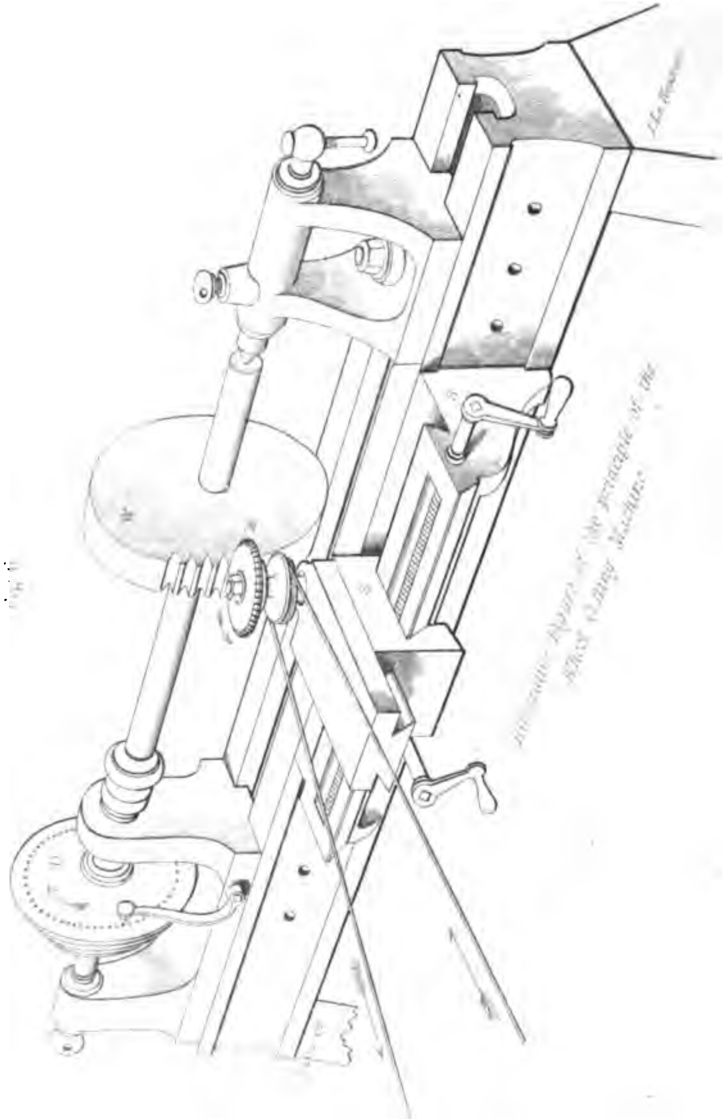
Fig 5

Illustrative Figure of the principle of the Screw Cutting Machine

Here we have the slide rest s, whose tool-holder is slid along by means of the screw s, which receives its motion from the work in the lathe, by means of the wheels w w, by which it is evident, that as the work x revolves in the lathe, a revolving motion will be transferred to the screw s, and the point of the tool will, on sliding along, have a spiral or screw on the work; and according to the respective diameters of the wheels w w, so shall we have a screw formed on x, more or less fine in the pitch of the thread, according to the proportions of the respective diameters of the wheels w w, as in the figure w or the work, is twice the diameter of w or the end of the slide screw. The pitch of the thread on x will be twice as wide as on s, and as s and x are revolving in opposite directions, we shall have a right hand screw on the one, and a left hand screw on the other, or the reverse, according to the nature of the guide screw s; and by placing an intermediate wheel between w and w, we shall then cause them to be either both right hand screws, or both left, as the case may be; the depth of cut is given in succession, by the set or transverse adjusting screw N.

502. Again, in the case of the wheel-cutting machine, we have the slide rest in full existence. See Fig. 6.

503. All wheel-cutting machines, however complex they may be in their minor arrangements, consist of two essential parts, the slide rest s, which holds the revolving cutter R, and the spindle T, on which the wheel w, which has to be cut, is fixed. This spindle is made part of the dividing wheel D, by fitting into a socket or chuck, so that when the head D is moved round in successive steps, or according to the required divisions on the face of it, which is set off or divided and held fast by the index point or holder E, it is evident that whatever be the divisions or fractions of the divided circle D, we move round step by step; the same will be most faithfully transferred to the wheel w, which we desire to cut or divide into teeth; and by means of the slide rest s, we slide the revolving cutter across the face or edge of the wheel w. It is likewise evident, that we must thereby cut a tooth every time we slide the cutter across, after each division is taken in succession by the shifting of the head or dividing wheel D.



Illustrative figure of the principle of the Screw Cutting Machine.

This is a very meagre description of the principle of a most important machine, in which, as in innumerable other instances, the slide principle enables us to produce with such facility, results in the form of workmanship, whose mathematical accuracy throws all *hand* work utterly into the shade, not only as to absolute precision, but also economy of production.

504. As before said, were I to endeavour to trace in detail the countless applications of the slide principle from its first appearance before the mechanical world, as introduced by the late celebrated Henry Maudslay, and follow it down to the present time, a thousand pages would not give space for all that might, with such truth and justice, be said on the advantages which mankind have been and are now deriving from the slide rest, and its lineal descendants.

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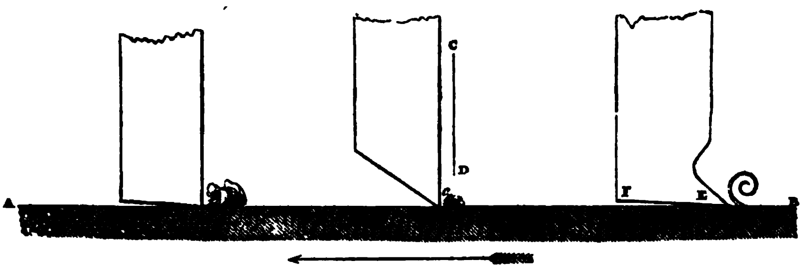
505. *Some Observations respecting the Form of Tools employed for Turning and Planing Iron, Brass, &c., together with some Remarks on the Hardening and Tempering of such Tools.*

Hitherto, so far as I am aware, the form of tools employed in turning or planing iron, &c., has not either received that attention which the importance of the subject calls for, nor has any attempt been made to reduce the subject to such plain and general principles of which it is not only capable, but when so treated, then only adapted to be of service to those in whose hands the management of such tools is for the most part entrusted. Indeed, so much practical importance attaches to this subject, that the quality as well as the quantity of work produceable from turning lathes and planing machines, entirely depends upon the skill of the operator in giving to his tools the proper form. There are many excellent workmen, who, by a species of intuition, have acquired the art of giving to the tools either the true form, or so near have they got to the true principle, that by holding to and repeating again and again that form which they found the best, they are enabled to produce the required result. But even with such, when a case occurs in which they have to go a little out of their usual routine, they are then as much "at sea" as if they knew nothing about the matter. This arises from no other cause than the want of the knowledge of the general *principle*, which would guide them to the true form, whatever be the case; and moreover, now that slide lathes and planing machines are becoming so very common in the workshops of engineering establishments, and that such machines, from their automaton power, no longer require regularly bred mechanics to attend them, it becomes more than ever necessary to reduce the subject to those simple principles to which it is capable, so that the subject may be brought within the range of the *supposed* inferior capacity of a humbler grade of men, from whom we want no more than careful attention to secure the best results from those surprisingly productive machines. We shall now proceed to the subject of these remarks, and with that view shall take, in the first place, the most simple case.

The chief, and indeed the only point which we require to consider, is the direction in which we wish to cut or penetrate the metal. Suppose, therefore, the plane A B is the surface of a plane of metal, from which we wish to cut off shavings, in the direction A B, either by A B moving against


No. 1. No. 2. No. 3.

the tool, or the reverse, namely, the tool moving against it, for it is the same action in either case. Suppose we were to employ such a tool as No. 1; in this case we should have little or no penetrating quality in the form of the tool, which would in consequence not cut, but *rub* off the particles, or crush them off by sheer brute force. The reason of this is, that we have given it so very blunt or obtuse an edge at the point of cutting, that by their coming against it at right angles to its face, the whole force which moves the plane A B will be consumed in merely rubbing off (not cutting) the particles of metal.

Next, in the case of No. 2, which *looks* more like a tool that would cut, we shall find that there again we should fail to produce the required result, and also encounter other evils. In this case we still have no more penetrating property in the direction A B, for the force of the tool is still in the same position, with regard to the surface to be cut, as in the instance of No. 1, that is, it is at right angles to it, so that we have no advantage here; and what is far worse, we have from this tool a penetrating quality, in a direction quite opposite to that which we desire, namely, in the direction C D. In moving the surface A B against this tool, No. 2, we should, on attempting to take a cut, find that the penetrating quality in the direction C D, would immediately exhibit itself in a series of saw, teeth-like marks, more or less deep, according to the strength of the cut and that of the tool, which indeed would, on account of its form, not preserve its point entire for a moment, but would be *snipped* off with little or no force, because the cross section of metal at its *point* is scarce measurable. This is the most usual error in the forming of tools, that is to say, because they *look* sharp, that is thought sufficient; forgetting altogether the direction in which the strain is to be applied, and in consequence not providing sufficient metal a cross section in the direction of the strain.

If we look to No. 3, we shall find that all these requisites are provided. In the first place we have a high degree of acuteness in the direction of the cut, namely A B; then as to strength, *behind* the point we have all the metal from E to F to give the point E the requisite support; in short, as regards strength, we have as much more strength in the case of No. 3, over No. 2, as the distance E F is greater than c, No. 2. Besides this great strength which we have in the case of No. 3, we have also another advantage of great moment, namely, the entire absence of all tendency to chatter or produce a rippled surface, F E acting as a most complete *stop* to any risk of digging into the surface which we are planing or turning, which would inevitably be the case with No. 2, supposing the point to be capable of resisting the force, which it could not. The very form of the shaving in the case of either of these tools, would exhibit the relative advantages of each. In the case of No. 3, they would be most complete curls, as may be evident from the form of the tool.

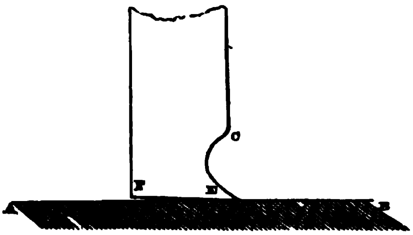
In No. 1, therefore, we have strength, but no acuteness in either direction.

In No. 2 we have acuteness, it is true, but in a direction quite opposite to that in which we require it, and *no strength.*

In No. 3 we have acuteness entirely in the direction in which we require it, and the greatest degree of strength.

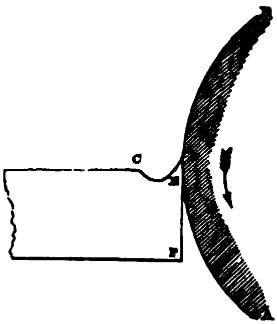
We may therefore establish from this attempt at investigation, the following principle, namely, that in forming and setting a tool to cut any surface, we have only to attend to placing it so that the *end of it forms the least possible angle with the surface to be cut,* and whatever degree of acuteness be considered proper, let the keenness be given by hollowing out the surface E C, as given here.

No. 3.



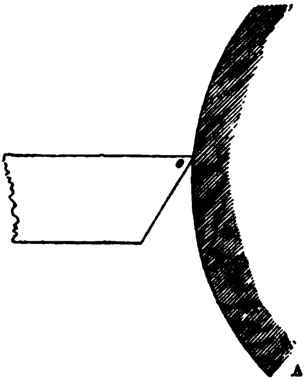
I again repeat the principle, namely, that in forming the cutting tool, what we have to attend to is, to let the end of the tool be as nearly parallel to the surface to be cut as possible, and any acuteness that may be required shall be given to the surface on which the shavings slide; the very same holds good in the case of turning tools, and indeed in every tool, from a razor or carpenter's chisel up to the most enormous and powerful tool in a lathe or planing machine. In the case of turning, we may see the application of the "principle" very clearly exemplified.

No. 3 as a turning tool.



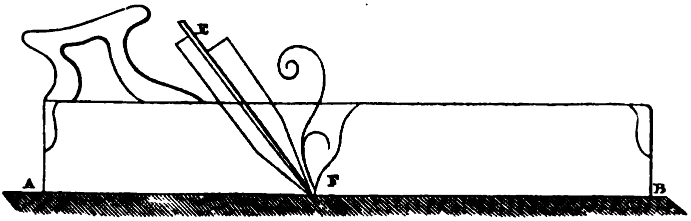
Here we see No. 3 as a turning tool, A B being a portion of a cylindrical bar in the lathe; E F should be as near as possible a tangent, that is, at right angles to the radius of the curve.

No. 2.

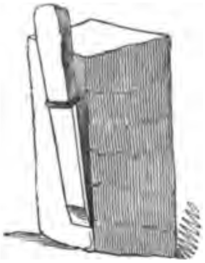


In the case of No. 2, employed as a turning tool, we should not be able to preserve its point for an instant, as will be evident from the small cross section at C.

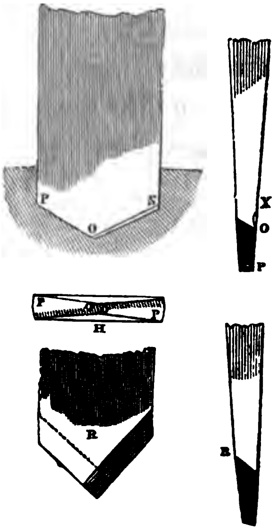
When *scraping* is all that is necessary, which is a last finish just before preparing the work to be polished, No. 1 may be employed with advantage, as in that case its low penetrating quality in both directions becomes of much service, but *then* it is not desired to employ it as a *cutting* tool.



In the instance of a common joiner's plane, we shall find the same principle carried out most fully; E F is the plane iron, A B being as before the surface to be cut. In the case of this tool, an artificial end is given to the cutting tool, by means of the sole of the plane, which gives the requisite non-penetrating quality in all directions, except that in which we require to remove the material, or take the cut, namely, A B.



The same again is seen in the action of a chisel or hatchet. It will be observed that the bevelled surface of the chisel is always placed *outwards*, and the flat surface placed next to the wood which we are about to cut, so that the angle between the face of the chisel *next* the wood, and the surface of the wood, shall form the *least possible* angle with it.



Also, in forming *drills*, we shall find the very same principle in action, as has been given in the foregoing examples. Thus, H being the end view of a drill, the edge O P should be the least possible prominent, or out of the plane of the surface of which they are the edges, O S being less prominent than O P, so that there may be as little penetrating quality at the edge O P as possible. A drill so formed, will cut the smoothest holes without any chattering, which is so commonly the case when the edges are bevelled very much back, as given at R. Such a drill would very soon lose its edge, and would cut a very rough hole besides.

In order to give great keenness to the edge of the drill, we have only to apply the same principle as before stated, in respect to turning tools, by hollowing out a groove at X, on each cutting face.

FIG. 1.

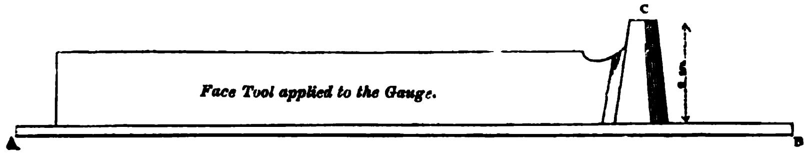
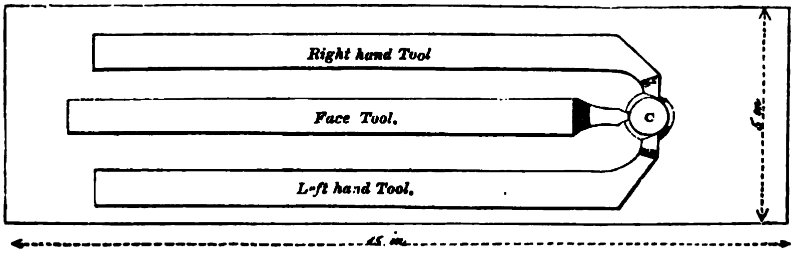


FIG. 2.



The above is a sketch of a very convenient and simple tool gauge, for enabling any one to ascertain whether a tool is ground or formed to the proper angle. It consists of a planed plate of metal, A B, on whose surface there is at one end fixed a conical steel pin C, whose taper or angle formed by the sides of the cone with the surface of the plate A B, is just that which is proper for the cutting face of the tool C, being a *cone* given in a very simple universal gauge for every kind of tool, such as seen in Fig. 2. By using this gauge, all difficulty of forming the tools to the proper angle, is at once removed.

And the same gauge will answer for every kind of planing or turning tool whatsoever, and of whatever size. A B may be about 15 inches long, by 5 wide, and about $\frac{3}{8}$ths of an inch thick; these dimensions are by no means absolutely requisite, but will be found generally useful.

The angle formed by the sides of the cone, and the surface of the plate, should be about three degrees.

GENERAL

EXPLANATION OF THE PLATES.

PLATE I.—ESSAY I.

ON THE TEETH OF WHEELS.

Chart shewing the horses' power to which the teeth of wheels of certain pitches, working under different circumstances, are equal,—is described on the Plate, and in Chap. V. Arts. 163—179.

PLATE II.—ESSAY II.

ON THE SHAFTS OF MILLS AND OTHER MACHINES.

Fig. 1.—No. 1, No. 2, and No. 3.—Old mode of fixing gudgeons called *laid-in-gudgeons*, Art. 187, pp. 178, 179.
Fig. 2.—No. 1 and No. 2.—Cross-tailed gudgeons, Art. 188, p. 179.
Fig. 3, Fig. 4, and Fig. 5.—Parts of cross-tailed gudgeons, Art. 188, p. 179.
Fig. 6.—A hollow cast iron shaft. No. 2. An end view, to shew the manner of fixing the gudgeons, Art. 192, p. 181.

PLATE III.—ESSAY II.

Fig. 7.—No. 1, No. 2, No. 3, and No. 4.—A feathered shaft, with different sections, Art. 193, p. 181.

Fig. 8.—No. 1 and No. 2.—Square shafts, Art. 193, p. 182.

Fig. 9.—No. 1 and No. 2.—Stress on shafts, Art. 194, pp. 182, 183.

Fig. 9.—Water wheel with teeth on the shrouding, to give motion to the mill, Art. 194, p. 183.

Fig. 10.—The chief strain on the vertical shaft is that which tends to twist it, Art. 194, p. 183.

Fig. 11.—Example of a compound strain on the shaft, Art. 194, p. 183.

Fig. 12.—Shaft of a water wheel, Art. 194, p. 183, and Art. 195, p. 183.

PLATE IV.—ESSAY II.

Fig. 1.—Effect of position in causing a greater or less degree of strain on shafts, Art. 195, p. 183.

Fig. 2.—Stress on shafts from the action of the moving power, Art. 196, p. 184.

Fig. 3, Fig. 4. No. 1, No. 2, No. 3; and Fig. 5.—Effect of position in causing more or less stress on the shafts, Art. 197, pp. 184, 185; and Art. 198, pp. 185, 186.

Fig. 6 and Fig. 7.—To explain the effect of the place of a wheel on a shaft, Art. 199, p. 186.

PLATE IV. A.—ESSAY II.

Fig. 1, Fig. 2, Fig. 3.—Represent an improved method of fixing gudgeons on wooden shafts, Art. 207, p. 193, &c.

Fig. 4.—Plan of a shaft and wheels to show how the stress on a shaft may be determined, Art. 207, pp. 193, 194.

Fig. 5, Fig. 6, and Fig. 7.—Figures to show the stress on shafts and gudgeons in different circumstances, Art. 210, p. 195.

PLATE V.—ESSAY III.

ON THE LONGITUDINAL CONNEXION OF SHAFTS, DENOMINATED COUPLINGS.

Fig. 1.—Represents three shafts connected by couplings, supported by double bearings, Art. 291, p. 266.

Fig. 2.—No. 1 and No. 2.—The square coupling, with double bearings, Art. 292, p. 267.

————No. 3.—A different modification of the square coupling, Art. 292, p. 267.

————No. 4.—A coupling box made in two pieces, Art. 292, p. 267.

Fig. 3.—No. 1, No. 2, and No. 3.—A variety of the couplings with double bearings, Art. 294, p. 268.

Fig. 4.—No. 1, No. 2, and No. 3.—The round coupling, Art. 295, pp. 268, 269.

Fig. 5.—No. 1 and No. 2.—Clutches or glands, Art. 297, p. 269.

Fig. 6.—No. 1 and No. 2.—Boring-mill clutch, first construction, Art. 300, pp. 270, 271.

Fig. 7.—No. 1 and No. 2.—Boring-mill clutch, second construction, Art. 303, pp. 271, 272.

Fig. 8.—No. 1, No. 2, and No. 3.—Coupling having circular plates, Art. 306, p. 272.

PLATE VI.—ESSAY III.

Fig. 9.—No. 1 and No. 2.—Coupling link used by Messrs. Boulton and Watt in their portable steam engines, Art. 308, p. 273.

Fig. 10.—No. 1, No. 2, and No. 3.—A coupling sometimes used to connect the fly-wheel shaft of a steam engine with the mill-work, and is so contrived, that in case the fly should turn in a wrong direction, the mill-work remains at rest, Art. 310, p. 273.

Fig. 11.—The universal joint, Art. 314, p. 276.

Fig. 12.—No. 1, No. 2, and No. 3.—Square coupling, having one bearing, Art. 318, p. 277.

Fig. 13.—No. 1 and No. 2.—Mode of coupling rollers used in machinery for spinning cotton, Art. 320, p. 278.

Fig. 13^2.—No. 1, No. 2, and No. 3.—Cylindrical coupling, having one bearing, Art. 321, p. 279.

Fig. 14.—Bolted coupling, used in some mills in Manchester, Art. 323, p. 279.

PLATE VII.—ESSAY III.

Fig. 15.—No. 1, No. 2, and No. 3.—A variety of the bolted coupling, Art. 325, p. 280.

Fig. 16.—Another modification of the bolted coupling, Art. 326, p. 280.

Fig. 17.—No. 1, No. 2, and No. 3.—Quadrant coupling, Art. 327, p. 280.

Fig. 18.—No. 1, No. 2, No. 3, and No. 4.—Coupling forming an universal joint, Art. 329, p. 281.

Fig. 19.—No. 1.—Elevation of coupling, executed at Manchester, Art. 331, p. 282. No. 2, and No. 3, are on the next Plate.

PLATE VIII.—ESSAY III.

Fig. 19.—No. 2 and No. 3.—Sections of couplings executed at Manchester, Art. 331, p. 282. No. 1, is on the preceding Plate.

Fig. 20.—No. 1, No. 2, and No. 3.—Cylindrical coupling, having projections, Arts. 334, 335, pp. 282, 283.

Fig. 21.—Square coupling (having a coupling-box) for upright shafts, Art. 337, p. 283.

Fig. 22.—No. 1 and No. 2.—Square coupling for upright shafts, having a socket instead of a box, Art. 338, p. 284.

Fig. 23.—Coupling for upright shafts, having projecting quadrants, Art. 340, p. 284.

Fig. 21^2.—Lying shafts, shewing the situation of the couplings with regard to the wheels, Art. 348, p. 286.

PLATE IX.—ESSAY IV.

ON METHODS OF DISENGAGING AND RE-ENGAGING MACHINERY, WHILE IN MOTION.

Fig. 1, and No. 2.—The sliding pulley, Art. 362, p. 294.

Fig. 2.—No. 1, No. 2, No. 3, and No. 4.—The bayonet, Art. 364, p. 295.

Fig. 3.—No. 1 and No. 2.—The lock pulley, Art. 366, p. 297.

PLATE X.—ESSAY IV.

Fig. 4.—No. 1 and No. 2.—Fast and loose pulleys, or dead and live pulleys, Art. 368, p. 297.

Fig. 5.—Sack tackle, Art. 372, p. 299.

Fig. 6.—No. 1.—Wheels, *in gear*, having a moveable bridge, Art. 371, p. 298. No. 2, is on the next Plate.

PLATE XI.—ESSAY IV.

Fig. 6.—No. 2.—Wheels *out of gear*, Art. 374, p. 299. No. 1, is on the preceding Plate.

Fig. 7.—No. 1 and No. 2.—Clutch for engaging the wheel A with the shaft B, Art. 376, p. 300.

Fig. 8.—No. 1 and No. 2.—Friction clutch, Art. 378, p. 301. No. 3 is on the next Plate.

PLATE XII.—ESSAY IV.

Fig. 8.—No. 3.—Hoops of friction clutch, Art. 378, p. 302. No. 1 and No. 2 are on the preceding Plate.

Fig. 9.—No. 1 and No. 2.—Friction cones, Art. 380, p. 302.

Fig. 10.—No. 1 and No. 2.—Wheels moving by contaction, Art. 382, p. 303.

Fig. 11.—Sack tackle, Art. 384, p. 304.

Fig. 12.—No. 1 and No. 2.—Self-disengaging coupling, Art. 386, pp. 304, 305.

PLATE XIII.—ESSAY V.

On Mechanism for Equalizing the Motion of Mills, denominated Lift-Tenters, Engine Governors, and Water-Wheel Governors.

Fig. 1.—Steam-engine governor, Art. 389, p. 308.

Fig. 2.—The balls A B and C being made to revolve, are all found on the same horizontal plane, Art. 392, p. 310.

Fig. 3.—Lift-tenters for wind mill, first construction, Art. 396, p. 311.

Fig. 4.—Lift-tenter, second construction, Art. 397, p. 312.

Fig. 5.—No. 1.—Elevation of water-wheel governor, first construction, Art. 399, p. 313. No. 2, No. 3, and No. 4, of Fig. 5, are on the next Plate.

PLATE XIV.—ESSAY V.

Fig. 5.—No. 2, No. 3, and No. 4.—Water-wheel governor, first construction, Art. 399, p. 314. No. 1, of Fig. 5, is on Plate XIII.

Fig. 6.—No. 1 and No. 2.—Water-wheel governor, second construction, Art. 401, p. 315.

PLATE XV.—ESSAY V.

Fig. 7.—Water-wheel governor, third construction, Art. 402, p. 315.

Fig. 8.—No. 1 and No. 2.—Water-wheel governor, fourth construction, Art. 403, p. 316.

Fig. 9.—No. 1, No. 2, and No. 3.—Water-wheel governor, fifth construction, Art. 404, p. 317.

PLATE XVI.—ESSAY VI.

On Changing the Velocity of Machinery while in Motion.

Fig. 1.—Turning lathe motion, Art. 419, p. 335.

Fig. 2.—Alternate cones, Art. 421, p. 336.

Fig. 3.—Wheels moving by contaction, Art. 423, p. 336.

Fig. 6.—No. 1 and No. 2.—Mechanism used in cotton spinning called a double speed, third construction, Art. 430, p. 340.

Fig. 4 and Fig. 5, are on Plate XVII.—Art. 426, p. 338.

PLATE XVII.—ESSAY VI.

Fig. 4.—Double speed, first construction, Art. 426, pp. 338, 339.

Fig. 5.—Double speed, second construction, Art. 426, pp. 338, 339.

PLATE XVIII.—ESSAY VII.

On the Framing of Mill-Work.

Fig. 1.—Pivot of upright shafts, Art. 439, p. 346.

Fig. 2.—No. 1, No. 2, and No. 3.—Pivot of upright shaft having a steel foot, Art. 440, p. 346.

Fig. 3.—Cylindrical steel pivot, Art. 441, p. 347.

Fig. 4.—No. 1, No. 2, and No. 3.—Journal of upright shaft supported by a breast, Art. 444, p. 347.

Fig. 5.—Millstone bush, Art. 445, p. 347.

Fig. 6.—No. 1, No. 2, and No. 3.—Headstock framing for supporting gudgeons of water wheel, Art. 446, p. 348.

Fig. 7.—No. 1, No. 2, No. 3, and No. 4.—Wooden framing for lying shafts, Art. 447, p. 348.

Fig. 8.—Framed post, Art. 448, p. 349.

Fig. 9.—Framing for lying shafts suspended from a ceiling, Art. 448, p. 349.

Fig. 10.—No. 1 and No. 2.—Bridge and pedestal of upright shaft, Art. 449, p. 349.

Fig. 11.—Wooden framing of an upright and lying shaft connected by bevel wheels, Art. 450, p. 349.

Fig. 12.—Framing having cloves, Art. 451, p. 349.

Fig. 13 and Fig. 14.—Transverse sections, showing forms of pieces of timber and of iron, Art. 456, p. 351.

Fig. 15.—Feathering, Art. 457, p. 352.

Fig. 16, Fig. 17, and Fig. 18.—Represent advantageous forms of sections, Art. 457, p. 352.

Fig. 19.—No. 1 and No. 2.—Bridge made of wood for sustaining lying shaft, Art. 458, p. 353.

Fig. 20.—No. 1 and No. 2.—Bridge made of cast iron for sustaining lying shaft, Art. 458, p. 353.

PLATE XIX.—ESSAY VII.

Fig. 21.—Headstock for water wheel, Art. 460, p. 353.

Fig. 22.—Mode of suspending lying shafts by cast iron, Art. 461, p. 353.

Fig. 23.—No. 1 and No. 2.—Cast iron framing for supporting 3 pair of flour-mill stones, Art. 462, p. 353.

Fig. 24.—No. 1 and No. 2.—Machine called *squeezers*, used in bleaching, having cast iron framing, Art. 462, p. 353.

EXPLANATION

OF

THE ADDITIONAL PLATES

OF THIS EDITION,

NUMBERED AND DESCRIBED FROM PLATE XIX.

PLATE XX.

Represents several diagrams, shewing the theoretical mode of describing the teeth of wheels; to accompany (the Appendix A.) Professor Willis's Paper on that subject, pp. 149—157, and 166—172.

PLATE XXI.

Bramah's original Slide Rest.

Fig. 1. is an elevation; Fig. 2. an end view; and Fig. 3. a plan of Bramah's slide tool, which was first used in the year 1794, and was the workmanship of the late Mr. Maudslay, made by himself when in the employment of the late Mr. Bramah.

The work to be turned is placed in the usual way between two centre pieces, one is shewn in Figs. 1 and 3, by *c*, where it is secured. The square bar *d*, on one end of which is fixed the tool, is then made to advance by means of the screw *e*, working through a nut, secured to the square bar *d* by a small set screw; this nut piece can be moved along the bar according to the length required, and by turning the handle *e*, the bar *d*, and consequently the tool, is brought close to the work which it is intended to turn. The frame for carrying this bar slides between two V's on the frame *b*, by the motion given to the handle and screw; thus it will be seen, this rest has two motions, the one at right angles to the other.

The slide rest and frame *b* are moveable along the traversing bar *a*, according to the length of the work, and when placed in any particular position may be secured by the handle and screw underneath.

This rest is very different to those in use at present, as will be seen hereafter.

PLATES XXII. AND XXIII.

BRAMAH'S LATHE FOR TURNING SPHERES.

The ball or sphere *j* is fixed in the usual way between the two centres of the lathe, having projecting pieces on it for that purpose. The mandril is driven by a leather strap working on the circumference of the pulleys or riggers *d*; on one end of this mandril is the chuck *f*, having a pin on its face, called the driver; when this begins to revolve, it comes into contact with a carrier fixed on one of the projecting pieces of the sphere, which must necessarily drag it round with it, and thus communicate the revolving motion; there is a set screw on this carrier to fix it by, as is shewn in Figs. 1 and 3.

The tool *i*, Fig. 2, is fixed to the moveable slide *b*, by two adjusting screws, and moves on its centres *h' h'*, describing the circle of the sphere; this slide works in a V on one side of the bed *a*, and on a flat smooth surface on the other. When the sphere is turned, the back centre screw *g* is loosened, which at once liberates it. There is also an adjusting screw for the back centre of the mandril, which at times requires tightening. By withdrawing the slide, used in spherical turning, and fastened to the sliding carriage by four screws, a common hand rest can be put in its place, when the lathe is required for ordinary purposes.

PLATE XXIV.

GREAT BORING LATHE, BY MESSRS. NASMYTH, GASKELL AND CO.

The two lathes already described are very much smaller than that shewn by this Plate; the work they are capable of turning is altogether of a different kind.

Figs. 1, 2, and 3, severally represent an elevation and two end views, where it will be seen that this lathe is suited to various speeds, depending on the diameters of the pulleys or driving riggers *d*, which are put in motion by a leather strap. The headstocks *a*, and the bed *b* are well secured to the stone foundation, by means of strong wrought iron holding down bolts for that purpose; on the face-plate *c*, to which is fixed the work to be turned, is an internal wheel worked by a small pinion hung on an intermediate shaft, which receives its motion from the driving shaft, by the spur wheel and pinion *e* on its opposite end; long mortices are shewn on the face-plate in Fig. 3, through which the work to be turned is clamped, and revolves with it; *f* is the slide rest for carrying the tool, placed on the two standards *g g*, the tool being made to advance, by a long screw the length of the slide rest. By the handle seen in Fig. 1, it is made to travel along the sliding bed. It has also a second motion at right angles to that already described. The tool is fixed to the rest by four screws, Fig. 1. The headstock *a* is raised on a carriage *b*, to which it is bolted. This lathe is of large dimensions, and is capable of boring cylinders of great diameters; it is also well adapted for turning locomotive engine wheels.

PLATE XXV.

FACE-TURNING LATHE, BY MESSRS. NASMYTH, GASKELL, AND CO.

This Plate differs but little from the last described. There is only on rigger or pulley *a*, on the mandril or shaft, and the face-plate *c* has four adjusting screws for securing the work to it. Instead of driving this plate from the pulley-shaft, it is put in motion by a small pulley worked by a strap, from the main shaft of the building, in a similar way to the pulley *d*, making the one quite independent of the other. The internal wheel, and the pinion, are similar in every respect to those described by Plate XXIV., which is also the case with regard to the slide, the slide-rest, and the headstocks *a a*.

The foundations to which the bed *b* is bolted are of stone bedded in concrete.

PLATE XXVI.

FOOT-LATHE, BY MR. F. LEWIS, MANCHESTER.

The bed *a*, Figs. 1 and 2, is placed on the two standards *b b*, forming at the same time centres for the crank-spindle *n* to work in, which receives its motion from the foot-board *n′* by the two connecting links *oo*. On the spindle *n*, is a pulley and also a fly-wheel, for regulating the motion; (the latter being used at times as a pulley for quick speeds;) these convey it through the upper pulleys *c*, by a leather strap, to the mandril or spindle *d*, on the end of which is a chuck or face-plate. The headstock *q* forms suitable bearings for this spindle to revolve in, having a back centre-screw to tighten it by. The following headstock *r* slides along the bed to any position, according to the length of the work to be turned, which is placed between the two centres *f f*; that in the latter headstock is adjustable, being fixed to a cylinder *g* working in it, and moved either backward or forward by the handle or wheel *g′*, which, as it revolves, works a screw fixed to it and kept steady by the bolt underneath, Fig. 1. When the work has been properly centered, the cylinder *g* is made secure in the headstock by the small handle and screw *h*.

The slide *i* for carrying the tool is advanced along the bed by a pinion working in the rack *i′*, put in operation by a small handle placed on the square end of the spindle *i″*; a motion at right angles to this is obtained by the slide *j*, worked in the same way by the screw *j'*, and lastly, a circular motion may be given to the tool by the worm and worm-wheel *l*. The tool is placed in the tool-box *s*, and there fixed by the two set screws; this box may be placed in any position, and then secured by a set-screw *m*.

An index *q'* is graduated on the headstock *q*, for the purpose of turning spheres.

The rack *i'* is secured to three small brackets bolted to the bed of the lathe.

This machine is capable of turning a bar 3 feet long, and 6 in. diameter, and spherical cups, balls, &c., &c., 5 in. diameter, having an index to turn either conically or longitudinally.

As will be seen by the title, this lathe is put in motion by the foot of the man working it, and unlike the last two, it is only adapted for turning small work.

PLATE XXVII.

SEVEN-FEET TURNING-LATHE, BY MR. F. LEWIS, MANCHESTER.

In the usual way, the top part of the fixed bed *a*, Fig. 2, is of the V form, upon which the headstocks *bb* and the slide *h* are moveable; that for carrying the driving apparatus is secured to it by strong bolts and nuts; the shaft of the driving pulleys *c* works in the two bearings of the headstock; one is conical, for the purpose of keeping it always tight and to avoid any play, which would be the case were it parallel, after having worked any length of time; it is tightened against the cone-piece of the headstock by nuts on the opposite end of the shaft. To the face-plate *g*, the work is fixed by four jaws *rrrr*, moving in V's on the back of the plate, and radiating to the centre, to which they can be approached or drawn from, by a small handle put on the square end of the screw, by which means they are made to slide in the V's according to the size and nature of the work to be turned; the motion is communicated to the face-plate by the spur pinions and wheels on the intermediate shaft *f*, and thence to the pinion working the internal wheel on the face-plate, consisting of six segments screwed to it. By this arrangement the speed of the face-plate is very much decreased, and it should be understood that though the plate *g* is on the main shaft, it is not fixed to it by keys, but runs loose, and revolves at a much decreased velocity; a triangular carriage is bolted to the headstock's side for supporting the bearings of the intermediate shaft.

In the opposite headstock the cylinder *q* and centre *n* moves, being slid through it by turning the wheel and screw *q'*, according to any required position for centering the work, which, when done, is firmly secured by the two tightening rings and nuts *q''*.

This headstock, and also the slide, is moved along the surface of the bed *a* by pinions working in a rack *o*, fixed to the three brackets *p*, on its side. The pinions are made to revolve by a lever turning the wheels next to them. Independent of the motion of the slide along the top of the bed, it has three others, viz.: the frame *j* can swivel round on the carriage *i*, to any angle; the slide *k* can travel along the frame *j* by a small handle fixed on either end of the screw *k'*, and lastly, a motion at right angles to this is obtained by the slide on which the tool is placed, being also worked by a screw; the tool is fixed by the bolts *l'*, which hold it firmly in its place.

On the end of the main driving shaft *e* is the centre *n*, and between this and the other, the work to be turned is centered in the usual way.

The tires for locomotive wheels, and also wheels of a diameter of seven feet, can be turned either internally or externally by this lathe.

PLATE XXVIII.

LATHE FOR TURNING GUN-BARRELS, BY MESSRS. G. AND J. RENNIE.

The object of this machine is to perform an operation entirely different from that produced by the several motions of the common turning lathe, where the work is either of a cylindrical or elliptical figure. In this case, the curve required to be turned, on the whole length of the musket barrel, is of an irregular form. This, as will be seen by Figs. 1, 2, 3, and 4 of this Plate, is effected by a very simple contrivance, a description of which follows:—

The bed *b* is secured by three bolts *a' a' a'* to the side frames or standards *a a*, which have also fixed to them a tank *o* for receiving the waste oil and the iron turnings from the barrel being turned; on the top of the bed is placed the headstock frame *i*, for carrying the several parts of the machinery; and on the mandril *d* are the tight and loose pulleys for giving motion to the machine: the centre *c'* of this mandril is adjusted by the back centre *c''*. By the two sets of spur wheels and pinions a double motion is produced, the forward by *f* and the backward by *g*, which are alternately worked by means of the small clutch *h*, thus giving a revolving motion to the square threaded screw *j* for advancing the tool frame *l* along the barrel to be turned; this motion is made self-acting by means of two small bars *p p* fixed to the clutch lever, having projecting pieces on their sides, with which the frame *l* comes in contact, and pressing against them, draws them and engages the clutch in the wheels for giving the opposite motion, while the object of the handle *h'* is to disengage it by hand. The spindle on which these small wheels are fixed runs freely in brass bearings fitted to the headstock *i*.

On the opposite end of the machine is the headstock *m*, having an adjustable centre *c′*, worked by the screw and handle *m″*, and may be fixed in any required position by a small set screw *m‴*; this head can be moved along the bed *b*, to suit the length of any barrel *n*, and by the cross bar and bolts *m′* may be secured to it. The musket barrel to be turned is slid on the long bar or mandril *n′*, which it fits very tightly. One end of this mandril is then dropped in the chuck *e* for carrying it round, while it is centred at both ends in the centre pieces *c′ c′*.

The most curious part of this machine is the frame *l* for holding the tools and likewise the barrel of the musket being turned; which latter operation is rendered rather difficult from the irregular shape of the barrel. Fig. 3 shews a section, looking at the frame *l*, which consists of a back plate, on the face of which is a brass plate with the eccentric grooves shewn by the dotted lines struck from different centres; between the latter plate and the back plate are four dies sliding in V's, having ribs working in the above grooves, which, not being concentric, press them against the barrel, and thereby hold it firmly while the tools *l′* are turning it; but as the diameter of the barrel alters, it is necessary to loosen the dies, in order to allow the frame *l*, of which they form part, to slide along the bed on its V's. This is thus effected: a curve bar or template *k*, suitable to the curve required by the barrel, is firmly screwed to the side of the bed, its upper edge *k*, Fig. 3 and 4, being of the shape of a V, on which the lower part of the rack *l‴* slides, the upper part working at the same time in the V's screwed to the projection given to the back plate *l* by three screws. Thus the motion produced by the curve of the bar *k* is transmitted through the rack and the segment *l″* to the grooved plate on which the latter is fixed; the result of this is the tightening or loosening the four dies, which must naturally be the case, as they have ribs working in the grooves, which they are compelled to follow. By the weight suspended from the vertical rack, any irregular motion that might take place is entirely obviated, it being kept firmly working on the upper surface of the curved bar *k*, while the small handle is provided for the purpose of raising it by hand when required. There are two tools *l′*, Fig. 3, fixed to one of the four dies already described; the one farthest from the frame *l* is for rough turning, while the other follows it, giving the finishing cut.

A small bearing is provided on the bed for supporting the outer end of the screw *j*, and the standards are well secured to the floor by bolts for that purpose.

This machine is capable of turning and finishing three barrels per hour; and similar machines are in use at the Royal Armoury of Enfield, fitted up by Messrs. Lloyd and Co., and twelve of them, in conjunction with a complete set of machinery for making muskets, are at work in the Imperial

Armoury at Constantinople, constructed by Messrs. Rennie, besides machines for the French Government and for his Highness the Pasha of Egypt.

PLATE XXIX.—Figs. 1 and 2.

Portable Hand Drill, by Messrs. Nasmyth, Gaskell, and Co.

The utility of the little machine shewn by Figs. 1 and 2, is its portability; and it is found very convenient in drilling holes in such pieces of machinery which if required to be brought under a larger machine would cause great delay and inconvenience.

The frame *a* carries the upright drilling spindle *b*, the top of which is a screw *e′* for raising it by the handle wheel *e* by hand, while the revolving motion is communicated to it by the two small bevel wheels for conveying it at right angles. When required to drill a hole in any piece of machinery, it is first of all set in its proper place; after this is done, the handle *c*, or small fly wheel, is turned round for working the drill, and by a slow revolving motion given to the upper handle *e*, the drill, while working, is made gradually to descend. It can, if required, be secured to its work.

Fig. 3.

Foot Drill, by Messrs. Nasmyth, Gaskell, and Co.

The difference between this machine and those described by Plates XXX. and XXXI., consists in the mode by which the upright drilling spindle is made to rise and fall. In those referred to, this is made self-acting, while in this case it is performed by the pressure of the foot of the man superintending the drilling of the hole.

This machine is driven by the riggers or pulleys *b*, the one running loose, while the other is fixed to the spindle for conveying the motion by means of the upper and lower sets of pulleys *c*, by which the speed can with great ease be made to vary considerably; this is done by altering the position of the leather strap shewn by the dotted lines. The motion is then carried at right angles, to the drilling spindle by the bevel wheels, for producing the requisite revolving motion.

A moveable table *g*, for carrying the work to be drilled, is fixed to the frame *a*, in which it slides, and can be raised or lowered by the wheel and screw *h′*; this is found of great use, as the size of work may very much vary.

By means of the footboard *f*, working as a lever on its fulcrum, the drilling spindle is made to rise and fall; the pressure of the foot on the board *f* causing the rod *f'* to rise, which by the upper lever fixed to the frame of the machine depresses the spindle *d*, while it is revolving; as soon as the pressure is taken off, the counterbalance weight *f''* causes the drill to ascend and take its former position, where it is kept until again used.

PLATE XXX.

Wall Side Drilling Machine, by Messrs. Nasmyth, Gaskell, and Co.

The several parts of this machine consist of a frame fixed to the side of the wall of the building, against which are bolted the frames *bb*, for carrying the upright drilling bar *e*, put in motion by the driving pulleys or riggers *c*, according to the speed required, which is regulated by the diameter of the pulley on which the strap works. Independent of this, it has also a double motion, obtained by the two spur wheels and pinions *d*; this second motion is fully explained in the reference to the Plates XXXIII. and XXXIII A.

This machine is made self-acting by the spur wheels and pinions *ff* raising the drilling bar as they revolve; the upper part of the screw *g* has a cross guide *h*, sliding up and down between the two upright parallel rods *i i*.

The table *k*, for carrying the work, is made to slide on the bed *j*, similar to the bed of a planing machine, one of its sides being of the V form, while the other is a flat smooth surface; this table is advanced by a chain fixed to it at one end, and works round the rollers *m*.

In the drawing, a small cylinder *n* is being bored.

This machine is well adapted for boring holes for the pivots of engine beams, and all parallel holes.

PLATE XXXI.

Double Pillar Drill, by Messrs. Nasmyth, Gaskell, and Co.

The principle of this machine is the same as that last described, with the exception of a few of its parts, which are of larger dimensions, and for larger purposes.

The frame-work *a* for supporting the different parts of the machinery, rests on two upright pillars; on this frame is a small shaft for carrying the driving pulleys *b* and pinion *c*, conveying the motion to the upper or intermediate shaft by the spur wheel, whence it is taken at right angles to the drilling bar by the bevel wheels *d*; the bar *e* is raised and lowered by the screw on the upper part of this bar, which screws itself up in a nut; when it is required to be lowered, the handle or wheel *j* is worked round by hand, setting in motion the upper spur wheels, thereby communicating it to the upright screw and drilling bar.

The moveable bed or table *h*, for carrying the work, travels along the bed *g;* it has two motions, the one at right angles to the other, and by the long screws and handles *i i* is brought in any convenient position.

Its principal adaptation is for boring the holes for receiving the tubes in locomotive boiler plates, and in such cases when any number of holes are required.

PLATE XXXII.

RADIAL DRILLING MACHINE, BY MESSRS. BENJ. HICK AND SON, BOLTON.

An entirely different arrangement of a drilling machine from those generally used, may be seen by this Plate; in all ordinary cases, the drilling bar or spindle is stationary, that is to say, it has no lateral motion, being only able to rise and fall in its bearings; in this case, the whole drilling tackle is made to slide along a radial bar or carriage, whereby the drill can be brought over the work into any required position within the limits prescribed by the radius of the arm *b*.

The arrangement of this machine consists of a strong upright column *a*, bolted in a most substantial way to the stone foundations; a screw *c* works up and down in the internal part of this column, according to the height required for the work; this is made secure when raised to its proper position by a nut *d*, tightened by the four pins on its circumference; the upper part of the screw has a collar, upon which the radial bar rests, which at the same time is capable of revolving on this centre; a carriage for supporting the small square shaft, the bevel wheels, and the fast and loose pulleys placed horizontally, is fixed to the upper part of the radial bar by bolts and nuts; on the other extremity of the shaft *g*, is a second pair of bevel wheels, for conveying the motion at right angles, thus causing the drilling bar *j* to revolve, on the lower end of which is fixed as usual the drill *k*. The two upright supports *h* are bolted to the travelling frame; these carry the apparatus for raising and lowering the drill, which consists of two small chains fixed to the top of the drilling bar, working round rollers, and also two others having weights suspended from them, and running over the chain pulleys, that with the large weight brings the drill down, while it is drilling, which, after it has performed its duty, is disengaged by a lever and rod, so that its weight is neutralized, when that of the smaller ball comes into operation for raising the drilling bar to its former position. On the face of the radial bar is a rack and pinion *l*, which, by means of the wheel handle *l′*, causes the drilling frame to slide along the surface of the bar *b*, which, as already stated, is regulated by the position required for the drill, while the square spindle or shaft *g* slides through the wheel, and also the bearing fixed on the bar. The motion is communicated by a leather strap on the fast pulley *e*, transmitting it through the bevel wheels to the drilling bar.

PLATES XXXIII. AND XXXIII. A.

UPRIGHT DRILLING AND BORING MACHINE, BY MR. F. LEWIS, MANCHESTER.

In most drilling machines, the motion is conveyed at once to the upright bar, by fixing the driving pulleys on it, which is the case in those already described, as will be seen by referring to the Plates; in this instance, an intermediate shaft or spindle, placed horizontally, receives the motion by a strap on the pulleys or riggers *c*, and is conveyed at right angles by means of a pair of small bevel wheels *h*; a double motion is provided to this machine whereby the speed can be much altered, independently of the different diameters given to the driving pulleys. The mode by which this alteration of speed is obtained is thus: there are two shafts, one having a wheel working in a pinion, and the other a pinion working in a wheel, each pair being of the same diameters, Fig. 2; it must be understood that the pulleys *c* and the pinion shewn in Fig. 1, run loose on the shaft on which the front wheel is keyed, and are there secured. Now supposing a slow speed be required, it is only necessary to throw in gear the back wheel and pinion, by sliding the shaft in a groove where it may be kept fixed by a pin *d''*, Fig. 2. The result of this is, that the pulleys, running loose on the front shaft, convey the motion through the pinion fixed to them to the back wheel, and hence through the back pinion to the front wheel, conveying it through the bevel wheels to the drilling bar *e*. An example will better illustrate this:—Let it be supposed that the pulleys and pinion are making twenty revolutions per minute; the wheels being 16 inches in diameter, and the pinions 6 inches in diameter; the speed of the back shaft would be 7¾ turns per minute, and consequently that of the wheel and front shaft would be reduced to about 3¾ turns, thus making the respective velocities about 5½ to 1. When a quicker speed is required, the back shaft, wheel, and pinion are disengaged, (by the same means it was engaged, and secured by the pin *d''*,) the front wheel is then made fast to the pulleys by a screw *d'* on its face, and both revolve at the same velocity. The back wheel and pinion should be always disengaged previous to fixing the front wheel to the pulleys, otherwise great damage would be done to the teeth of the wheels.

The self-acting motion in this machine is the same as those generally used, which is thrown off and on by the small clutch *k* and lever *i*, and may be worked by hand by the wheel *l*; the screw *g* works in the nut *g′*, and by the small spur wheels *f*, raises the drill bar.

The table *m* for carrying the work to be drilled or bored, is raised or lowered according to circumstances, by the two pinions *n* working in the racks fixed to the wall plate; this is effected by the handle *o*, wheel and pinion *p*, and is kept from returning by the paul and ratchet wheel *q* provided for that purpose; the plate *r* is bolted to the wall through four snugs. The frames *b* for supporting the several parts of the machinery have suitable bearings on them for carrying the shafts and spindles; they are well secured to the wall by strong bolts and nuts.

This machine can bore from half an inch to 14 inches in diameter, with a depth of two feet, and the table *m* will admit of a wheel 3 feet 8 inches in diameter being placed on it.

PLATE XXXIV.

SLOTTING OR KEY GROOVING MACHINE, BY MESSRS. NASMYTH, GASKELL, AND CO.

This machine is of a very simple construction; the motion being underground, and conveyed to it by the small shafts to a dog or crank wheel *b*, giving the slotting bar a reciprocating motion by means of the connecting rod *c*, to which it is fixed; the lower part of the slotting bar works in a cross guide. To the sliding plate *d*, the wheel *e* to be grooved is fixed; the cut is performed by the descent of the slotting bar, the length or depth of which is regulated by a moveable crank pin on the face of the plate *b*. It is necessary to have access to the underground work, and for that purpose the stairs *f* are shewn. A machine upon this principle for cutting the teeth of wheels was, we believe, used by Messrs. Bolton and Watt, at Soho.

PLATE XXXV.

SLOTTING MACHINE, BY MESSRS. SHARP AND ROBERTS, OF MANCHESTER.

The perspective view shewn by this plate represents a machine for slotting or paring small work, such as the straps of connecting rods, or cutting the key grooves of wheels of small diameters.

The table *l* for carrying the work, is a circular plate of about two feet in diameter. It has two horizontal motions at right angles the one to the other, working between V's, and these are made self-acting by means of the ratchet wheels *o*, *o*, worked by the cam *d* on the main driving spindle *c*, which, as it revolves, strikes against the lever *k*, and thereby gives them a progressive motion; the pauls *p p* advance it in proportion to the leverage or distance from the fulcrum given to lever *k*; this may be altered at pleasure by changing the position of the rod *r*. The self-acting progress of the table *l* is thus made to vary from $\frac{1}{16}$ to $\frac{1}{8}$ of an inch per revolution of the crank wheel *g*. The spiral spring *j* serves to pull up the rod after it has been forced down by the cam *d*, and bring back the pauls for a new stroke. The use of the other pauls is to prevent the ratchet wheel from running back while this latter operation is being performed. Another ingenious contrivance in this machine consists in a third or circular motion of the plate *l*, which turns on its centre, and having its circumference equally divided by the notches *m*, may be moved round an equal angular distance at a time, and by the spring or catch *n* is retained in a fixed position while the tool is at work. This arrangement is found to be very advantageous in cutting the key grooves of wheels, being required to be perfectly equidistant.

The riggers *f* are worked by a strap from the main shaft, and next to them is hung a fly wheel *e*, to regulate the motion; these and the crank wheel *g* are securely keyed to the spindle *c*. The reciprocating motion is then carried to the slotting bar *b*, by a small connecting rod *h*; *i i*, are groove-pieces fixed to the frame in which this bar works; the length of the stroke may be altered by sliding the crank pin *s*, in a groove provided on the plate *g*, but its longest stroke cannot exceed 8 inches. The different parts of this machine are fitted to the frame or standard *a*, having on its two internal sides V's, in which the table frame slides, which may be either raised or lowered by the handle underneath. The screw handle *q* and also the handle on the ratchet wheel, serve to bring back the tables after they have been advanced by the self-acting apparatus. The action of this machine is similar to that of the morticing machine at Portsmouth.

PLATE XXXVI.

Machine for cutting Key Grooves in Wheels, by Messrs. Nasmyth, Gaskell, and Co.

Another description of a slotting machine is represented by this Plate, differing merely in detail from that last described, which is limited to the nature of the work it is capable of performing by the two sides of the standard frame preventing the admission of large wheels. In the present machine, a wheel of any size can be grooved, the whole of the machinery being underneath instead of above the table.

The four small columns *a*, support the table *b*, on which there is a bed for the slide to work in; on this, the dividing plate *m* is placed, having its circumference notched in a similar way to the machine described in Plate XXXV. The spindle *e*, is driven by the rigger *c*, and has a fly wheel *d*, to regulate its motion; on the end of this same spindle is a pinion, working in the crank spur wheel *f*, connecting the motion to the slotting bar by the rod *h*; the position of the crank pin *g*, may be altered to any convenient length of stroke by sliding it along the groove shewn on the face of the spur wheel *f*. Thus a reciprocating motion is given to the bar *i*, having on its upper end the tool for cutting the grooves, represented by the dotted lines on the large spur wheel being cut. This machine is made self-acting by an eccentric or cam on the spindle *f*, which raises and lowers the small levers by means of upright rods working the ratchet wheels *j*, and pauls *k*; the ratchet wheels are fixed to long square threaded screws (the length of the bed), and work through a nut fixed to the underside of the sliding table, which may be inclined or placed at an angle suitable to the taper required for the key. The handle *l* prevents the circular dividing plate *m* from changing its position while the groove is being cut, after which it is disengaged from the notch by hand, till it meets a second one, equidistantly divided.

Between the columns the diagonal stays are placed, to give strength and stability to the machine, there is also a cross plate running from one side frame to the other, through which an aperture is made to allow the slotting bar *i* to work through.

The tool on the top of the slotting bar can be altered at pleasure, by unscrewing the small screw which secures it in its place.

It is from the advantages derived by the principle of this machine, which admits wheels of unlimited diameters, from its underneath motion, that Messrs. Nasmyth, Gaskell, and Co. have founded their patent.

PLATE XXXVII.

LARGE SLOTTING MACHINE, BY MESSRS. NASMYTH, GASKELL, AND CO.

Among the variety of slotting and key-grooving machines already described, there has been none similar to this, either in the general arrangement of its parts, or its capabilities as to the magnitude of the work it is capable of slotting; it is altogether a much larger machine, and of simple contrivance. A crank is shewn in the drawing, having its sides pared off by the tool; however, it is not solely confined to cranks, but may be used for any machinery that can be placed within the limits prescribed by the two columns.

The arrangement of this machine consists of a rectangular frame *a*, Figs. 1, 2, and 4, upon which are placed the plummer blocks for carrying the shaft bearings; the two columns *a' a'* resting on the top of the bed *b* support this upper entablature frame *a*. Like most slotting machines, the plate for carrying the work has three different motions, the two first at right angles the one to the other, while the third consists of a circular motion, which is required when the work to be operated upon is circular. The first is longitudinally along the bed *b*; this is done by working the ratchet wheel and screw *e'*, thus drawing the slide *e*; the second at right angles by the ratchet wheel and screw *d'* working the slide *d* in a transverse direction; and lastly the third, which consists of a circular plate having on its circumference the worm wheel *c*, made to revolve on its centre by working the worm or endless screw and the ratchet wheel *c'*. The screws *e'* and *d'* severally work in nuts fixed to the under side of the slides, one of which may be seen, by the dotted lines shewn by Fig. 3 in the plan of the bed-plate. Fig. 4 shews a plan of the driving gear for working the tool or slotting bar *k*; the motion being conveyed to the riggers or pulleys *i* from the driving shaft, is communicated to another set of pulleys upon which is a spur wheel and pinion, conveying it through an intermediate shaft to the large spur wheel *h*, to which is fixed the connecting rod *j*; by this connection the wheel *h* answers both this purpose and that of the crank, for conveying the alternate or up and down motion to the slotting bar *k*. By different arrangements that may be given to the wheels and driving riggers or pulleys *i*, independent of the double motion, similar to that described by Plate XXXIII. and XXXIII. A., a great variety of speeds may be obtained, which is of importance, some parts of machinery requiring a much greater velocity than others, while being operated upon.

The hollow bar *k* is guided at one end by passing through the upper part of the frame *a*, which is all one piece; this may be seen in Figs. 1 and 4; and the length of the stroke may be altered to suit the work by changing the position of the connecting rod *j* on the slotting bar *k*, sliding it in a groove provided for that purpose.

The three different motions already described are rendered self-acting by a pin on the spur wheel *h* striking as it revolves against the lever shewn by the dotted lines in Fig. 1, which communicate with the levers, rods, and small bevel wheels *f*, and ultimately give a self-acting motion to the different slides. Handles are placed on the three ratchet wheels, for working them by hand.

Fig. 2 shews the tool *l* in the act of paring the circular part of a crank, for which the third motion is put on, and the other two are consequently thrown off.

The frame is strongly secured to the stone foundations Figs. 1 and 2, by holding-down bolts.

PLATE XXXVII. A.

SLOTTING AND PARING MACHINE, BY MR. F. LEWIS, MANCHESTER.

The machine represented by Figs. 1 and 2 of this Plate, though different in detail from those preceding it, is the same in principle; this necessarily must be the case, where either the operation of slotting, paring, or key-grooving is to be performed, which severally require an alternate or reciprocating motion, whereby the introduction of the crank or dog wheel is indispensable.

The drawing shewn by this Plate, is a slotting machine of great power, by which work of considerable size may be operated upon, the table for carrying the work being about 3 feet in diameter, while the stroke, which may be varied, can be extended to 2 feet 6 inches.

In the side elevation will be seen the frame *a*, to which is connected the whole of the machinery for working the different parts, according to the nature of the work required. On the front of this frame are two faces, Fig. 2, to which are fixed the V pieces, two of which can be adjusted by the set screws, according to the wear of the slide *g*. The speed may be altered to suit the proper velocity for the work, by changing the strap on the riggers *b*, these receiving the motion from the main shaft; the fly wheel regulating that motion, without which it would be liable to jerks, and much variation. By the wheel and pinion *d*, the speed of the spindle on which is placed the crank or dog wheel *e*, is much reduced; the short rod *f* connects the up and down motion to the slide *g*, for carrying the tool *i*; and by loosening the screws *h h*, any tool can be applied. The spindles for carrying both the wheel and pinion *d d* run freely in bearings fixed to the frame *a*. On the top of the bed *j*, the sliding table *k* is moveable, and on its upper surface, the circular table *l* slides in a contrary direction, this latter table has also a revolving motion on its centre, by working the worm on the circumference of the worm wheel, which is done by turning the handle shewn for that purpose. A self acting motion is given to this machine, by a pin on the rim of the upper spur wheel *d* striking as it revolves against the lever seen in Fig. 2. by the dotted lines. On the lower end of this lever, is a paul for advancing the ratchet wheel, the boss of which is a brass nut, which, as it turns, screws up the square-threaded screw, and draws with it the table *k* on the adjustable bed *j*. At a convenient distance on this lever is placed a small rod, connected at the bottom with a series of levers, which being put in motion, work the slide frame, and give also the circular motion connected to it by the two small wheels and paul, thus making the three motions connected with the sliding frames for carrying the work entirely self acting, which is also the case in those machines already described by the preceding Plates.

That part shewn as circular on the main frame *a*, is an aperture through which the spindle *e* is passed.

PLATE XXXVIII.

Screw-Cutting Machine, by Mr. Fox.

The machinery required for this purpose consists of a bed *a* supported by two end frames; upon these are fixed the requisite bearings or plummer blocks, *b* and *d*, for carrying the two spindles *c* and *e*,—the latter being driven by the riggers *i*, which, by the small double conical friction clutch *f*, are made to work or revolve in opposite directions, thus keeping the machine in constant operation. By means of the spur-wheel and long pinion *h*, the motion is conveyed to the upper or working spindle of the chuck or box *m*, for holding the screw or bolt to be cut, while passing through the dies of the outer frame, (which are tightened by the handle and screw *l* pressing against the two springs *j*,) thus working itself through the dies, and drawing with it the spindle *c* and upper wheel *h*, the length required. On the box *m*, Fig. 2, is shewn a screw and handle for the purpose of tightening or slackening the screw before and after the operation of cutting. The machine is put in motion by turning the small handle and eccentric *g*, on the top and bottom of the upright rods, which press the friction clutch against either the one or the other of the two riggers, which, from running loose on the spindle *e*, can be made to revolve in either direction.

When it is required to tap nuts, they are placed in the outer head, and secured by the wheel *l*, which in the operation described above contained the dies, the tap is then placed in the box *m*, and as it revolves cuts the proper thread in the nut.

This machine may also be used as an horizontal drilling machine, the work to be drilled being secured in the head, Fig. 3, and the drill in the box *m*, the screw above the riggers being then tightened, presses forward the drill, which as it advances drills the required hole.

The spindle which carries the box *m*, might be made to slide independently of the wheel and pinion *h h*, simply by having a longitudinal groove or keyway cut on its circumference.

PLATE XXXVIII. a.

Large Screw-Cutting Machine, by Messrs. Nasmyth, Gaskell, and Co.

This machine is a contrivance for the purpose of cutting screws of large diameters, and consists merely of two side frames or standards for carrying the several parts of the required machinery. Unlike most machines, this has neither bed nor table, the frames being connected together by the two strong wrought iron bolts *dd*, (which also answer the purpose of guides,) and the two stretching bolts for steadying the lower part of the standards, the upper part of the standards *aa* having bearings in which the driving shaft works by means of the pulleys *b*. This machine has a double motion, similar to that described in the drilling and boring machine, Plates XXXIII. and XXXIII. A., which, as it is there shewn, is obtained by two pairs of spur wheels and pinions *c*. In this case, when the quick speed is required, the lower shaft, pinion and wheel is slid through its bearings in a similar way to the second motion of a crane or crab, a projection being given to the end of the shaft for that purpose; and when the slow motion is used, the lower shaft is again brought into the position shewn by Fig. 1, the wheel coming against the collar on the upper pinion, and the collar of the lower pinion against the upper wheel; these collars prevent them from going any further, while the moveable stop, Fig. 4, keeps the lower spindle in its proper position.

The chuck *f*, Figs. 1 and 2, forms part of the shaft for carrying the pulleys; and in it are fixed the dies for cutting the thread on the screw, which may be taken out at pleasure, and others put in their place, by loosening the set screws *g*; a second chuck or frame, Fig. 5, (for supporting the screw, at its head, between the dies,) slides along the guide bolts *dd*, as the thread is being cut on the screw; when this operation is finished, the round headed screw, Fig. 5, is slackened, and consequently the screw is liberated and another put in its place.

In cases where nuts are to be tapped it is only necessary to reverse the operation by putting the nut in the chuck *f*, and the tap in that marked *e*, where they are secured as already stated in reference to the screw.

The arrangement of this machine is extremely simple in construction, and it is capable of cutting the threads of screws of considerable diameters.

PLATE XXXVIII. B.

BOLT-SCREWING MACHINE, BY MESSRS. BENJ. HICK AND SON, BOLTON.

The object of this machine is the same as that described by Plate XXXVIII. A., and its arrangement and principle are in every respect similar, with one exception; instead of a double motion, as is the case in that above referred to, this has a backward and forward motion given to the hollow shaft, which is thus effected:—there are three driving pulleys, *c*, *e*, *d*; when the strap runs on *c*, the machine is put in motion by the spur-wheel and pinion *c′*, thus communicating it to the chuck *f*, for holding the steel dies or cutters *g*; as soon as the whole length of the screw has been cut, the machine is required to be reversed, which may be done by running the strap from the pulley *c* on to that of *d*, whereby the internal wheel *d′* is set in motion by a pinion on the cylinder, revolving freely on to the lower driving spindle or shaft, thus drawing the screw back to its former position; and lastly, when the machine is at rest, the strap is passed on to the circumference of the centre pulley *e*, which, being loose on the spindle, revolves without producing any effect on the working parts of the machine. The upper bolts *b*, for steadying the two frames, form also guides for the sliding frame. These and the other parts, as already said, are the same as the machine last described, which renders all further description useless.

PLATE XXXIX.

Self-acting Nut-cutting Machine, by Messrs. Nasmyth, Gaskell, and Co.

The machine shewn by this Plate is supported on a frame similar to that described by Plate XXXVI., where the table or bed, *f*, is fixed to the four columns, which are much strengthened by the diagonal crosses.

The spindle driven by the riggers or pulleys *e* from the main shaft runs freely in brasses fitted in the heads *d d*; on one end of this spindle the steel cutter *a* is fixed; the slide *f′*, on which is placed the dividing plate or block, is advanced between the two V's by a screw, having at one of its extremities a ratchet wheel *i*, worked from the main spindle by means of a small strap, the two small bevel wheels conveying the motion at right angles, thus advancing the ratchet wheel, which, as it turns, works the screw through a nut fixed to the under side of the slide *f′*, thereby making it self-acting; *j* is a small handle for the purpose of bringing back the slide by hand, after it has performed its work. By this machine a very great saving of time is effected in planing or siding the faces of nuts, with the utmost accuracy, and with an almost incredible saving of time, for in ordinary cases the work performed by this machine was entirely done by chipping and filing. The mode of working it is simply by fixing the nut as it comes from the forge on a mandril, and the latter in a hole on the block *b*, where it is securely fastened by a small nut above, it is then advanced to the cutter by the slide *f′*, where it receives a perfectly smooth face; the block is then disengaged from the handle *c* in the notch *g*, and made to revolve till it comes to the following notch, where it is again secured. The block is divided into six and eight equidistant portions, suitable to either square or six-sided nuts.

The small tank *l* contains water, kept constantly falling on the cutter *a*, for the purpose of keeping it cool.

The spindle upon which the nuts are to be cut may be suited to any size, as shewn by the spindle *m*.

PLATE XL.

Machine for Cutting the Teeth of Wheels, by Mr. F. Lewis, Manchester.

By this machine, wheels of the following description may be cut: 1st, common spur-wheels; 2ndly, conical or bevel wheels; and 3dly, worm-wheels.

a a represent the two side frames, the one of a V shape for the slide *g* to work on, while the other is a flat smooth surface. The machinery supported by these frames may be divided under two different heads, viz.: 1st, that required for giving the revolving motion to the wheel to be cut; and 2ndly, that which is requisite for working the cutter in the various positions it assumes.

1st. Machinery for turning the wheel. The spindle *b* has on one extremity the handle *b′*, which works round the circumference or rim of a plain wheel, shewn by the dotted lines; this wheel or rim has on its circumference two notches, equal to the width of the handle *b′*, in one of which it falls, and is there kept fixed during the operation of cutting one tooth. On the opposite end of this spindle is a small spur change-wheel *c*, working through a second or intermediate one, that fixed to the worm spindle, the worm *d* then conveys the motion through the worm-wheel *e* to the upright spindle *f*, on the upper end of which is placed the wheel to be cut. This spindle revolves in brass bearings fitted to the end frame.

The change-wheels and pinions *c c c* may be altered to regulate the speed and consequently number of teeth and pitch given to any required wheel, this, as will be seen, is very easily done by unscrewing the screws which connect the bearing pieces of the two pinions and intermediate wheel to the side frame, when wheels of different diameters can be put in their places according to the required motion. This at once regulates the distances of the bearing pieces, after which they must be well secured to the frame by again tightening the nuts.

2dly. Machinery for working the cutter *m*, consists of a cross slide *g* travelling on the top of the frames *a*, as already described; this is moved backwards and forwards by a screw working in a nut, and a small handle, which is not seen in the drawing; on this slide the frame with the two headstocks *h h* is securely screwed by a strong bolt and nut; *i′ i′* are the centres by which the frame *i i* is fixed to that last described, having also two heads similar to those marked *h h*; on the back of this frame *i i*, is a bolt and nut which becomes a centre for the sliding frame *k*, and by which it is fixed to it. The following motions are the result of these different combinations: 1st. The frame *g* is advanced by the screw. 2dly. The two heads forming the headstock frame *h* can be moved horizontally on the frame *g* either to the right or left on its centre. 3dly. In the same way the frame *i i* can be made to hang forward or fall back on its two centres *i′ i′*. And 4thly. That marked *j* assumes a position either to the right or left on its centre, while the slide *k*, which in fact carries the cutter and its necessary connections, is made to rise and fall by a small pinion working in the rack *k′* and receiving its motion by the handle *k′′*, there are two V pieces on the frame *j*, in which the slide works up and down; these are fixed by four screws. The pulley *l* receives its motion from the main shaft by a band, and conveys it to the cutter by two pair of wheels and pinions, by which the speed of the cutter *m* is regulated; on the slide *k* is fixed the bearing pieces for the three spindles; *o* and *p* represent an adjusting screw and index for regulating to very great accuracy the position of the cutter *m*, which may easily be taken out and replaced by another by unscrewing the small screw *n*.

The frame *a a* is firmly stayed by bolts and nuts shewn on its sides.

It must not be forgotten that when the headstock frames *h* and *j* have assumed their new positions described by the second and fourth motions, there is fixed on both the sides of the screws or bolts upon which they turn, two smaller bolts working in slots or grooves, by which they are firmly bolted, otherwise the frames might slip, and this would cause great damage to the work being operated upon.

Any one of the different positions given to these frames may be obtained independently of the others, or they might be used all together.

The following description will shew the mode by which this machine is worked for the different kinds of wheels:—

In its present position it is regulated for cutting the teeth of a common spur wheel, which is securely fixed to the top of the upright spindle *f* and made to revolve as already described by the handle *b′*, which works from notch to notch for every tooth, the change-wheels *c c c* being regulated to the required number; by this operation of the handle, the distance moved by the wheel to be cut is always the same, thereby ensuring the utmost accuracy. The whole of the machinery resting on the frame *g* is advanced till the cutter comes close to the wheel; the sliding frame *k*, to which is connected the cutter, (which is now put in motion by the pulley and small spur wheels,) is lowered by the handle working the rack and pinion *k′* till it comes in contact with the tooth to be cut; after the performance of this operation the frame *g* is drawn back by a screw and handle already described, when the wheel is again made to revolve the distance of one tooth, and the operation of the cutter repeated.

In the case of a bevel-wheel, the working of the machine is similar, the only difference being the position assumed by the cutter-frame, which is made to incline forward at any angle suited to the bevel by the third motion described; this is again altered when it is required to cut the teeth of a worm-wheel, similar to that shewn in this Plate by *e*, where the teeth are seen at the angle of the worm; for this purpose the machine assumes the position described by the fourth motion; and lastly, when used to cut the teeth of skew bevel-wheels by the second and third motions combined.

It is unnecessary to describe the utility of this machine, or its accuracy, when small wheels with an extremely fine pitch are required. One has now been in use for some length of time at the Bank of England, where it is found to be very useful in cutting the teeth of the small wheels required in that complicated and ingenious machine for marking the numbers on the bank notes, the numbers changing as fast as a man can feed the machine.

The following Plate shews a larger machine of the same description, which may be better understood, and has the same letters of reference.

PLATE XL. A.

MACHINE FOR CUTTING THE TEETH OF LARGE METAL WHEELS,

BY MR. F. LEWIS, MANCHESTER.

This Plate represents two geometrical views of the last described machine, by which the different motions will be understood with greater facility. It is in every respect similar as to its working parts, with an additional self-acting motion for working the cutter, which operation in the last case was performed by hand. It is also adapted for cutting the teeth of much larger wheels from the length of the bed or frame *a*, upon which the cutter frames slide. The letters of reference and description are the same as those described in Plate XL.; consequently a repetition will be useless, it will only be necessary to describe its additions, consisting of the self-acting motion obtained by the bevel wheels *s* communicating the motion of the driving pulley *l* to the upright spindle *t′*, on the top of which is the spur wheel and pinion working the worm *u*, and consequently the worm wheel *u′* fixed on the same spindle as the handle *k″* and pinion *k′*, thus raising the cutter frame *k*, on the back of which is fixed the rack. The worm wheel *u′* is seen in Fig. 2 by a dotted line behind the spindle *t′*. The handle *k″* serves to raise the frame *k* by hand after the tooth is cut by the self-acting motion, which is found to be a great improvement in the capabilities of this machine. The two side frames *aa* are connected together by the stretching bars *v* provided for that purpose.

From the great diameters of the wheels this machine is capable of carrying, which are fixed to the face plate *o* by the bolts and nuts *o′o′*, it is evident that were it not for the adjusting screw or stay *p*, a considerable degree of motion would be felt at that part of the circumference where the tool is operating, and great inaccuracy would be the result; by the stay *p*, this evil effect is entirely obviated, as it may be adjusted to any required height.

Wheels of the following description and sizes may be considered within the limits of this machine: spur, bevel, worm, and skew bevel wheels, five feet diameter in iron, and 10 feet in wood, having a breadth of 14 inches with any pitch or number of teeth.

PLATE XLI.

MACHINE FOR CUTTING THE TEETH OF WOODEN WHEELS MODELS OR PATTERNS, AND ALSO THOSE OF IRON, BY MESSRS. NASMYTH, GASKELL, AND CO.

This machine is beautifully drawn in perspective with the usual felicity of Mr. Nasmyth, and intended for the same purpose as those described by Plates XL. and XL. A. Instead of the machinery being fixed to a table, it is here made to slide on the bed *b*, supported at both ends by the standards *a a*.

It is driven by the leather strap *c* on the rigger, which receives its motion from the main shaft. The bevel mitre wheels *d* then convey it at right angles to the large band pulley *f*, whence it is carried to the small pulley fixed on the cutter spindle *j*, adjustable by the set screws *j'* both at top and bottom. The spindle *e*, and also the driving spindle upon which the rigger is fixed, are fitted to carriages bolted to the bed *b*. The small pulley *l* is for the purpose of keeping the band *f* tight, as the position of the cutter frame alters; it consists of a weight hung over a tightening pulley, the weight *l* falling as the band *f* slackens; the small column for carrying the pulley being fixed on the slide *g*, which can be moved the whole length of the bed *b*; on this slide is shewn a circular plate, having a centre on which the upright frame *h* is made to turn. By means of the wheel and handle *k*, the cutter frame *i* can be raised or lowered in the frame last described, by loosening the two nuts shewn on the back. The strength of the frame *h* is much increased by the rib shewn in the drawing.

It is obvious from the foregoing description that by this arrangement the following motions are obtained: 1st, a longitudinal motion along the bed; 2dly, a circular motion; 3dly, a transverse motion, which is required for cutting the whole width of the tooth, the frame being worked by the screw *h'* and handle *h''*; and lastly, the motion necessary to adjust the position of the cutter to the centre of the wheels whose diameters vary; this is performed by the handle *k*.

The dotted line shewn in the drawing represents a very large spur wheel pattern of mahogany being cut; it is securely fixed to the chuck plate *m* on the shaft *o*, which is made to revolve in the two heads by a worm wheel on the same shaft, worked by the worm *n* and handle for that purpose.

The mode of working this machine is almost similar to those last described; the wheel being accurately clucked, the cutter frame is advanced along the bed till it approaches the wheel, when the handle and screw *h′* being turned, will bring the cutter to the surface of the teeth. This done, the handle and screw *h′* are moved in a contrary direction, till the wheel, being cut, revolves the distance of one tooth, by the small handle and worm *n*.

The frame *p* in the adjoining figure may be put in the place of that shewn by *i*, when the machine is required to cut the teeth of iron wheels, in which case a quicker velocity is required, as will be seen by the band wheel and also the spur wheels and pinions; the spindle for connecting these are all adjusted by small set screws.

PLATE XLI. A.

MACHINE FOR CUTTING THE TEETH OF WHEELS, BY M. M. GLAVET.

In the year 1839 a patent was taken out by the inventors of this machine, for the purpose of cutting the teeth of iron and wooden wheels, either spur or bevel, by mechanical means. It is, as will be seen by the Plate, altogether on a different principle from those described, which are certainly very superior in capabilities.

In the arrangement of the plan and section, Figs. 1 and 2, the cast iron table *a* is supposed to be supported on suitable standards for that purpose. On this table or bed are fixed two plates *bb*, with bearings for the spindles or cylindrical bars *cc*; a cap is fitted to these bearings and secured to each of them by two screws. The two cylindrical bars are united together at their extremities by the two connecting links *dd*; these bars are made square in the middle part, and the two V pieces *ii* are fixed to this square part of the bars by screws, in which the rest *h* for carrying the tool *n* is made to slide backwards and forwards; it is adjustable by screws. On the square part of the smaller spindle is fitted a frame in which a curve *k* is dropped, while the steel spring *l* presses against a pin, and consequently the curve *k* against a screw *j*, directing the tool to the required shape of the tooth. This is done by the bar *h*, Fig. 2, on one extremity of which the tool is fixed, and the screw *j* at the other. By a contrivance in this machine, the advance of the tool is thus made self-acting; two stops *pp* are adjustable in the mortices Figs. 1, 2, and 3, and are regulated according to the advance required, while the long square-threaded screw *m* works in a boss on the frame in which the curve *k* is situate, its other extremity being fixed to the frame *o'* for carrying the ratchet wheel *o*, prevented from turning either one way or the other by the springs or pauls *qq*.

This machine is thus put in motion; the wheel to be cut being fixed on its proper centre, which in this case differs from those already described, from its being secured to a part of the machine itself. An alternate motion is conveyed either by a crank or other suitable means to the rods *f*, and consequently the two cylindrical bars slide in their bearings *b b b b*, drawing along with them the cutter on the surface of the tooth, the form of which is regulated, as before mentioned, by the curve *k*; with these parts the screw *m* to which it is attached, and also the ratchet frame and ratchet wheel are moved; till the lower part of the frame *o'*, Fig. 3, comes in contact with the stop *p*, against which it strikes, thereby throwing it from its vertical position to that of an angle, carrying with it the ratchet wheel to which it is fixed, thus working the square-threaded screw and ultimately advancing the tool *n*; the opposite stop *p* is so regulated as to bring it to its original vertical position when in its turn it strikes against it. By the handle *m'* the tool is brought back by hand.

Figs. 5 and 6 shew the adaptation of this machine for the purpose of cutting the teeth of conical or bevel wheels; consisting of an adjustable or sliding centre pin in a long mortice fixed to the bed.

From the above description, it is doubtful whether this machine has ever been put into practice or not, owing to its evident imperfection in producing any thing like accuracy of workmanship.

PLATE XLII.

Vertical Boring Machine, by Messrs. Nasmyth, Gaskell, and Co.

The many advantages derived by this arrangement of a vertical boring machine over those where the work is placed in an horizontal position, may perhaps be unknown to persons unacquainted with the general character of machinery; it may not be unadvisable to point out a few of the principal features shewing the superiority of this machine.

In the first place, the arrangements of its parts; the manner in which the cylinder is placed, namely, its vertical position, thereby doing away entirely with all the injurious effects produced by the weight of the body being planed or bored; thus obviating all tendency to distort its figure, which is the case where the operation is performed by the horizontal system, and where the sides are bulged out from the weight of the upper part; this may be better understood by forming a cylinder of thin paper, which will be found to widen in the middle and assume an oval form from its own weight.

This alteration of form is found to be quite sensible when the cylinders are of large diameters. Another great advantage of this system of vertical boring, is avoiding all risk of flexure in the boring bar, upon which the cutter wheel or head is fixed for carrying the boring tools; this bar has a tendency to bend down in the centre to a curve, instead of keeping a perfectly straight line, transferring the figure assumed by the bar to the surface of the cylinder; but this will much depend both on its length and diameter.

Another advantage of this machine is, that the cutters are kept clear of the borings, which fall to the bottom of the cylinder as fast as they are cut. By this superior arrangement all these objections are entirely removed, thus avoiding all the tendency gravity has in altering the trueness of the cylinder or the bar; added to these, the power requisite to bore the cylinder is found to be much less than in those placed horizontally, a very desirable object in a large establishment.

A short description of its several parts will enable the reader more fully to understand the advantages already alluded to.

Fig. 1 represents a cross section of this machine, and Fig. 4 a plan shewing its position in a corner of the building where it is placed. In these two views it will be seen that the driving part of the machinery is situated below the ground line on suitable strong foundations, in which it is inclosed. These parts are rendered accessible by the steps *t*, which are found to be necessary in cases where the machinery is likely to get out of order, a precaution never to be neglected.

The two riggers *kk* receive their motion from the main shaft by means of a leather strap, one of these runs loose on the shaft, and the strap is thrown on it when the machine is not at work; this is done at pleasure with the greatest possible facility; by a bevel wheel and pinion *j* it is then conveyed through the shaft *i* to the endless worm *n*, working in a large worm wheel *o*, which is fixed on the great vertical boring bar *a*, whereby a very easy motion is obtained, and all jerks avoided. It will be seen by the series of wheels in Fig. 4, how much the speed of the boring bar is reduced. The shaft *i* is placed at an angle, and works in a bearing or plummer block and a step *h*, both of these being made of brass.

The vertical bar is made in two parts *a* and *c*, the upper one *a* for carrying the cutter head or boring wheel *r*, while to the lower one is connected the driving apparatus; they are coupled together by the upper one resting, as is shewn in Fig. 3, in a socket on the top of the lower one; a steel key *l* is then driven in, which entirely prevents it from turning; the toe of the bar *c* rests in a step or socket shewn by Fig. 5; the entire weight of this bar and its appendages is thrown on the hardened cast steel disks *s*, which are constantly kept supplied with oil. Both extremities of the bar *c* are rendered adjustable to the greatest possible accuracy by means of the small set screws *qq*, Figs. 3 and 5, which, by being tightened, press against the conical brass segments, the upper one forming part of the great base or floor plate *b*, which is materially strengthened by six strong ribs on its under side. The cross beam *g* is well fitted to the sockets *f*, built into the wall of the building, where they are bolted by strong bolts, Figs. 1, 2, and 4. It has an additional stay in the bolt *u*.

There are four standards or supports, *dd*, Fig. 1, for carrying the cylinder to be bored, which can be altered to any convenient position by unscrewing the bolts which fix them to the base plate. After the cylinder has been properly placed in its right position, it is fixed to these supports by clamps *e* and bolts; and thus rendered quite immoveable.

In the boring bar *a* is a deep socket *m*, Fig. 1, which allows the bar to slide up and down by means of the screw *p* and the nut *l*; upon the lower side of this socket is a flange *m*, upon which the cutter head or wheel *r* rests, receiving its motion from the bar by means of a nut, answering both the purpose of nut and key. By the different arrangements of the sun and planet motion of the wheels on the upper part of the bar, any degree of motion can be given to the screw for the descent of the cutter wheel. After the cylinder has been once bored through, the cutter wheel is raised by means of a small crane, and the chains, Fig. 2, and by the peculiar arrangement of the nut *l* in the socket *m*, the cutter wheel can be drawn up the cylinder without turning the screw *p*, as it leaves the nut behind, which is afterwards screwed up, there being no other weight to raise but that of the nut. The cutters are then set afresh to the new or finishing cut, after which the cylinder may be considered perfectly true. The position occupied by the crane enables the cylinder to be placed, and the bar lifted in and out with most perfect ease; while the space occupied by this machine is very small compared with those where the work is performed horizontally; it is, however, important that the base plate *b* should be well secured to the foundations by strong bolts.

The speed of this machine may very easily be varied, by having different sized riggers or pulleys on the driving shaft which conveys the motion to the riggers *kk*, Figs. 1 and 4.

One of these machines may be seen in the erecting shops at her Majesty's Dock Yard, Woolwich.

PLATE XLIII.

GREAT BORING MACHINE, BY MESSRS. NASMYTH, GASKELL, AND CO.

The machine represented by this Plate is, with few exceptions, the same as that last described, where the cylinder to be bored is placed in a vertical position, whereby numerous advantages are derived, as already explained.

The motion is communicated by the driving pulley *c* to a bevel pinion working the bevel wheel *d*; the shaft on which this wheel is fixed, has on its opposite end a worm for communicating the motion through the worm wheel to the upright shaft *f* and boring bar *a*, having on its circumference the grooves *a′* in which the cutter head is moveable, sliding up and down according to the progress of the work; *k* is a tool carrier fixed to the cutter head. The foundation plate *h* forms a bearing for the upright shaft, the lower end of which rests in the step *g*, while the cylinder *l* is secured by the clamps *j j* to the supports *i i* fixed to the foundation plate. These parts are in every respect similar to the boring machine shewn by Plate XLII., by which they are more fully described.

Two strong piers of masonry *m′* support the entablature *m*, (for carrying the self-acting apparatus for raising and lowering the cutter head *b*,) to which it is bolted by strong holding-down bolts *m″*. This apparatus consists of a rack *n* worked by a pinion, the motion being transmitted from a trullion wheel through two spur wheels and pinions *o*; the whole of this upper machinery revolves with the boring bar, with the exception of the internal wheel or screwed hoop *p*, the consequence of which is the small trullion wheel is made to turn on its axis by the tread of the wheel *p* in which it works, and thereby ultimately raises the cutter head *b*, the two side slings connecting it to the upper frame *q′*, to which is fixed the rack *n*.

This machine is of the largest dimensions, and was made for the purpose of boring the large cylinders, 10 feet in diameter, for the Great Western Steam Navigation Company's vessel the Mammoth, now in progress at their works at Bristol.

PLATE XLIV.

VERTICAL BORING MACHINE, BY MESSRS. BENJAMIN HICK AND SON, BOLTON.

By this combination of three distinct machines, the following different operations may be performed, viz.: boring, drilling, and face-grinding; it is so contrived that the entablature *b*, supported by the four columns *a a a a*, carries the upper parts of the three different machines, consisting of the requisite driving machinery for communicating to them their respective motions.

That in the centre, A, is a vertical boring machine for boring cylinders of large diameters, which are fixed in the usual way on the six moveable supports *h* by the clamps *i*; in addition to which it is rendered perfectly steady by the circular frame or ring *j*, sliding up and down in grooves on the back of the two middle columns, the adjusting screws *j* being tightened when the cylinder is properly placed under the centre of the boring bar *c*, which receives its motion from the leather strap and pulleys *d*, whence it is conveyed through the bevel pinion and wheel *e* on the upright shaft *f*, upon which is also keyed the spur pinion for driving the wheel *g* fixed to the lower part of the boring bar *c*, and working in the step *m*; the rack and wheel *k* gives the cutter head the requisite feed while boring out the cylinder. The six supports *h* are made to slide in grooves on the foundation plate *n*, according to the different diameters of the cylinders being operated upon; these, when properly placed, are bolted to the plate *n*.

The second machine, B, is a vertical drilling and boring machine for work of smaller dimensions than the machine A. It is shewn in the drawing boring out the centre of a crank *x*, fixed to the travelling table *p*, and slides on V's on the frame *o′*, which has also a motion at right angles, on the bed *o* fixed to the foundation plate. The drilling bar *u′* is lowered by the screw *u″* according to the feed, its motion being conveyed to it from the pulley *q* and strap *q′* to a spur wheel and pinion not shewn in the drawing, the pinion is on the same spindle as the pulley, and the wheel on that of the reversed cone *u*, by which it is carried to the square-threaded screw *u″* by two pairs of small bevel wheels *u*, fixed on an horizontal spindle and working in suitable bearings on the carriages *u‴ u‴*. The apparatus for raising the drilling bar is the same as that described by Plate XXXII., which consists simply of two small chains fixed to the bar *u′* and working round the pulleys, its opposite end being attached to a weight *v*.

The third machine, C, is for the purpose of grinding up the faces of rings for metallic pistons, conical valves, &c.; the travelling table *p* of this machine is in every respect similar to that of B, and upon it is placed the piston *y* to be ground; the upright rod, receiving its motion from the pulley and strap *q′*, is kept in a vertical position by the cross frame *s*, while the shaft *r* and grinding plate are connected to the lower end of the rod, and are occasionally raised for examining the surface being ground.

The different motions given to these machines are quite independent the one of the other, by which means any one of them can be worked separately. The whole is placed on a suitable strong foundation of stone. After the large cylinder is bored, it is raised from its position by a crane placed on the floor above.

PLATE XLV.

MACHINE FOR PLANING IRON, BY NICHOLAS FORQ, CLOCKMAKER.

This Plate represents an elevation of a planing machine, which was invented as early as the year 1751, for the purpose of planing the pump barrels used at the Marley Water Works near Paris. These pumps consisted of wrought iron segments bound together by strong hoops or straps; for this purpose M. Forq erected this machine near Mauberge in France.

It consisted of a small spur wheel *e* placed between two trundle wheels *h* and *i* in the same vertical plane. The lower trundle *i* was put in motion by manual labour being applied to the winch handle *g*; on the same spindle was fixed a fly wheel *c* to regulate the motion, while the upper trundle wheel *h* communicated it to a large spur wheel *b* above, one half of its circumference only being furnished with teeth. On the axis of this wheel *b* a wooden pulley *a* of a very large diameter was fixed, over its periphery an endless band or cord *f* was made to pass, and also over the smaller pulleys *k l m n o p*, to which was fixed the cutter or tool frame. This rabot or cutter was then drawn backwards and forwards, between two large parallel bars of square iron placed horizontally, for giving the alternate motion to the tool, which formed a kind of cross, the cutting surface being suited to the circular figure of the interior of the barrel, and bored or planed it regularly.

The bearings of the spindles were fixed on cross timbers, in a similar way to the frame for carrying the work being planed.

The segments forming this barrel had been previously planed separately, by a similar process to that already described; its parts having been well forged to the required shape previous to their admittance in the planing machine, when all the joints were made perfect.

By this machine M. Forq seems to have prepared all the barrels he required, as he says, with the greatest accuracy, which varied in their diameters from 10 inches to 4 feet, and from 7 feet to 10 feet long. The inventor asserts that he had seen nine barrels 7 feet long, of which eight had a diameter of 10 inches, and the remaining one 15 inches, filled with water for three months, and were perfectly water-tight.

A pump barrel, 10 inches diameter and 7 feet long, consisted of nine staves or segments, held together by 12 wrought iron hoops three inches wide, the extremities terminating with collars two inches broad. The hoops were half an inch thickness of metal.

This apparatus is evidently incomplete for want of the attachment of the planing tool, and the reciprocatory motion, which is not shewn in the original drawing.

PLATE XLVI.—Fig. 1.

Millwright's Planing Machine, by Messrs. Nasmyth, Gaskell, and Co.

This Plate represents two different modes of planing. Fig. 1 shews the tool moveable; Fig. 2, the tool fixed. By the first arrangement many advantages are derived, such as planing the parts of very large and heavy objects, whose great weight and size would otherwise render them quite inadmissible in the machines of ordinary construction, where the work has to be fixed on a sliding bed and moved along the machine with it. It is evident by this contrivance that a very great saving of power is effected, since the only moving part of this machine consists of a small traversing frame for carrying the slide and tool.

By referring to Fig. 1, it will be seen that this machine rests in a slip or pit made for the purpose, with steps to descend into it. The two side frames *a* are bolted to the ground, and stretching bolts connect them above. There are also two cross carriages for carrying the shaft *b*, and a bed plate having a series of apertures through which any piece of work can be secured, they can be raised or lowered to any height by four side screws. The frame *c* for carrying the slide works backwards and forwards along the edges of the bed *a*, and by means of a screw and handle *d*, its position can easily be altered.

It is evident from the above description, that in cases, such as planing the key ways on large heavy shafts, this machine is found to be very useful and economical, for the surface to be planed is but very small compared with the length and bulk of the work which otherwise would be travelling along the bed, occasioning both waste of time and power. The length given to the cut can be altered at pleasure.

PLATE XLVI.—Fig. 2.

Planing Machine, by Messrs. Nasmyth, Gaskell, and Co.

The perspective drawing shewn by Fig. 2, represents a planing machine of ordinary construction, where the tool is fixed and the work moveable.

It consists simply of a bed *a* about nine feet three inches long, upon which the travelling table *b* works backwards and forwards. The wheels for working this table are so arranged as to bring it back after the work has been planed at a much greater speed, thereby saving time to a very great extent. The two side frames *cc* have on their faces the upright slots or grooves *d′ d′*, in which the cross frame *d* is raised and lowered at pleasure, to regulate its height according to the magnitude of the work being planed. This is easily performed by the handle wheel and large screw *h* working in a nut on its back. The sliding frame *e* for carrying the tool *g* is then brought to its proper situation by means of the screw and handle *f*, after which the machine is set to work by running the leather strap from a loose pulley or rigger to one fixed on the shaft.

The table *b* has the usual mortices, by which the work is clamped to it by strong bolts.

PLATE XLVII.

Planing Machine, by Messrs. Nasmyth, Gaskell, and Co.

The only difference between the principle upon which this machine works, and that last described by Fig. 2, Plate XLVI., is in the motion by which the sliding table is brought back after it has travelled the length of the bed. From its great similarity, it is termed the mangle motion, and by the great speed obtained, is found very useful.

The bed *a* rests on six columns, connected together by diagonal frames to increase its stability; and the sliding table *b* running backwards and forwards on this bed, has the usual mortices in it. The upright frames *k k* are fixed to the bed by bolts; in the two grooves of this frame, the cross frame *j* is made to slide up and down to suit the work to be planed, by means of the long screw and handle *l*, after which it is securely fixed to them. The sliding frame for carrying the tool can be placed in any situation along the frame *j*, by turning the handle and screw *h*, by which it may be brought in close contact with the work; it can also be placed at an angle by loosening the screws in the short mortices shewn on its face. The tool can be taken out and replaced by another, by two small adjusting screws, by which it is fixed.

The motion is communicated to the machine by a leather strap working on the riggers *c*. The spindle for carrying them being supported on the top of a small column by its side; on the opposite end of the spindle is the small pinion working alternately inside and outside of the trullions on the large wheel *d*, the distance travelled by the table being regulated by a stop on its periphery; this is effected by the spindle sliding backwards and forwards in the groove *e*, situated on the side of the machine. On the shaft of the trundle wheel, is a large wheel, round which an endless chain is placed, passing over two small pulleys *f'*, which pulleys are fixed to the frame by a screw; one of these only can be seen, the other being on the opposite end of the machine; the chain is fixed to both ends of the bed, and communicates the alternate motion to the sliding table; close to the groove bearing *e*, (which forms part of the diagonal frame,) is a small rod *g* connected to a spindle by a series of levers and bell cranks, by which the upright rod is made to rise and fall alternately, working the ratchet wheel fixed to the screw *h*, thereby giving a self-acting motion to the cutter along the frame *j*. This machine is capable of planing work about 8 feet long, by 3 feet 6 inches wide. The length being regulated by the distance travelled by the sliding table.

The frame for carrying all the machinery is well bolted to the floor.

PLATE XLVII. A.

PLANING MACHINE, BY MESSRS. NASMYTH, GASKELL, AND CO.

This description of planing machine is the same as that represented by Fig. 2, Plate XLVI., where the work to be planed is fixed to a travelling table *d*, moveable along the surface of a bed *a*, and made to travel both backwards and forwards by a self-acting apparatus, which is also connected with the tool *o*, the motion being in the first instance communicated to either one or the other of the two pulleys *e* or *f*; that marked *e* is fixed on a shaft *q'*, and conveys the motion through the wheels and pinions *e'* to a pinion working the rack *h*, screwed to the under side of the table *d*. This travelling table has a number of mortices, by means of which the machinery to be operated upon is clamped, while the bed *a* on which it slides has one of its sides of a V form, and the other a flat smooth surface, Fig. 2. The two upright side-frames *bb* are secured to the bed *a*, and are connected together at the top by a cross frame *c*; these two frames have long vertical mortices in which the sliding cross frame *l* is moveable according to the dimensions of the work placed on the table, and can be raised or lowered by the wheel and square-threaded screw *l'*; a second frame *m* is also moveable along the cross frame *l* by the ratchet wheel, square-threaded screw, and wheel *m'*, suitable to any required position, while a third motion is obtained by the slide *n* (for carrying the tool *o*) being moveable on the frame *m* by the wheel and screw *n'*. By the following arrangement the reversing motion of the table and that of the frame *m* and tool are made self-acting. On one side of the travelling table are two moveable studs *jj*, (regulated according to the distance the table is intended to travel,) which coming in contact with the lever *i''* strike against the upper part *j'*, and communicate the motion on one side to the rod and lever *k*, and also to the upright rod *k*, to which is fixed the moveable stud and paul *k'* for working the ratchet wheel *m'* and screw, as already described, the counterbalance weight *k* causing the paul to rise for a renewed stroke. Two guide pieces are screwed to the frame *b* for the upright rod *k* to work in. On the opposite side of the lever *i''* is a short connecting rod (for reversing the motion of the travelling table) fixed to the bell crank working the rods shewn by the dotted line in Fig. 3 under the table, which ultimately transmit the motion to the disengaging apparatus, consisting of an upright lever, counterbalance weight, and cross bar *i'*, with two short projecting bars for the leather strap to work between, and by which it is passed from the pulley *e* to that of *f*, whereby the motion is immediately reversed, the spur wheel and pinion *f'* causing the pinion to work the rack *h* in a contrary direction, there being three wheels in one instance and two in the other. Both the pulley *f* and pinion *f'* are fixed to a cylinder working freely on the spindle *q'*.

The travelling table is generally made to return at a much greater velocity than that required while the tool is acting, by which means considerable time is economized.

When the machine is at rest, the leather strap runs on the loose pulley *g* without producing any effect on the working parts of the machine, in which case the disengaging bar *i'* is situate in the position shewn by the plan Fig. 3.

A frame *p* and standard *q* for supporting the shafts *q'* and the several parts of the machine is fixed to the side of the bed *a*, and the tool *o* can be changed at pleasure by unscrewing the screws which hold it in its place.

PLATES XLVII. B. AND XLVII. C.

PLANING MACHINE, DRIVEN BY STEEL BELTS, BY BENJAMIN HICK AND SON, BOLTON.

These two Plates represent a planing machine of a different description from those generally used, and of unusual size, and owing to the peculiar motion given to the tool, it cuts both ways, on Whitworth's patent plan. It is shewn in Figs. 1 and 2, planing the faces of the ports of a cylinder for a steam engine.

The arrangements of its parts consists of two upright side frames *a a*, fixed below the surface of the ground, on which are placed the cross and longitudinal bearers *b b*, for carrying the work to be planed. On the top of the frames *a a* are bolted two long side frames *d d* supported also by the three standards *e e e*; the shape of these two frames *d d*, as will be seen by Fig. 2, is of the V form, and upon them the cross table *c* for carrying the tool *j* works; there are two bearings on this table for supporting the hollow cylindrical bar, upon which the frame *i'* is moveable, (by means of a screw passing through the hollow bar,) according to the position of the tool *j* on the face of the work being planed; a long key is provided on the cylindrical bar, to prevent the carriage *i'* from altering its position by turning on it.

On the front part of the frame *i'* is a circular box, which moves at each end of the cut; in it the small cylinder for carrying the tool revolves; this front part of the frame *i'* can be placed at any angle, by means of grooves on its faces, two small bolts and nuts on either side fixing it, when properly adjusted, while the tool is kept in contact with the face of the work by the handle and screw above it.

The working of this machine is as follows: the motion being communicated to the pulleys by the leather straps *ff*, the one for working the machine forward, and the other for working it in a contrary direction, convey it through the two spur wheels and pinions *gg*, to the pulleys *h*, fixed on two strong shafts extending across the extreme ends of the machine under the floor, one of these pulleys being situate at each corner of the machine, and round them the steel belts work, the ends of which are attached to the projecting levers *m* on the sides of the long cross bar *i*, and secured by tightening screws; the operation of reversing can be effected at any point by levers and a rod, extending the whole length of the bed, and has a moveable stop for that purpose.

The friction of these belts upon the pulleys, drives the machine, and is sufficient for the heaviest cuts, its only moveable parts being the carriage *c* and its appendages, while the object under operation of planing is a fixture, by which means the same amount of power is necessarily required for either a heavy or a light casting; the surface it is capable of planing being 30 feet long, and 9 feet 6 inches wide; when large surfaces such as lathe beds are to be planed, the platform *b* is raised to a level with the floor.

Note.—A similar tool was also contrived, some years ago, by the editor of this work.

PLATE XLVIII.

PUNCHING AND PLATE-CUTTING MACHINE, BY MESSRS. NASMYTH, GASKELL, AND CO.

The peculiarity of this machine is its adaptation to two different purposes, so closely connected to one another, that in the construction of boilers of any description, or in fact to any work, where wrought iron plates are used, either to be cut or punched, it is found very useful; by the arrangement of the parts of this machine, this is effected at the same time and by the same motion.

The large cast iron frame or standard *d*, carries the shaft *e*, on one end of which is fixed an eccentric, Fig. 1, for raising and lowering, or giving the alternate motion to the slide *f*; on the other extremity of this same shaft, is the large spur wheel *c*, put in motion by the pinion on the driving shaft on which is hung the fly wheel *b*, for steadying the motion, and also the two pulleys or riggers *aa*, the one keyed to it, while the other is allowed to run loose, when the machine is not working. The outer carriage *p* supports the bearing of this latter shaft.

On the upper part of the frame *d* is the steel cutter *g*, secured to it; a similar one is placed in the reverse position on the top of the sliding frame *f*, the lower part having fixed to it the punches *h* for punching the plates, which work freely in the dies *i* screwed to the frame *d*; there is a stop *j* on the under side of the jaws of the frame, for preventing the plate from rising, as it has a tendency to remain fixed to the punches, which it is obliged to quit, when it comes in contact with the stop.

From the above description, the mode of operation will be easily understood. The motion being given to the fixed pulley from the driving shaft of the building, is then communicated to the slide by the pinion and wheel, causing the eccentric shaft *e* to revolve, thereby giving the up and down motion to the punches, and also the cutters *gg* alternately, which in the side elevation, Fig. 2, is shewn cutting a plate. The slide *f* works in a V slide fixed by six screws, the steel cutter being also fixed by three of a similar size.

The punching operation performed by this machine is made self-acting, by the following arrangement. The plate to be punched is secured in the usual way by clamps to the travelling table *k*, its four wheels running on a railway of triangular bars, on the bed plate; two carriages *o* are fixed to the under side of the travelling table, for supporting the notched bar *n* bolted to them.

As the large spur wheel *c* revolves, the pin *m* comes in contact with the lever connected to the rod *l*, by which the bar *n*, and consequently the table, is advanced. This done, the table is again drawn back to its former position, by the chain shewn in Fig. 1, when the operation is repeated.

Messrs. Nasmyth, Gaskell, and Co. have made subsequent improvements in this machine, by altering the position of the apparatus for cutting the plates of boilers, to the opposite side of the punching apparatus, while the frame work is so adapted as to carry the driving parts in the centre, the same shaft working both machines.

PLATE XLIX.—FIGS. 1 AND 2.

PLATE-BENDING MACHINE, BY MESSRS. FAIRBAIRN AND CO.

The variety of forms given to different boilers, according to the nature of the steam for which they are intended, renders the machine shewn by

Figs. 1 and 2 of this Plate quite indispensable to a boiler manufacturer. It will easily be seen by examining these drawings, how a flat wrought iron plate may be brought to any required curve, by passing it between the rollers *b b*, which are regulated by the large adjusting screws *g g*.

The driving and loose riggers *a a* have the motion communicated to them by a leather strap, from a shaft worked by the engine connected to the establishment, the spur wheel *c* on an intermediate shaft, then conveys it by a pinion to the large wheel, on which is fixed one of the rollers *b*, the other being worked by the two pinions on the opposite side of the machine. The fly wheel *d* is placed on the driving or rigger shaft, and by its great weight gives a regular steady motion to the different working parts of the machine. The roller bearings are adjusted by the large square-threaded screws working in the two side frames *e* for supporting them. Much additional strength is given to this machine, by the stretching bolts *f f*, which bind it together.

By placing a handle on one of the arms of the fly wheel *d*, this machine might be put in motion by manual power.

PLATE XLIX.—Figs. 3 and 4.

Vice for cutting Boiler or other Wrought Iron Plates, by Messrs. Nasmyth, Gaskell, and Co., Manchester.

Figs. 3 and 4 shew a simple contrivance, by which wrought iron plate can be held secure in a frame, while the edges are cut with a chisel. The frame consists of the upper and lower parts of a vice *a* and *c*, the faces of which are hardened steel, between these a wrought iron plate *d* is placed, where it is securely clamped by tightening the two nuts on the large square-threaded screws *b*, keyed through the frame, which also connect the machine to the foundation of the building.

The chisel *e* for cutting this plate is shewn by Fig. 4, by which the work is performed in a very perfect way.

The lower vice frame *c* is much strengthened by the ribs shewn by Fig. 4, which give it a firm bearing on the ground.

PLATE L.—Figs. 1 and 2.

Punching Machine, by Messrs. Kinmond, Hutton, and Steel, Dundee.

These two figures represent an elevation and an end view of a machine for punching holes in wrought iron plates for boilers and other purposes.

The strong frame *d*, which carries the whole of the machinery, is made of cast iron, and is very firmly fixed to the floor of the building. The pulleys *a* receive the motion from a line of shafts; one of these runs loose, and the strap is thrown on it when the machine is at rest. The speed given to the punching spindle *g* is reduced by two pairs of spur wheels and pinions *c c*; upon the end of the spindle *g* is fixed the eccentric or cam, which, as it revolves, raises and lowers the slide *h* for carrying the punch *e*, which works in a die *f*, large enough in diameter to fit it, and through this die passes the circle of the plate cut out by the punch.

A very uniform motion is obtained by placing the heavy fly wheel *b* on the main driving shaft.

The different parts of this machine can, without difficulty, be taken out and replaced by other suitable to larger or smaller work.

PLATE L.—FIGS. 3 AND 4.

MESSRS. B. HICK AND SON'S MANDRIL FOR EXPANDING RINGS.

The mandril in a lathe, is that part upon which the work to be turned is placed or fixed, consequently different mandrils are required to suit the various kind or forms of machinery to be turned; by this contrivance of Mr. Hicks, rings of very different diameters may be turned; a few words will give an idea of the mode in which this is performed. The spindle or mandril *a* is of a cylindrical form, having a square-threaded screw at one end; on the larger or middle part of this mandril are four grooves, in which the conical pieces *c* are made to slide, and have on their circumference the ring *d* to be turned; a tightening cone *b* of a cylindrical shape is then placed on the screw, and by screwing the nut, presses it against the four pieces *c*, thus expanding them till the ring becomes quite securely fixed on their circumference.

By this means, rings of various diameters may be turned, within the limits intended by the mandril.

PLATES LI. AND LII.

MACHINE FOR PUNCHING BOILER PLATES, BY MESSRS. MAUDSLAY, SONS, AND FIELD.

Fig. 1 is an elevation, Fig. 2 a plan, and Fig. 3 a side view of a punching machine on a very improved principle, whereby the plates required for boilers and other purposes may be punched with the greatest possible accuracy, insuring at the same time very superior workmanship and great dispatch. It is usual in all ordinary punching machines, first of all to mark out the rivet holes in the plate, by a template, with white paint, and then to place it as near as the eye will permit under the punch; by the contrivance of this machine, this operation is entirely dispensed with, it being only necessary to fix the plate to a travelling table, and then to adjust the various parts of the machine to the proper distance required between the rivets.

The large cast iron frame *p* carries the several parts of the machinery, and also the two plummer blocks for supporting the bearings of the lying shaft *a* running the whole length of the building, for the purpose of working other machines; this frame is securely bolted to the wall, which, added to its own weight, gives it great stability.

On the shafts *a*, connected together by the two coupling boxes, is placed the crank *b*, the motion being communicated through the crank pin to a connecting rod *c*, to which is attached the upper lever *d*, being always at work, while the shaft *a* is revolving; the fulcrum of this lever is on the frame *p*. A lower lever *e* for raising and depressing the punching frame *f* has also its fulcrum on the same frame; this last lever working only when the punching operation is being performed. On the top of the frame *p* is a lever and long rods *e′* for engaging and disengaging the machinery. The side view represented by Fig. 3, shews the machine at work, and by drawing down the lever *e′*, connected by the rods to the counterbalance weight, which will allow it to remain steady in any position, they are disengaged from the pin on the lever *d*, which at once ceases to communicate the motion to the lower or punching lever *e*; thus it will easily be understood that those parts of the machine constantly at work, are the lying shaft *a*, the crank *b*, the connecting rod *c*, and the upper lever *d*.

The slide *f* for carrying the punches, works between V's, Fig. 2; one is fixed to the frame *p* by adjusting screws, both on its face and sides, and by the curious shape given to the end of the lever *e*, this slide is made to rise and fall, the counterbalance weight and lever *g* being connected by two short links; on the bottom of the slide is screwed a small frame for carrying the punches, which may be taken out and replaced by others. The dies *i* in which the punches work are placed in a frame bolted to the frame *p*, and by unscrewing the small adjusting screws, these may be taken out and replaced by others suitable to the different sized punches that may be required. On the under side of the frame *p* is screwed a small stop, by which the circle punched from the plate is forced out as the slide rises.

In the front of the machine is placed a long table, supported by the carriages *o*, consisting of two columns and diagonal frame, having bolted to them two long bars, upon which the moveable table is made to slide; this table, as will be seen by Fig. 2, has a number of holes by which the plate to be punched is secured by clamps; it is advanced by the rope or chain *n* passing under the pulley, Figs. 1 and 2, and over a second one hung from the ceiling of the building, to which is connected a weight sufficiently heavy to draw forward the table and plate fixed to it; after travelling the length of the table *o*, it may be brought back by turning the winch handle and spindle *l*, on which is a pinion, for working the rack, fixed to the under side of the moveable table *m*.

By a very ingenious contrivance, forming part of this machine, the rivet holes of boilers may be punched to very different pitches, a thing very much wanted in such cases as the repairing of old boilers, or replacing an old plate by a new one, where it is of the utmost importance to have the rivet holes coinciding with the greatest accuracy, which may be better understood by supposing that in the length of a plate, one, two, or three additional rivet holes may be required, which distance would have to be equally divided in the whole length of the plate. A description of this part of the machine will fully shew how this operation is performed. On the end of the punching lever *e* is screwed a small plate with a pin *j′*, adjustable by a screw working in the short mortice, Fig. 3; on the frame *p* is bolted a fulcrum piece for the lever and counterbalance weight *j*; to this lever a long rod is connected at its lower end, passing under the table *m*, and the lever *r* is fixed to it, as will be seen by the dotted lines in Fig. 2; it has also two stops or projecting pieces *r′ r′* on its surface. A second lever *s* has its fulcrum fixed to the table *o*, one end of this lever being connected to the lever *r*, while the opposite end has on its surface two small pins *s′ s′*; the fulcrum *s″* of this lever is adjustable by the two small screws shewn in the plan, Fig. 2. A long notched bar *k* is fixed to the under side of the table *m* by screws, and the bar *t* has its fulcrum *t′* also fixed to it; this latter bar *t* is moveable on its centre, and may be placed at any angle by an adjusting screw working in a mortice *t″*, where it may be fixed in any required position, as will be seen; it is by the angle given to this bar, that the regular increase of distance is obtained between the rivets. This is effected in the following manner: as the lever *e* works on its centre, the pin *j′* to which it is fixed, strikes as it descends on the end of the lever *j*; its motion is then communicated to the long rod, passing under the table, for working the lever *r* backwards and forwards, it being fixed to the rod; as the latter lever moves, it alternately engages and disengages the stop or projecting pieces *r′ r′* from the notched bar, allowing it at the same time to slide forward the distance between the rivets; while the bar *t* placed at the required angle, and working between the pins *s′ s′* on the lever *s*, immediately affects the distance travelled by the lever *r*, which it shortens; by this arrangement the machine is made quite self-acting, but it may also be worked by the handle on the end of the rod *j*.

When it is required to change the punches *h* for different sized holes, it is necessary, when replaced, to adjust them to the greatest possible accuracy in the dies *i*; this could not be done without stopping the whole line of shaft *a*, were it not for a provision made for that purpose; in such a case, the key in the upper part of the connecting rod *c* is withdrawn, a block of wood is then placed between the frame and the lever *d*, whose fulcrum is on the side of the frame *p*, upon which it rests; it is of course understood that the punching lever *d* is at rest, being disengaged from the pin *e'*, the result of this is the working of the long mortice of the connecting rod up and down on the pin, without communicating any motion whatever; and by the lever *q*, worked gently by hand, and shewn dotted in Figs. 2 and 3, the punching lever *e* can be raised and lowered, communicating its alternate motion to the slide frame, by which means the punches are with great facility adjusted in the dies *i*.

The jaws in the frame *p* are much strengthened by placing a square wrought iron bar between them, in places provided for the purpose.

From the above description, it will be seen, that the advantages possessed by this machine, more than compensate for the increased number of its parts, which are but few when compared with its superiority over those of ordinary construction; there are several at work in the boiler shop at Her Majesty's Dock Yard, Woolwich.

PLATE LII. A.

Steam Punching Machine, by M. Cavé, Paris.

The mode of applying the motive power to this machine is altogether on a different principle from the others contained in this work, to which it is either conveyed through wheels, pulleys, straps, or bands, driven from the shafting running through the building. In this case it is worked by a small steam-engine, connected to and forming part of the machine. In the steam cylinder *a* is a solid piston and piston rod *b*, accurately fitted, the cylinder being bored out in the usual way; *c* is a slide valve, worked by the rod *d* on the faces of the ports *c'* and *c''*, the former for the admission of steam to the cylinder, and the latter for the exhaustion, whence it is carried through a pipe to any convenient outlet; the rod *d* works steam tight through the stuffing box of the slide case, which contains the steam brought by the steam pipe *e* from the boiler. This latter pipe is connected to the valve casing, by flanches bolted together.

On the top of the piston rod *b* is a cross head *b'*, on either side of which are the links *f' f'* connected to the punching lever *f*; two small rollers *b' b'*, Fig. 3, are placed on the outer ends of the cross head which slide up and down in the guides *f'' f''*, whereby the parallel motion of the piston is kept perfectly true. On one end of the punching lever *f* is the connecting rod *k*, for conveying the alternate or reciprocating motion of the piston through the crank *l* to the fly wheels *m*, by which it is regulated. On the opposite end of the lever *f* (whose fulcrum is on the frame *h*) is the punching cylinder for holding the punch, both of which have projecting pins on them for connecting them together by the wrought iron links *g′ g′*, for preserving the parallelism of the punch, these links being adjustable by the screws above. The punch *n* is connected to the lower end of the cylinder *g* by a key, as shewn by Fig. 8, while the die *o* in which it works can be altered, according to the size of the punch used, by the two small adjusting screws shewn in Fig. 1. A stop *p* prevents the plate from rising after it has been punched, the circular pieces punched out falling through the aperture *s* provided for that purpose.

The strong frame *h* for carrying the several parts of the machine, is securely fixed to the stone foundation; on the front part of this frame is a cap (fixed by six bolts and nuts, Fig. 4, representing a sectional plan of the frame) which is tightened according to the wear of the cylinder *g*, which becomes considerable after having been at work any length of time; by this means, any irregularity in the motion is entirely obviated.

The long lever *j* is fixed to the rod *d* for alternately opening and shutting the ports *c′* and *c″* by the slide valve, which operation is performed by the lever *f*, as it rises and falls, striking against the pins *d′ d′*, the counterbalance equalizing the weight of the opposite side of the lever; whose fulcrum is screwed to the upright guide frames *f″*, and projects considerably over them, Fig. 1. The handle *j′* is connected to the lever *j*, for starting or stopping the engine by hand; and by means of the pins *j′* on the two uprights fixed to the upper side of the frame *h*, the lever *j* is secured when the machine is at rest.

In its present position the working of this machine is as follows: steam being admitted by the pipe *e* to the slide casing, passes through the steam port *c′* to the under side of the piston, which it presses up, causing the lever *f* to rise between the guides *f″ f″*, and assume the position represented by the dotted lines, the connecting rod, crank, and consequently fly wheel following their respective motions. In this new position, the lever *f* moving on its fulcrum, must necessarily depress the punching cylinder *g* and punch *n*, thereby communicating to it the requisite alternate motion. In its ascent, the lever *f* strikes against the upper pin *d′*, and raises the valve over the steam port *c*, which is no sooner done than the steam that has performed its duty rushes through the exhaust port (the communication being made between the two) and through it makes its escape, when the piston, rod, lever, &c., fall by their own gravity to their original position, (having merely to overcome the pressure of the atmosphere,) till the lever on its descent again strikes on the lower pin, which immediately opens the steam port *c′* to the position shewn by Fig. 1, which has already been described,—when the operation is again repeated.

This machine may be used for cutting the edges of plates, by adopting the cutter shewn by Figs. 5, 6, and 7.

All the parts of an high pressure steam-engine are necessary, as already described, to work this description of punching machine, and a higher velocity to the punch must necessarily be given than by the ordinary method.

PLATE LIII.

Riveting Machine, by Messrs. Fairbairns and Co.

The drawings shewn by Figs. 1 and 2 very much resemble the common punching machine, the motion being in every respect similar. The work performed by this machine is in all ordinary cases done by manual labour, which operation necessarily requires the services of three men, one the holder on, to hold his hammer or tool inside the boiler against the head of the rivet, while the other two beat out the iron to the conical form given to its opposite end; this is attended with a very disagreeable noise, quite unavoidable, but which is entirely done away with by the use of this machine, which performs with almost instantaneous pressure, what is done in all ordinary cases by a long series of impacts. Fig. 1 is an elevation, and Fig. 2 a plan, where it will be seen that the motion is communicated to the tight and loose riggers *a a* by a leather strap; the speed at which these revolve is much too great for the purpose intended by this machine, consequently a pinion and large spur wheel *b* is required, by which it is reduced as 6 is to 1, that is while the pinion shaft is making six revolutions, that on which the large wheel is hung, and also the cam, is revolving at only one. On the pinion and rigger shaft is placed the fly wheel *c*, for giving a uniform motion to the working parts of the machine. The riveting lever *e* is then made to rise and fall by the action of the cam *d*, the face of which is steeled, while on the end of the lever *e* is the steel roller *f*, revolving as the cam works against it; by this contrivance the friction, which would be considerable, is materially obviated; the riveting lever works easily on its fulcrum *h*, the two short links connecting it to the riveting tool *j*, sliding backwards and forwards in the socket bearing, by which it is kept working in a perfectly true and horizontal direction.

The various parts of this machine are connected to the side frame *g*, made of cast iron, and on it are placed the plummer blocks for carrying the shafts.

On the sole plate is fixed the riveting block *i*, against which that part of the boiler, chimney, or other work *k* being riveted is placed; it is hung by the block and chain shewn in Fig. 1.

The operation is thus performed: the rivet being put through the holes by the attendant workman, previously punched by the punching machine, is brought round so that its head will fall into the recess on the projecting part of the riveting block; the machine is then put in motion by changing the position of the strap from the loose to the fixed pulley, which being communicated to the riveting tool *j*, gives the required shape to the rivet.

The whole is placed on a suitable foundation *l* for giving solidity and stability to the machine, and a pit is made to allow the wheels to work in.

It is stated by Messrs. Fairbairn, that with the attendance of two men and two boys to the plates and rivets, this machine can fix in the firmest manner eight rivets of three quarters of an inch diameter in a minute, whereas by the common process, three men and a boy can only rivet up 40 per hour, the quantity then done in the two cases being in the proportion of 480 to 40, or as 12 is to 1, exclusive of the saving of one man's labour.

PLATE LIV.

DOUBLE GRINDING MACHINE, BY MESSRS. NASMYTH, GASKELL, AND CO.

The object of the machine represented by this Plate, is to grind up the faces of the different parts of machinery, when a great surface is required to be made perfectly smooth; to accomplish which, this contrivance has been used.

Figs. 1, 2, and 3, severally shew a side elevation, an end elevation, and a plan of a double face grinding machine, the one side being a repetition of the other. To the two cast iron cross frames *a a*, are bolted two large plummer blocks for carrying the main shaft, having at each extremity the circular frames divided into twelve compartments, in which are placed the grinding stones *f*, each being adjustable by the small set screws *m* round its circumference. On the top of the cross frames *a*, are placed two longitudinal frames *b b*, made also of cast iron, for supporting the long bed frames *c c*, and also the self-acting apparatus furnished to this machine. Two motions, the one at right angles to the other, are given by the slides *d* working along the beds *c*, and also the face plates *e* for carrying the work, by which it is brought into contact with the grinding stones. Pits are made to allow the wheels, for carrying the stones, to work in.

The self-acting motion given to the work being faced, by means of which it slides along the bed *c* while the grinding stones are revolving on their axes, is thus obtained: on the main shaft next to the driving riggers or pulleys *g*, is a worm *l*, which, as it revolves, works a worm wheel represented by the dotted lines in Fig. 2, thus communicating the motion to the upright spindle; from this it is carried by the bevel wheels to the spindle running horizontally the whole length of the machine, having at each extremity three small bevel wheels. The action of this apparatus is thus, supposing the slide to be travelling in the direction towards the small bevel wheels, two of which are required for the purpose, while the third or outer one runs freely on the spindle, without producing any effect, the small clutch being disengaged from it; on the travelling slide *d* is fixed a stud or pin *h'*; a long rod *h* of the same length as the bed *c*, is moveable in two stud bearings fixed to it. As the slide *d* travels, the pin *h'* comes into contact with a second stud or pin adjusted to any position on the rod *h*, according to the length of the motion required, which must naturally press it forward, and thereby throw out the clutch on the end of the spindle, which being shifted from one bevel wheel to the other, disengages that which had been at work before, while it engages the outer one, that had been running loosely on the spindle; by this curious contrivance, the screw for working the slide revolves in a contrary direction, and instead of drawing the slide *d* towards it, sends it back. A counterbalance weight *n* is connected to the extremity of the rod *h*, for keeping it in a steady position while this operation is being performed.

The tappet wheel *k* fixed on the end of the screw for advancing the other slide *e*, is also worked by a pin on the same rod *h*, whereby the work is advanced to the face of the grinding stone; it is on the upper part of the slide *e* that the work is fixed. A substantial foundation, consisting of stone work, is prepared for receiving the two frames *a a*, and to which they are firmly bolted down by strong holding bolts.

Another mode of performing this same operation might be adopted, by fastening a whole grindstone into the chucks, and passing a bolt through two surface plates of two feet diameter each, one on the middle part of each face of the grindstone, by which means they would be more effectually secured in their places.

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